Dufour and Soret Effects on the Thermosolutal Instability of Rivlin–Ericksen Elastico–Viscous Fluid in Porous Medium

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Dufour and Soret effects on the convection in a horizontal layer of Rivlin–Ericksen elastico–viscous fluid in porous medium are considered. For the porous medium, the Darcy model is used. A linear stability analysis based upon normal mode analysis is employed to find a solution of the fluid layer confined between two free boundaries. The onset criterion for stationary and oscillatory convection has been derived analytically, and graphs have been plotted, giving various numerical values to various parameters, to depict the stability characteristics. The effects of the Dufour parameter, Soret parameter, solutal Rayleigh number, and Lewis number on stationary convection have been investigated.

Key words: Thermosolutal Instability; Rivlin-Ericksen; Dufour; Soret Parameter.

1. Introduction

The onset of convection in a Newtonian fluid under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the stability of convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3]. McDonnel [4] suggested the importance of porosity in the astrophysical context. The onset of double diffusive convection in a fluid saturated porous medium heated from below is now regarded as a classical problem due to its wide range of applications in saline geothermal fields, agricultural product storage, soil sciences, enhanced oil recovery, packed-bed catalytic reactors, and the pollutant transport in underground. A detailed review of the literature concerning double diffusive convection in binary fluids in porous media is given by Nield and Bejan [5, 6], Trevisan and Bejan [7], Mojtabi and Charrier-Mojtabi [8, 9], Malashetty and Kollur [10]. Thermal convection in a binary fluid driven by the Soret and Dufour effect has been investigated by Knobloch [11]. He has shown that the equations are identical to the thermosolutal problem except the relation between the thermal and solutal Rayleigh numbers.

The above literature deals with Newtonian fluids. But in technological fields, there exists an important class of fluids, called non-Newtonian fluids, which are also studied extensively because of their practical applications, such as fluid film lubrication, analysis of polymers in chemical engineering etc. An experimental demonstration by Toms and Trawbridge [12] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd [13]. There are many visco–elastic fluids which cannot be characterized by Maxwell's constitutive relations. One such fluid is the Rivlin–Ericksen fluid [14]. Parkash and Kumar [15] and Sharma and Kumar [16] studied the thermal instability of the Rivlin–Ericksen elastico–viscous fluid in a porous medium. While Prakash and Chand [17] examined the effect of kinematic visco–elastic instability of a Rivlin–Ericksen elastico–viscous fluid in porous medium and found that the kinematic visco–elasticity stabilizes the fluid layer.

In the present study, we investigated the Dufour and Soret effects on the thermal instability of a Rivlin–Ericksen elastico–viscous fluid in a porous medium.

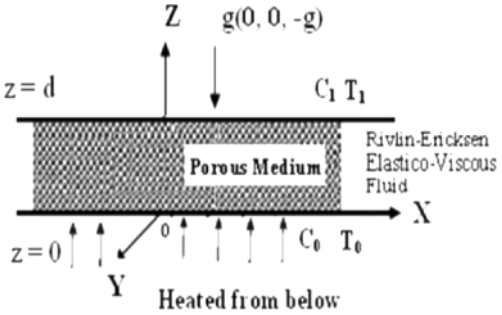

Fig. 1. Physical model.

2. Mathematical Formulations

Consider an infinite horizontal layer of a Rivlin–Ericksen elastico–viscous fluid of thickness d bounded by plane $z = 0$ and $z = d$ in a porous medium of porosity ε and medium permeability k_1 which is acted upon by gravity $\mathbf{g}(0,0,-g)$ as shown in Figure 1. This layer is heated and soluted from below such that a constant temperature and concentration distribution is prescribed at the boundaries of the fluid layer. The temperature T and concentration C are taken to be T_0 and C_0 at $z = 0$ and T_1 and C_1 at $z = d$, $(T_0 > T_1, C_0 > C_1)$. Let ΔT and ΔC be the differences in temperature and concentration across the boundaries.

Let $\mathbf{q}(u,v,w)$, p, ρ, T, C, α, α', μ, μ', κ, and κ' be the Darcy velocity vector, hydrostatic pressure, density, temperature, solute concentration, thermal coefficient of expansion and an analogous solvent coefficient of expansion, viscosity, kinematic visco–elasticity, thermal diffusivity, and solute diffusivity of fluid respectively.

