Effects of Non-Newtonian Rheology on Curved Circular Squeeze Films: the Rabinowitsch Fluid Model

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On the ground of the non-Newtonian Rabinowitsch fluid model, the study of Newtonian squeeze film problems of curved circular plates [P. R. K. Murti, Trans. ASME 97, 650 (1975)] is extended in this paper. According to the results, higher load capacities and longer approaching times are obtained for the curved plates lubricated with dilatant fluids, but the pseudoplastic lubricants result in reversed trends. The influences of non-Newtonian dilatant and pseudoplastic properties on the squeeze film characteristics are further emphasized for curved circular plates operating under a smaller minimum film height and a larger curved shape parameter. Numerical results of the non-Newtonian load capacity are also provided for specific values of the curved shape parameter, the nonlinear parameter, and the minimum film height.

Key words: Non-Newtonian Fluids; Rabinowitsch Fluid Model; Squeeze Films; Curved Circular Plates.

1. Introduction

Squeeze film mechanisms of curved plates play an important role in matching gears and artificial joints. The squeeze film behaviour results from two lubricated surfaces approaching each other with a normal velocity. Since the lubricant contained between these surfaces cannot be instantaneously squeezed out, it provides a certain approaching time for these surfaces to come into contact. Conventionally, studies of squeeze film performances between approaching surfaces are presented by using a Newtonian fluid. Some contributions can be observed in, for example, rectangular surfaces and braking plates by Jones and Wilson [1] and Wang [2]; circular disks by Hamrock [3], Murti [4], and Lin [5]; and curved circular and annular plates by Murti [6] and Vora [7]. According to their results, the load capacity increases with decreasing values of the film height. Comparing with the case of flat plates, the concave films provide an increase in the load capacity. However, the reverse results are obtained for the convex plates. Owing to the requirement of modern machine elements operating under severe situations, the addition of small amounts of long chained polymer solutions has received great attention. Experimental evidences of Spikes [8] show that the use of additives can stabilize fluid properties and minimize the sensitivity of change in shearing strain rate. However, the viscosity of these kinds of lubricants displays a nonlinear relationship between the shearing stress and the shearing strain rate. In the literature, some non-Newtonian fluid models have been applied to investigate the performance characteristics of squeeze film systems, for example, the couple stress fluid model in human joints by Ahmad and Singh [9], sphere–plate mechanisms by Al-Fadhalah and Elsharkawy [10], and partial bearings by Lin et al. [11]; the power law fluid model in conical bearings by Shukla [12], and journal bearings by Singh and Sinha [13]; and the micropolar fluid model in hemispherical bearings by Sinha and Singh [14] and journal bearings by Naduvinamani and Santosh [15]. From their results, the non-Newtonian couple stress effects result in higher values of the squeeze film load as compared to the Newtonian-lubricant case [9 – 11]. In addition, the load capacity of squeeze film bearings with power law fluids increases with increasing values of the flow behaviour index [12, 13]. Furthermore, the non-Newtonian effects of micropolar fluids

are found to increase the squeeze film pressure and the load capacity as compared to the corresponding Newtonian case [14, 15]. On the other hand, according to the experimental study of Wada and Hayashi [16], the nonlinear stress–strain rate relationship for lubricants blended with additives can be described by an empirical cubic stress model or the Rabinowitsch fluid model. In this fluid model, following nonlinear relationship between the shearing stress τ and the shearing strain rate $\partial u/\partial r$ holds for one dimensional flow:

$$\tau + \alpha\tau^3 = \eta_0 \frac{\partial u}{\partial r}, \tag{1}$$

where η_0 denotes the initial viscosity of a Newtonian fluid, and α represents a nonlinear factor responsible for the effects of non-Newtonian fluids. The Rabinowitsch fluid model can be applied to dilatant fluids for $\alpha < 0$, Newtonian fluids for $\alpha = 0$, and pseudo-plastic fluids for $\alpha > 0$. Some ranges of experimental values of the nonlinear factor α for some non-Newtonian fluids are provided by Wada and Hayashi [16]. On the ground of the Rabinowitsch fluid model, the effects of non-Newtonian rheology on various lubrication problems have been analyzed by many investigators, such as journal bearings by Wada and Hayashi [17] and Sharma et al. [18], slider bearings by Hashimoto [19] and Lin [20], hydrostatic thrust bearings by Singh et al. [21], circular disks by Hashimoto [22], and annular plates by Lin [23]. According to their results, the effects of non-Newtonian Rabinowitsch fluid properties provide significant influences on the bearing performances [17–23]. Since the investigations of squeeze films between exponential curved plates (including the special circular plates) are important in many applications such as the clutch plates, approaching gears, curved rollers, artificial joints, and synovial joints, a further study using non-Newtonian Rabinowitsch fluids as lubricants is therefore motivated.

