The Averaged Null Energy Condition and the Madelung Constant for Cold Dark Matter and Energy

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To explain the relative abundance of the dark energy and non-baryonic cold dark matter (74% and 22% respectively), making up 96% of the material content of the universe, it is proposed that space is filled with an equal amount of positive and negative mass particles, satisfying the average null energy condition, and with it the smallness of the cosmological constant. This assumption can explain the relative abundance of the dark energy and cold dark matter by the Madelung constant for the gravitationally-interacting positive and negative mass particles.

Key words: General Relativity; Negative Masses; Cosmology; Madelung Constant.

1. Introduction

The standard model of elementary particle physics only describes about 4% of the material content in the universe. But little is known about the remaining 96%, made up of 22% non-baryonic cold dark matter and 74% dark energy. It is interesting that both the cold dark matter and dark energy are of the same order of magnitude, suggesting a common root for both, and it is the purpose of this communication to propose a possible connection, by making the assumption that the vacuum of space is occupied by a kind of plasma, made up of gravitationally interacting positive and negative masses [1, 2]. The assumption of the existence of negative besides positive mass particles somehow replaces supersymmetry which has never been observed.

2. The Averaged Null Energy Condition

In the general theory of relativity, negative masses as sources of the gravitational field are normally excluded by the strong energy condition, because they lead to causality-violating closed world line solutions. However, the Casimir effect [3] is an example where the strong energy condition is violated in the region between conducting plates, in which the quantum mechanical vacuum energy is negative. Another example is the negative energy of gravistatic fields even though their source is a positive energy mass [4].

These cases can be covered by the average energy condition. For the universe as a whole, it means that the sum of all energies must be equal to zero. There then, the negative gravitational potential energy has a negative mass which has to be equal to the positive kinetic energy of the cosmic expansion.

In physics, looking at analogies has often served as a heuristic principle. Very much as the sum of all electric charges in the universe is zero, one should expect the same to be true for all masses. This assumption is suggested by the conservation of mass and energy, going back to the beginning of the universe where the entropy was very small. This assumption also satisfies the averaged null-energy condition of general relativity.

With the Planck energy as the highest energy reached at the Planck length, it is assumed that space is densely filled with an equal amount of positive and negative Planck mass particles, making the vacuum of space a kind of plasma, which one may call a Planck mass plasma. It is superfluid, and like a superfluid it has phonons, rotons, quantized vortices, et al., as quasiparticles, but of both positive and negative mass.

3. Negative Masses in Einstein's Gravitational Field Equation

As it was shown by Hund [4], Einstein's gravitational field equations lead to the existence of negative masses. For this it is sufficient to consider the gravitational field outside a spherical symmetric mass distribution. For Schwarzschild's solution one can there set for the line element in spherical coordinates

$$\mathrm{d}s^2 = f^2 c^2\,\mathrm{d}t^2 - h^2\,\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right) \tag{1}$$

expressing the components of the metric tensor g_{ik} in space-time by two functions $h(r)$ and $f(r)$. Inserting these given components of the metric tensor given by (1) in Einstein's vacuum field equation

$$R_{ik} = 0\,, \tag{2}$$

one obtains

$$\begin{aligned} &(hf)' = 0\,,\\ &h\left(f'' + \frac{2}{r}f'\right) - h'f' = 0\,. \end{aligned} \tag{3}$$

For the gravitational field F, if measured in eigen-time $f\,\mathrm{d}t$, and eigen-length $h\,\mathrm{d}r$, one obtains for the acceleration, and hence the force F in the radial direction,

$$F = \frac{\mathrm{d}}{f\,\mathrm{d}t}\left(\frac{h}{f}\frac{\mathrm{d}r}{\mathrm{d}t}\right) = -\frac{c^2 f'}{hf}\,. \tag{4}$$