We assume that the medium is homogenous, isotropic, and Darcy's law is valid. Applying the Oberbeck–Boussineque approximation, the governing equations for the Rivlin–Ericksen elastico–viscous fluid in a porous medium are

$$\nabla \mathbf{q} = 0\,, \tag{1}$$

$$0 = -\nabla p - \frac{1}{k}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\mathbf{q} + \rho_0\left(1 - \alpha\left(T - T_0\right) - \alpha'\left(C - C_0\right)\right)\mathbf{g}\,, \tag{2}$$

$$\sigma\frac{\partial T}{\partial t} + \mathbf{q}\nabla T = \kappa\nabla^2 T + D_{\mathrm{TC}}\nabla^2 C\,, \tag{3}$$

$$\varepsilon\frac{\partial C}{\partial t} + \mathbf{q}\nabla C = \kappa'\nabla^2 C + D_{\mathrm{CT}}\nabla^2 T\,. \tag{4}$$

where D_{TC} and D_{CT} are the Dufour and Soret coefficients; $\sigma = \frac{(\rho c_{\mathrm{p}})_{\mathrm{m}}}{(\rho c_{\mathrm{p}})_{\mathrm{f}}}$ is the thermal capacity ratio, c_{p} the specific heat, and the subscripts m and f refer to the porous medium and the fluid, respectively.

We assume that temperature and concentration are constant at the boundaries of the fluid layer. Therefore boundaries conditions are

$$\begin{aligned} &w = 0\,,\ T = T_0\,,\ C = C_0 \ \text{ at } z = 0 \\ &\text{and } w = 0\,,\ T = T_1\,,\ C = C_1 \ \text{ at } z = d\,. \end{aligned} \tag{5}$$

3. Steady State and its Solutions

The steady state is given by

$$\begin{aligned} &u = v = w = 0\,,\ p = p(z)\,, \\ &T = T_{\mathrm{s}}(z)\,,\ C = C_{\mathrm{s}}(z)\,. \end{aligned} \tag{6}$$

The solution of the steady state is

$$\begin{aligned} &T_{\mathrm{s}} = T_0 - \frac{\Delta T}{d}z\,,\ C_{\mathrm{s}} = C_0 - \frac{\Delta C}{d}z\,, \\ &p_{\mathrm{s}} = p_0 - \rho_0 g\left(z + \alpha\frac{\Delta T}{2d}z^2 + \alpha'\frac{\Delta C}{2d}z^2\right), \end{aligned}$$

where the subscript 0 denotes the value of the variable at the boundary $z = 0$.

4. Perturbation Equations

Let the initial steady state as described by above equation be slightly perturbed so that the perturbed state is given by

$$\begin{aligned} &q = q'\,,\ T = T_{\mathrm{s}} + T'\,, \\ &C = C_{\mathrm{s}} + C'\,,\ p = p_{\mathrm{s}} + p'\,, \end{aligned} \tag{7}$$

where the prime denotes the perturbed quantities. Substituting (7) into (1)–(4) and neglecting higher-order terms of the perturbed quantities, we get

$$\nabla \mathbf{q}' = 0\,, \tag{8}$$

$$0 = -\nabla p' - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\mathbf{q}' + \rho_0\left(\alpha T' + \alpha' C'\right)\mathbf{g}\,, \tag{9}$$

$$\sigma\frac{\partial T'}{\partial t} - w'\frac{\Delta T}{d} = \kappa\nabla^2 T' + D_{\mathrm{TC}}\nabla^2 C'\,, \tag{10}$$

$$\varepsilon\frac{\partial C'}{\partial t} - w'\frac{\Delta C}{d} = \kappa'\nabla^2 C' + D_{\mathrm{CT}}\nabla^2 T'\,. \tag{11}$$

Now introducing dimensionless variables as

$$(x'',y'',z'') = \left(\frac{x',y',z'}{\mathrm{d}}\right),$$
$$(u'',v'',w'') = \left(\frac{u',v',w'}{\kappa}\right)d,$$
$$t'' = \frac{\kappa}{\sigma d^2}t\,,\ \ p'' = \frac{k_1 d^2}{\mu\kappa}p'\,,$$
$$T'' = \frac{T'}{\Delta T}\,,\ \ C'' = \frac{C'}{\Delta C}\,.$$

There after dropping the dashes ($''$) for simplicity.

Now (8)–(11) can be written in non-dimensional form as

$$\nabla\mathbf{q} = 0\,, \tag{12}$$
$$0 = -\nabla p - \left(1 + F\frac{\partial}{\partial t}\right)\mathbf{q} + R_\mathrm{a}T + R_\mathrm{s}C\,, \tag{13}$$
$$\frac{\partial T}{\partial t} - w = \nabla^2 T + D_\mathrm{f}\nabla^2 C\,, \tag{14}$$
$$\frac{\varepsilon}{\sigma}\frac{\partial C}{\partial t} - w = \frac{1}{L_\mathrm{e}}\nabla^2 C + S_\mathrm{r}\nabla^2 T\,, \tag{15}$$