On the ground of the non-Newtonian Rabinowitsch fluid model, the study of Newtonian squeeze film problems of curved circular plates [6] is extended in this paper. By applying a small perturbation technique to the nonlinear non-Newtonian Reynolds equation, a first order solution for the film pressure, the load capacity, and the approaching time is obtained. Through the variation of the curved shape parameter and the non-Newtonian parameter, the non-Newtonian squeeze film characteristics of this study are presented and compared to those of the Newtonian-lubricant case.

2. Analysis

Figure 1 describes the squeeze film geometry between curved circular plates of radius a lubricated with a non-Newtonian fluid, in which the film shape h is taken to be an exponential curved type as Murti [6],

$$h = h_m \exp(-\beta r^2/a^2), \tag{2}$$

where h_m is the minimum film thickness, and β is the curved shape parameter. The values of the shape parameter are applied to convex films for $\beta < 0$, to circular films for $\beta = 0$, and to concave films for $\beta > 0$. The upper plate is approaching the lower fixed plate with a squeezing velocity of $-\mathrm{d}h_m/\mathrm{d}t$. For the present study, it is assumed that the film thickness is thin as compared to the plate radius, the material of plates is rigid, the body force is negligible, the fluid inertia force is small as compared to the viscous shear force, and the thin-film lubrication theory of Hamrock [3] is applicable. Then, the continuity equation and the momentum equations governing the one-dimensional motion of an incompressible non-Newtonian Rabinowitsch fluid used by Wada and Hayashi [16] and Lin [23] in an axial-symmetry cylindrical coordinate system are

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \tag{3}$$

$$\frac{\partial p}{\partial r} = \frac{\partial \tau}{\partial z}, \tag{4}$$

$$\frac{\partial p}{\partial z} = 0. \tag{5}$$

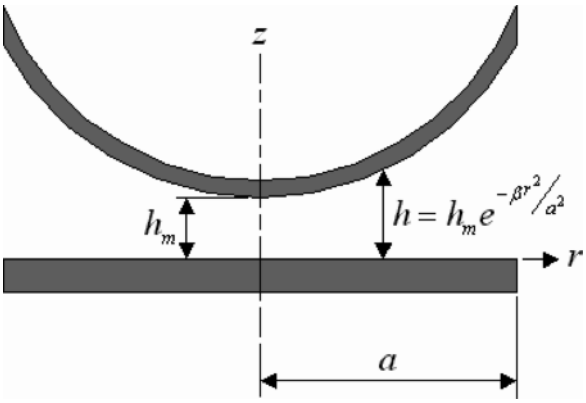

Fig. 1. Squeeze film geometry between curved circular plates lubricated with a non-Newtonian fluid.