With F given by (4), one can write for the second equation (3):

$$\frac{1}{hr^2}\left(r^2F\right)' - \frac{F^2}{c^2} = 0\,. \tag{5}$$

The first term in (5) is identical to the definition of the divergence of a radial vector $\mathbf{F} = F\mathbf{r}/r$, by the increase $\mathrm{d}(4\pi r^2 F)$ of the flux of $\mathbf{F}$ through a spherical surface of radius r, divided by the increase in the volume of this sphere $4\pi r^2 h\,\mathrm{d}r$. One therefore can write for (5)

$$\operatorname{div}\mathbf{F} - \frac{1}{c^2}\mathbf{F}^2 = 0\,. \tag{6}$$

Comparing this result with Newtonian gravity where (G is Newton's constant)

$$\operatorname{div}\mathbf{F} = -4\pi G\rho\,, \tag{7}$$

one concludes that the gravitational field $\mathbf{F}$ has a negative mass density

$$\rho_{\mathrm{g}} = -\frac{F^2}{4\pi G c^2}\,. \tag{8}$$

Let us first test this result for the linearized approximation of (6) putting

$$F = -\frac{Gm}{r^2} \tag{9}$$

the gravitational field of a spherical mass of radius R, for $r > R$. One there finds

$$\rho_{\mathrm{g}} = -\frac{Gm^2}{4\pi c^2 r^4}\,. \tag{10}$$

To obtain the total amount of negative mass m_{g} outside the mass M, we integrate and obtain

$$m_{\mathrm{g}} = \int_R^\infty \rho_{\mathrm{g}} 4\pi r^2\,\mathrm{d}r = -\frac{Gm^2}{c^2 R} \tag{11}$$

or

$$m_{\mathrm{g}} c^2 = -\frac{Gm^2}{R} = E_{\mathrm{pot}}\,, \tag{12}$$

where E_{pot} is the negative gravitational potential energy of a spherical shell of radius R and mass M.

This example shows that to obtain the gravitational field mass m_{g}, one simply has to equate the gravitational potential energy with $m_{\mathrm{g}}c^2$.

Including the nonlinear term in (6), Hund obtains the exact solution [4]

$$F = -\frac{Gm}{r^2}\frac{1}{\sqrt{1 - \frac{2Gm}{c^2 r}}}\,, \tag{13}$$

and one finds that there

$$\rho_{\mathrm{g}} = -\frac{F^2}{4\pi G c^2} = -\frac{Gm^2}{4\pi c^2 r^4}\frac{1}{1 - \frac{2Gm}{c^2 r}} \tag{14}$$

or provided that $r \gg \frac{2Gm}{c^2}$,

$$\rho_{\mathrm{g}} = -\frac{Gm^2}{4\pi c^2 r^4}\left(1 + \frac{2Gm}{c^2 r}\right)\,. \tag{15}$$

As before, integrating from $r = R$ to $r = \infty$, one obtains

$$m_{\mathrm{g}} c^2 = -\frac{Gm^2}{R}\left(1 + \frac{Gm}{c^2 r}\right) \tag{16}$$

which for $r \gg \frac{2Gm}{c^2}$ is the same.

4. The Gravitational Field Energy of a Mass Dipole

With the gravitational field mass of one particle given by its negative Newtonian potential (12), we make the assumption that the gravitational field mass of a mass dipole where $m_1 = m^+$ and $m_2 = -|m^+|$, and hence $m_1 m_2 = -|m^\pm|^2$, is given by

$$m_g c^2 = \frac{G|m^\pm|^2}{r}, \tag{17}$$

where r is the distance of separation between m_1 and m_2. The justification for this assumption is obtained by the weak field approximation in quantum gravity, which gives for the potential energy of two masses m_1 and m_2 [5]

$$E = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{2rc^2} + \frac{41}{10\pi}\frac{r_p^2}{r^2}\right]. \tag{18}$$

For $m_2 = -m_1$, the second term in the bracket on the r.h.s. of (14) vanishes, and one has

$$E = m_g c^2 = \frac{G|m^\pm|^2}{r}\left[1 + \frac{41}{10\pi}\frac{r_p^2}{r^2}\right]. \tag{19}$$

Therefore, as long as $r \gg r_p$, where $r_p = \sqrt{\hbar G/c^3} \sim 10^{-33}$ cm is the Planck length, the approximation (17) is justified.