where the non-dimensional parameters are

$$R_\mathrm{a} = \frac{g\rho\alpha k\Delta T d}{\mu\kappa} \quad \text{(thermal Rayleigh number)},$$
$$R_\mathrm{s} = \frac{g\rho\alpha' k\Delta C d}{\mu\kappa'} \quad \text{(solutal Rayleigh number)},$$
$$L_\mathrm{e} = \frac{\kappa}{\kappa'} \quad \text{(Lewis number)},$$
$$F = \frac{\mu'\kappa}{\mu\sigma d^2} \quad \text{(kinematic visco–elasticity parameter)},$$
$$D_\mathrm{f} = \frac{D_\mathrm{TC}\Delta C}{\kappa\Delta T} \quad \text{(Dufour parameter)},$$
$$S_\mathrm{r} = \frac{D_\mathrm{CT}\Delta T}{\kappa\Delta C} \quad \text{(Soret parameter)},$$

and the non-dimensional boundary conditions are

$$w = T = C = 0 \ \text{ at } \ z = 0 \ \text{ and } \ z = 1\,. \tag{16}$$

5. Normal Modes

Analyzing the disturbances in normal modes and assuming that the perturbed quantities are of the form

$$[w,T,C] = [W(z),\Theta(z),\Gamma(z)] \cdot \exp\left(\mathrm{i}k_x x + \mathrm{i}k_y y + nt\right), \tag{17}$$

where k_x, k_y are wave numbers in x- and y-direction, and n is the growth rate of the disturbances.

Using (17), (12)–(15) become

$$\left(1 + F\frac{\partial}{\partial t}\right)\left(D^2 - a^2\right)W + R_\mathrm{a}a^2\Theta + R_\mathrm{s}a^2\Gamma = 0\,, \tag{18}$$
$$W + \left(D^2 - a^2 - n\right)\Theta + D_\mathrm{f}\left(D^2 - a^2\right)\Gamma = 0\,, \tag{19}$$
$$W + S_\mathrm{r}\left(D^2 - a^2\right)\Theta + \left(\frac{1}{L_\mathrm{e}}\left(D^2 - a^2\right) - \frac{\varepsilon}{\sigma}n\right)\Gamma = 0\,, \tag{20}$$

where $D = \frac{\mathrm{d}}{\mathrm{d}z}$ and $a^2 = k_x^2 + k_y^2$ is the dimensionless resultant wave number.

The boundary conditions for free–free boundary surfaces are thus

$$\begin{aligned} &W = 0\,,\ D^2W = 0\,,\ \Theta = 0\,,\\ &\Gamma = 0 \ \text{ at } \ z = 0\\ &\text{and } \ W = 0\,,\ D^2W = 0\,,\ \Theta = 0\,,\\ &\Gamma = 0 \ \text{ at } \ z = 1\,. \end{aligned} \tag{21}$$

We assume the solution to W, Θ, and Γ is of the form

$$\begin{aligned} &W = W_0 \sin\pi z,\ \ \Theta = \Theta_0 \sin\pi z\,,\\ &\Gamma = \Gamma_0 \sin\pi z\,, \end{aligned} \tag{22}$$

which satisfies the boundary conditions (21).

Substituting solution (22) into (18)–(20), integrating each equation from $z = 0$ to $z = 1$, and performing some integrations by parts, we obtain the following matrix equation:

$$\begin{bmatrix} J(1+nF) & -a^2R_\mathrm{a} & -a^2R_\mathrm{s} \\ -1 & (J+n) & D_\mathrm{f}J \\ -1 & S_\mathrm{r}J & \left(\dfrac{J}{L_\mathrm{e}} + \dfrac{\varepsilon n}{\sigma}\right) \end{bmatrix} \begin{bmatrix} W \\ \Theta_0 \\ \Gamma_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

where $J = \pi^2 + a^2$.

The non-trivial solution of the above matrix requires that

$$R_\mathrm{a} = \frac{(1+nF)\left(J(J+n)\left(\frac{J}{L_\mathrm{e}} + \frac{\varepsilon n}{\sigma}\right) - S_\mathrm{r}D_\mathrm{f}J^2\right)}{a^2J\left(\frac{1}{L_\mathrm{e}} - D_\mathrm{f}\right) + \frac{\varepsilon n}{\sigma}}$$

$$+\frac{S_r J-(J+n)}{J\left(\frac{1}{L_e}-D_f\right)+\frac{\varepsilon n}{\sigma}}R_s\,. \tag{23}$$

For neutral instability $n=\mathrm{i}\omega$, (where ω is a real and dimensional frequency), (23) reduces to

$$R_a=\Delta_1+\mathrm{i}\omega\Delta_2\,, \tag{24}$$

where Δ_1 and Δ_2 are given in the Appendix.