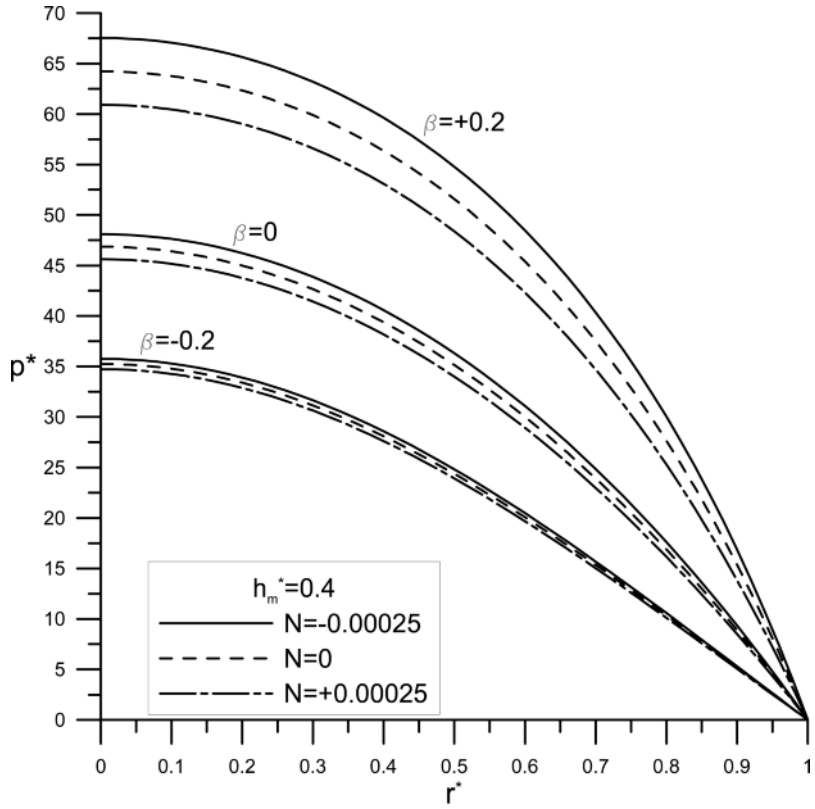

Fig. 2. Film pressure p^* as a function of the radial coordinate r^* for different β and N under minimum film height $h_m^* = 0.4$.

The non-slip boundary conditions are $u = 0$ at $z = 0$ and $z = h$; $w = 0$ at $z = 0$; and $w = -\mathrm{d}h_m/\mathrm{d}t$ at $z = h$. Following the procedure by Hashimoto [22] and Lin [23], the radial velocity component can be solved from (1) and (4).

$$u = \frac{1}{8\eta_0}\left[4z(z-h)\frac{\partial p}{\partial r} + \alpha\left(2z^4 - 4hz^3 + 3h^2z^2 - h^3z\right)\left(\frac{\partial p}{\partial r}\right)^3\right]. \tag{6}$$

Integrating the continuity equation (4) and using the expression of radial velocity component, one can derive the non-Newtonian squeeze film Reynolds equation of curved circular plates lubricated with a Rabinowitsch fluid:

$$\frac{\partial}{\partial r}\left\{r\left[\mathrm{e}^{-3\beta r^2/a^2}\frac{\partial p}{\partial r} + \frac{3}{20}\alpha h_m^2 \cdot \mathrm{e}^{-5\beta r^2/a^2}\left(\frac{\partial p}{\partial r}\right)^3\right]\right\} = 12\eta_0 r h_m^{-3}\frac{\mathrm{d}h_m}{\mathrm{d}t}. \tag{7}$$

Expressed in a non-dimensional form, one can achieve

$$\frac{\partial}{\partial r^*}\left\{r^*\left[\mathrm{e}^{-3\beta r^{*2}}\frac{\partial p^*}{\partial r^*} + \frac{3}{20}N h_m^{*\,2} \cdot \mathrm{e}^{-5\beta r^{*2}}\left(\frac{\partial p^*}{\partial r^*}\right)^3\right]\right\} = -12 r^* h_m^{*\,-3}, \tag{8}$$

where the non-dimensional parameter and variables are defined by

$$r^* = \frac{r}{a},\quad h_m^* = \frac{h_m}{h_{m0}},\quad p^* = \frac{p h_{m0}^3}{\eta_0 a^2(-\mathrm{d}h_m/\mathrm{d}t)},\quad N = \alpha\left[\frac{\eta_0 a(-\mathrm{d}h_m/\mathrm{d}t)}{h_{m0}^2}\right]^2. \tag{9}$$

In these definitions, h_{m0} is the initial minimum film thickness, and N depicts the nonlinear parameter dominating the effects of non-Newtonian fluids. The pressure boundary conditions are $\mathrm{d}p^*/\mathrm{d}r^* = 0$ at $r^* = 0$ and $p^* = 0$ at $r^* = 1$.