5. The Cold Dark Matter Mass as a Gravitational Field Mass

Very much as one can estimate the ground state energy for the Bohr atom model with the uncertainty principle $\Delta p \Delta q \sim \hbar$, we can here do the same by combining (17) with the uncertainty principle. As in Bohr's model we set $\Delta q \sim r$, but for Δp we have to take the sum of m^+ and m^-, hence $\Delta p \sim 2|m^\pm|c$, and thus have

$$2|m^\pm|cr \sim \hbar. \tag{20}$$

A more detailed derivation of this equation by Papapetrou and Hönl [6] leads to the same if the angular momentum in the groundstate is set equal to $\frac{1}{2}\hbar$. A wave mechanical treatment by Bopp [7] confirms this result.

By setting $m_g = M_D$, where M_D is the mass of the cold dark matter particles, one obtains by eliminating r from (17) and (20)

$$M_D = \frac{2G|m^\pm|^3}{\hbar c} \tag{21}$$

or

$$\frac{M_D}{m_p} = 2\left(\frac{|m^\pm|}{m_p}\right)^3, \tag{22}$$

where $m_p = \sqrt{\hbar c/G}$ is the Planck mass.

Assuming that $|m^\pm|$ is equal to the positive and negative roton masses of the superfluid Planck mass plasma, one would have to set $|m^\pm| \approx 0.1 m_p$, in analogy to the experimental data for superfluid helium, where the roton mass is about $0.1 m_D$, with m_D the Debye mass, respectively energy, here replaced by the Planck mass m_p, respectively the Planck energy. We thus have $M_D \sim 10^{-3} m_p \sim 10^{16}$ GeV, which is of the same order of magnitude as the grand unified theory (GUT) mass.

6. Madelung Constant Interpretation of the Cold Dark Energy

In condensed matter physics, the Madelung constant [8] determines the electrostatic potential of a single ion in a crystal with all other ions by approximating them as point charges. Similarly, in the Planck mass plasma one can approximate the positive and negative mass rotons by mass points. With Newton's law having the same form as Coulomb's law, the Madelung constant determines there the gravitational potential energy of one roton with all other rotons, expressed in terms of the gravitational potential energy of one roton with its nearest neighbour of the opposite sign of its mass. The rotons are positioned in a three-dimensional cubic lattice, the Madelung constant is $M = 3.49$, which has to be compared with the ratio of the dark energy (74%) to dark matter (22%), which is $74/22 = 3.36$. That this ratio turns out to be quite so close to the Madelung constant for a cubic lattice gives strong support for the positive–negative mass hypothesis. The agreement may turn out to be even better for non-cubic lattices obtained from a variational principle.

But there still remains a problem. With each positive mass forming a positive–negative mass dipole with a neighbouring negative mass, the Madelung constant calculation makes the assumption that all the positive and negative masses form a cubic lattice, as in a crystal the positive and negative electric charges. But this is not what we presumably have in the distribution of cold dark matter and dark energy. There, the cold dark matter particles which are positive–negative mass dipoles are separated from each other by large distances, with the space in between occupied by dark energy. However, at the beginning of the cosmic expansion the positive and negative masses could all be separated by small distances as the electric charges in a crystal. But the distribution of the energy, averaged over a large volume, would not be changed if this positive–negative mass configuration expands. This is similar to the "swiss cheese" solution of Einstein's gravitational field equation for a cubic lattice of Schwarzschild black holes with vacuum in between, which is globally equivalent to a solution where the masses of these black holes are uniformly distributed over space.

7. Conclusion

For the universe, the averaged null energy condition can only be satisfied if there are an equal number of positive and negative masses. Under this assumption, the ratio of dark energy to non-baryonic cold dark matter must be of the order of the Madelung constant for an assembly of gravitationally interacting positive and negative masses of the same magnitude. That the amount of dark energy and dark matter is equal within an order of magnitude suggests a common origin for both.

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