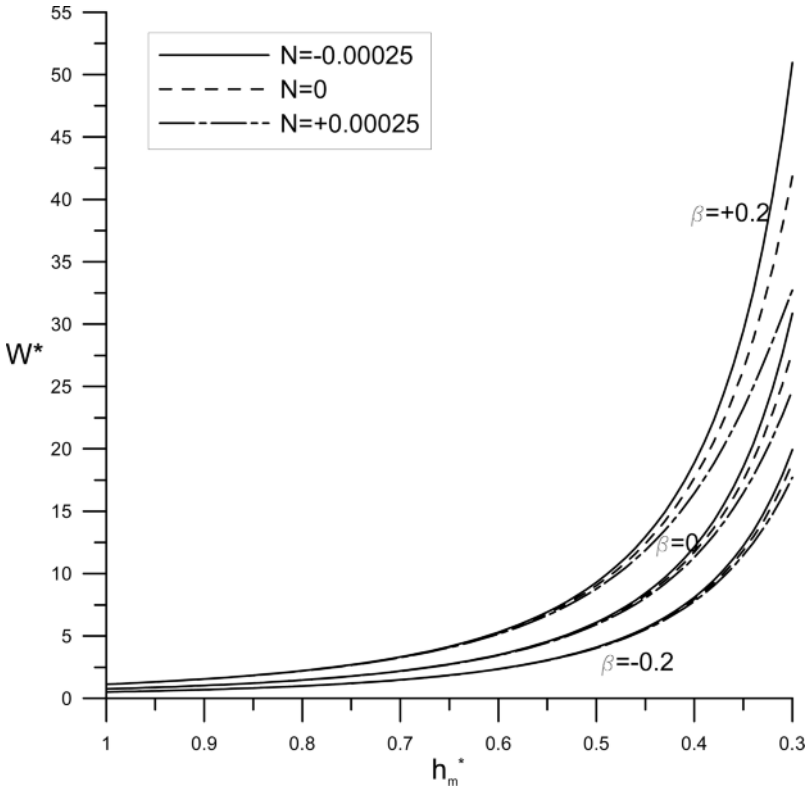

Fig. 3. Load capacity W^* as a function of the minimum film height h_m^* for different β and N.

It is difficult to obtain the analytical solution for the film pressure by solving the highly nonlinear equation (8). However, to simplify the problem, a first-order solution is expected by applying a small perturbation technique as Hashimoto [22] and Lin [23]. In this procedure, the non-dimensional pressure is perturbed for small values of the nonlinear parameter, $0 \leq |N| \ll 1$:

$$p^* = p_0^* + N p_1^* + O(N^2)\,. \tag{10}$$

Substituting the above equation into (8) and neglecting the higher-order terms $O(N^2)$, one can obtain two Reynolds equations, respectively, governing the film pressure p_0^* and p_1^*:

$$O(N^0):\ \frac{\partial}{\partial r^*}\left\{r^* \mathrm{e}^{-3\beta r^{*2}} \frac{\partial p_0^*}{\partial r^*}\right\} = -12 r^* h_m^{*\,-3}\,, \tag{11}$$

$$O(N^1):\ \frac{\partial}{\partial r^*}\left\{r^*\left[\mathrm{e}^{-3\beta r^{*2}} \frac{\partial p_1^*}{\partial r^*} + \frac{3}{20} h_m^{*\,2} \cdot \mathrm{e}^{-5\beta r^{*2}} \left(\frac{\partial p_0^*}{\partial r^*}\right)^3\right]\right\} = 0\,. \tag{12}$$

By application of the pressure boundary conditions, the film pressure can be obtained after solving (11) and (12):

$$p_0^* = \frac{1}{\beta h_m^{*\,3}}\left[\mathrm{e}^{3\beta} - \mathrm{e}^{3\beta r^{*2}}\right], \tag{13}$$

$$p_1^* = -\frac{81}{5 h_m^{*\,7}}\left\{\frac{\mathrm{e}^{7\beta}}{7\beta} - \frac{1}{49\beta^2}\left[7\beta r^{*2}\mathrm{e}^{7\beta r^{*2}} + \mathrm{e}^{7\beta} - \mathrm{e}^{7\beta r^{*2}}\right]\right\}. \tag{14}$$

Integrating the film pressure over the film region yields the load capacity

$$W = 2\pi \int_{r=0}^{a} p r \,\mathrm{d}r\,. \tag{15}$$

Now we introduce the non-dimensional form

$$W^* = \frac{W h_{m0}{}^3}{2\pi\mu a^4(-\mathrm{d}h_m/\mathrm{d}t)}\,. \tag{16}$$

The load capacity can then be obtained after performing the integration as

$$W^* = \frac{1-\mathrm{e}^{3\beta}(1-3\beta)}{6\beta^2 {h_m^*}^3} - \frac{81N}{35{h_m^*}^7}\left(\frac{\mathrm{e}^{7\beta}}{2\beta} - \frac{7\beta\,\mathrm{e}^{7\beta}-\mathrm{e}^{7\beta}+1}{49\beta^3}\right). \tag{17}$$

For $N = 0$, the above equation reduces to the load capacity for Newtonian curved circular squeeze films derived by Murti [6] when using the same non-dimensional definitions as in the present study:

$$W^* = -\frac{3}{6\beta^2 {h_m^*}^3}\cdot\left[\mathrm{e}^{3\beta}\cdot(1-3\beta)-1\right]. \tag{18}$$

To obtain the approaching time, a non-dimensional form is introduced as

$$t^* = \frac{W {h_{m0}}^2}{\pi\mu a^4}\cdot t\,. \tag{19}$$

Substituting the expression of the non-dimensional approaching time into (16), one can derive the differential equation governing the film height:

$$\frac{\mathrm{d}h_m^*}{\mathrm{d}t^*} = -\frac{1}{2}\left[\frac{1-\mathrm{e}^{3\beta}(1-3\beta)}{6\beta^2 {h_m^*}^3} - \frac{81N}{35{h_m^*}^7}\cdot\left(\frac{\mathrm{e}^{7\beta}}{2\beta} - \frac{7\beta\,\mathrm{e}^{7\beta}-\mathrm{e}^{7\beta}+1}{49\beta^3}\right)\right]^{-1}. \tag{20}$$

The initial condition for the non-dimensional film height is $h_m^* = 1$ at $t^* = 0$. Separating the variables and integrating the equation, we obtain the approaching time varying with the film height:

$$t^* = \frac{1-\mathrm{e}^{3\beta}(1-3\beta)}{6\beta^2}\left({h_m^*}^{-2}-1\right) - \frac{27N}{35}\cdot\left(\frac{\mathrm{e}^{7\beta}}{2\beta} - \frac{7\beta\,\mathrm{e}^{7\beta}-\mathrm{e}^{7\beta}+1}{49\beta^3}\right)\left({h_m^*}^{-6}-1\right). \tag{21}$$

3. Results and Discussion

According to the above analysis, the non-Newtonian curved circular squeeze film performances are influenced by two parameters. First, the nonlinear parameter N characterizes the non-Newtonian effects of the

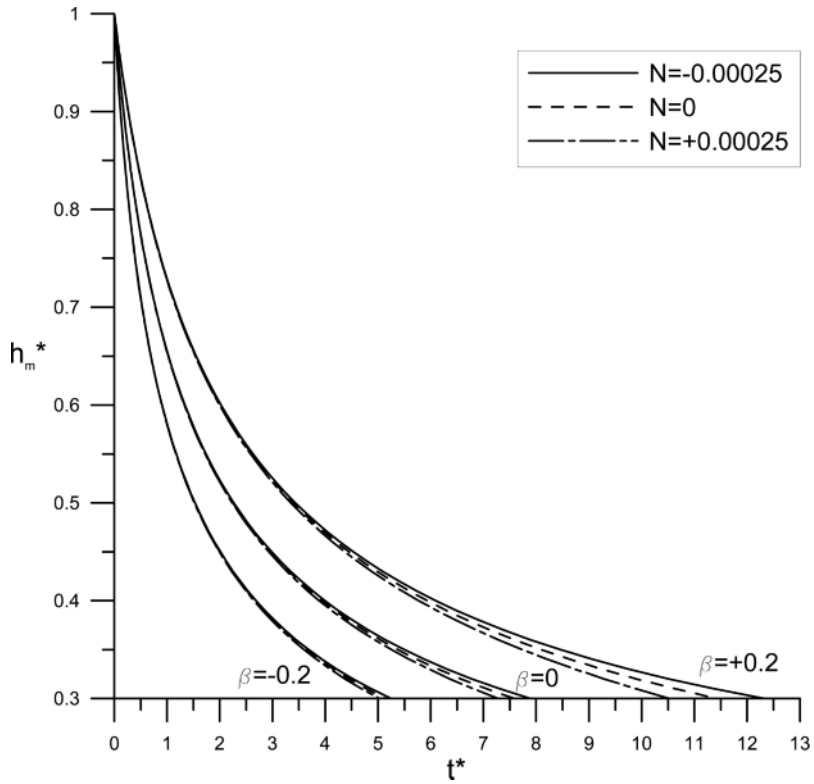

Fig. 4. Minimum film height h_m^* as a function of the approaching time t^* for different β and N.

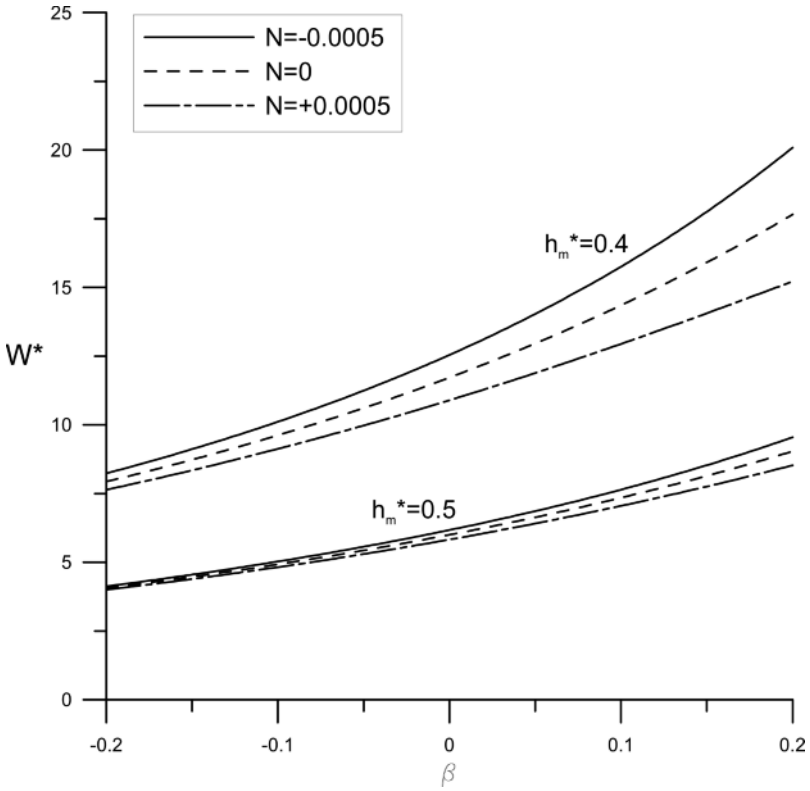

Fig. 5. Effect of variation of β on the load capacity W^* for different N.

Rabinowitsch fluid model. It is applicable to Newtonian lubricants for $N = 0$, pseudo-plastic fluids for $N > 0$, and dilatant lubricants for $N < 0$. The other curved shape parameter β generates convex films for $\beta < 0$, circular films for $\beta = 0$, and concave films for $\beta > 0$.

For $N = 0$ and $\beta = 0$: it is the Newtonian circular squeeze film as Hamrock [3].

For $N = 0$ and $\beta \neq 0$: it is the Newtonian curved circular squeeze film as Murti [6].

For $N \neq 0$ and $\beta = 0$: the present study reduces to the non-Newtonian circular squeeze film. It is the limiting case of non-Newtonian annular-disk squeeze film as Lin [23] when the inner-outer radius ratio approaches zero.

For $N \neq 0$ and $\beta \neq 0$: it is the non-Newtonian curved circular squeeze film (present study).

Figure 2 presents the film pressure p^* as a function of the radial coordinate r^* for different β and N under minimum film height $h_m^* = 0.4$. It is observed that the concave film ($\beta = +0.2$) generates a higher pressure than those of the circular film ($\beta = 0$) and the convex film ($\beta = -0.2$). Comparing with the case of Newtonian lubricants ($N = 0$), the influences of pseudoplastic lubricants ($N = +0.00025$) decrease the film pressure, but the dilatant effects ($N = -0.00025$) increase the value of p^*.

Figure 3 shows the load capacity W^* as a function of the minimum film height h_m^* for different values of β and N. The load capacity is observed to be similarly influenced by the nonlinear parameter of the non-Newtonian Rabinowitsch fluids model. In addition, the effects of non-Newtonian properties on the load capacity are more emphasized for the squeeze film operating under a smaller film height and a larger curved shape parameter ($\beta = +0.2$).

Figure 4 describes the minimum film height h_m^* as a function of the approaching time t^* for differ-

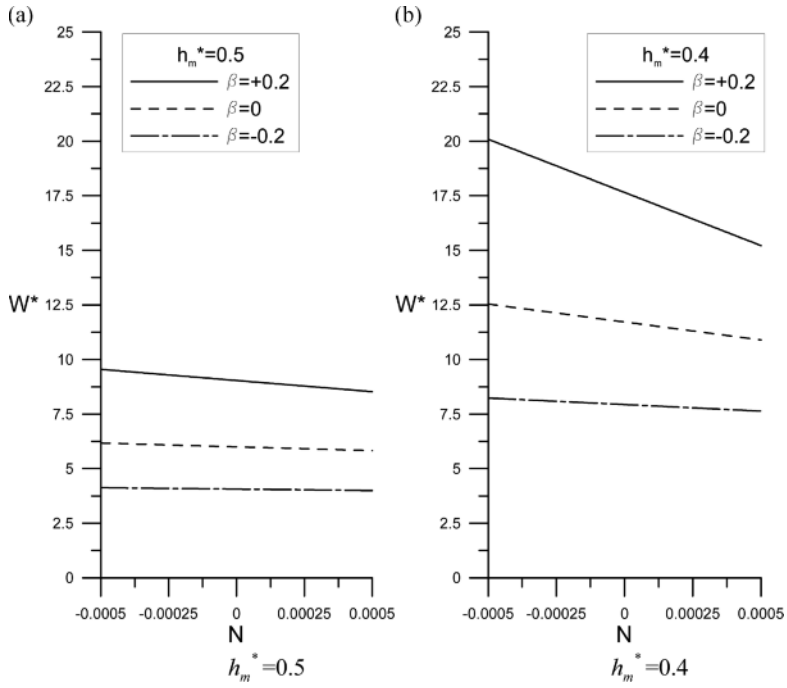

Fig. 6. Effect of variation of N on the load capacity W^* for different β.

ent values of β and N. Since the effects of non-Newtonian dilatant fluids ($N = -0.00025$) result in a higher film pressure such that the squeeze film is capable of supporting higher loads, the approaching time t^* is expected to be increased. Therefore, the approaching time required to reduce a given height for the squeeze film with non-Newtonian dilatant fluids ($N = -0.00025$) is extended as compared to the case of Newtonian lubricants ($N = 0$). However, the effects of non-Newtonian pseudoplastic fluids ($N = +0.00025$) reveal a reversed influence on the approaching time.

Figure 5 displays the effect of variation of the curved shape parameter β on the load capacity W^* for different N. Under the minimum film heights $h_m^* = 0.5$, the load capacity is observed to increase with the value of β. Decreasing the minimum film height down to $h_m^* = 0.4$ increases the effects of the curved shape parameter and the non-Newtonian properties ($N = -0.00025$, $N = +0.00025$) on the load capacity when comparing with the case of a Newtonian lubricants ($N = 0$).

Figure 6 presents the effect of variation of the nonlinear parameter N on the load capacity W^* for different β. Under the minimum film heights $h_m^* = 0.5$, the load capacity is observed to decrease slightly with the value of N. However, the effects of the curved shape

Table 1. Illustration of exponential curved circular squeeze films lubricated with non-Newtonian Rabinowitsch fluids.

Exponential curved circular squeeze films		
Radius of the lower circular plate	a	$1 \cdot 10^{-2}$ m
Initial minimum film thickness	h_{m0}	$1.00 \cdot 10^{-4}$ m
Curved shape parameter	β	$-2, -1, 0, +1, +2$
Squeeze velocity	$-\mathrm{d}h_m/\mathrm{d}t$	0.20 m/s
Initial viscosity of a Newtonian fluid	η_0	$1.00 \cdot 10^{-4}$ Pa · s
Nonlinear factor for dilatant fluids, a Newtonian fluid, and pseudo-plastic fluids	α	$-1.350 \cdot 10^{-6}$ m^4/N^2 $-0.625 \cdot 10^{-6}$ m^4/N^2 0.00 m^4/N^2 $+0.625 \cdot 10^{-6}$ m^4/N^2 $+1.350 \cdot 10^{-6}$ m^4/N^2

Table 2. Comparison of numerical values of the load capacity W^* with the Newtonian lubricant results of Murti [6] under $h_m^* = 1$.

Newtonian lubricant results of Murti [6]					
$h_m^* = 1$	$\beta = -2$	$\beta = -1$	$\beta = 0$	$\beta = +1$	$\beta = +2$
	0.508	0.616	0.750	0.918	1.130
Non-Newtonian lubricant results of the present study					
$h_m^* = 1$	$\beta = -2$	$\beta = -1$	$\beta = 0$	$\beta = +1$	$\beta = +2$
$N = -0.0005$	0.508	0.616	0.751	0.921	1.134
$N = -0.00025$	0.508	0.616	0.751	0.919	1.132
$N = 0$	0.508	0.616	0.750	0.918	1.130
$N = +0.00025$	0.508	0.615	0.749	0.917	1.128
$N = +0.0005$	0.507	0.615	0.749	0.916	1.126

Table 3. Comparison of numerical values of the load capacity W^* with the Newtonian lubricant results of Murti [6] under $h_m^* = 0.6$.

Newtonian lubricant results of Murti [6]					
$h_m^* = 0.6$	$\beta = -2$	$\beta = -1$	$\beta = 0$	$\beta = +1$	$\beta = +2$
	2.351	2.850	3.472	4.251	5.231
Non-Newtonian lubricant results of the present study					
$h_m^* = 0.6$	$\beta = -2$	$\beta = -1$	$\beta = 0$	$\beta = +1$	$\beta = +2$
$N = -0.0005$	2.369	2.879	3.520	4.334	5.373
$N = -0.00025$	2.360	2.864	3.496	4.293	5.302
$N = 0$	2.351	2.850	3.472	4.251	5.231
$N = +0.00025$	2.343	2.836	3.448	4.210	5.159
$N = +0.0005$	2.334	2.821	3.424	4.169	5.088

parameter and the non-Newtonian properties on the load capacity are further emphasized for the curved circular squeeze film operating under a smaller minimum film height ($h_m^* = 0.4$).

In order to guide the use of the present study, the data of squeeze films are illustrated in Table 1. The exponential film shapes for the curved shape parameter $\beta = -2$, -1, 0, $+1$, and $+2$ can be observed as Murti [6]. Based upon these data, the nonlinear parameter can be calculated, $N = -0.0005$, -0.00025, 0, $+0.00025$, and $+0.0005$. Tables 2 and 3 provide the comparison of numerical values of the load capacity W^* with the Newtonian lubricant results of Murti [6] under the minimum film heights $h_m^* = 1$ and $h_m^* = 0.6$, respectively. For the nonlinear parameter $N = 0$, the present results agree well with the Newtonian lubricant results of Murti [6]. According to the above results and discussion, the influences of non-Newtonian Rabinowitsch fluids on the curved circular squeeze film performances are apparent and depend on values of the nonlinear parameter N, the curved shape parameter β, and the minimum film height h_m^*.

Based on small values of the nonlinear parameter, the perturbation method is applied to solve the governing equation in this paper. From Figure 3, the values of the load capacity are presented for the minimum film height down to $h_m^* = 0.3$. According to the equations derived for the film pressure (13) and (14), the load capacity (17) and the approaching time (20), the condition still holds for the nonlinear parameter $|N| \leq 0.00069$ when the curved shape parameter are within the range $-2 \leq \beta \leq +2$.

4. Conclusions

Applying the non-Newtonian Rabinowitsch fluid model, a nonlinear non-Newtonian Reynolds equation has been derived, which can be applied for the study for curved circular squeeze films. Higher load capacities and longer approaching times are obtained for curved plates lubricated with dilatant fluids ($N < 0$); but the pseudoplastic lubricants ($N < 0$) yield reversed results. The influences of dilatant and pseudoplastic properties on the load capacity and the approaching time are further emphasized for curved circular plates operating under a smaller minimum film height and a larger curved shape parameter. Some numerical values of the non-Newtonian load capacity are also provided for engineering references.

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