Since R_a is a physical quantity, it must be real. Hence, it follows from (24) that either $\omega=0$ (exchange of stability, steady state) or $\Delta_2=0$ ($\omega\neq 0$ overstability or oscillatory onset).

6. Stationary Convection

For stationary convection $\omega=0$ $(n=0)$, (23) reduces to

$$(R_a)_s=\frac{J^2}{a^2}\left(\frac{D_f S_r L_e-1}{D_f L_e-1}\right)+\frac{(S_r-1)L_e}{1-D_f L_e}R_s\,. \tag{25}$$

We find that for the stationary convection the kinematic visco–elasticity parameter F vanishes with n and the Rivlin–Ericksen elastico–viscous fluid behaves like an ordinary Newtonian fluid. This is the same result as obtained by Motsa [18].

The critical cell size at the onset of instability is obtained from the condition

$$\left(\frac{\partial R_a}{\partial a}\right)_{a=a_c}=0\,,\ \text{which gives}\ a_c=\pi\,.$$

This is the same result as obtained by Lapwood [2] for a Newtonian fluid.

The corresponding critical Rayleigh number R_{ac} for steady onset is

$$(\mathrm{R}_{ac})_s=4\pi^2\left(\frac{D_f S_r L_e-1}{D_f L_e-1}\right)+\frac{(S_r-1)L_e}{1-D_f L_e}R_s\,. \tag{26}$$

This is also the same result as obtained by Motsa [18].

If $R_s=D_f=S_r=0$ then

$$(\mathrm{R}_{ac})_s=4\pi^2\,.$$

This is the exactly the same result as obtained by Nield and Bejan [5].

7. Oscillatory Convection

For oscillatory convection $\omega\neq 0$, we must have $\Delta_2=0$, which gives

$$\begin{aligned}\omega^2=&\left[J^4F\left(\frac{1}{L_e}-D_f\right)^2+J^2F\left(\frac{1}{L_e}-D_f\right)^2\right.\\&+\frac{\varepsilon}{\sigma}J^2F\left(\frac{1}{L_e}-D_f\right)-J^3\frac{\varepsilon}{\sigma}\left(\frac{1}{L_e}-D_f\right)\\&+J^3(S_r-1)D_f\frac{\varepsilon}{\sigma}-a^2\left(J\left(\frac{1}{L_e}-D_f\right)\right.\\&\left.\left.+J\frac{\varepsilon}{\sigma}(S_r-1)\right)R_s\right]\cdot\left\{\frac{\varepsilon}{\sigma}J^2F\left(\frac{1}{L_e}-D_f\right)\right.\\&\left.-J^2F\left(\frac{1}{L_e}-D_f\right)\frac{\varepsilon}{\sigma}-\frac{\varepsilon^2}{\sigma^2}J(1+JF)-J^2D_f\frac{\varepsilon}{\sigma}\right\}^{-1}.\end{aligned} \tag{27}$$

Equation (27) indicates the frequency of the oscillatory mode. If there is no positive ω^2 then an oscillatory instability is not possible. If there exist positive values of ω^2, then the thermal oscillatory Rayleigh number is obtain by inserting the positive values of ω^2 in (24).

The thermal oscillatory Rayleigh number is given by

$$\begin{aligned}(R_a)_{osc}=&\frac{1}{a^2}\left[J^4\left(\frac{1}{L_e}-D_f\right)^2-J^4(S_r-1)D_f\right.\\&\cdot\left(\frac{1}{L_e}-D_f\right)-\omega^2\left\{\frac{\varepsilon}{\sigma}J^2\left(\frac{1}{L_e}-D_f\right)\right.\\&+J^3F\left(\frac{1}{L_e}-D_f\right)^2+J^3FD_f\left(\frac{1}{L_e}-D_f\right)\\&\left.\left.+\omega^2JF\frac{\varepsilon^2}{\sigma^2}-\frac{\varepsilon}{\sigma}J\left(\frac{1}{L_e}-D_f\right)-J\frac{\varepsilon^2}{\sigma}\right\}\right]\\&\cdot\left[J^2\left(\frac{1}{L_e}-D_f\right)^2+\omega^2\frac{\varepsilon^2}{\sigma^2}\right]^{-1}\\&+\frac{J^2(S_r-1)\left(\frac{1}{L_e}-D_f\right)-\omega^2\frac{\varepsilon}{\sigma}}{J^2\left(\frac{1}{L_e}-D_f\right)^2+\omega^2\frac{\varepsilon^2}{\sigma^2}}R.\end{aligned} \tag{28}$$

From (28), it is clear that oscillatory convection depends on the visco–elasticity parameter F.

8. Results and Discussion

The expression for the stationary thermal Rayleigh number is given in (25) and the oscillatory thermal

Rayleigh number is given in (28). We discuss our results analytically and graphically.

Figure 2 shows the variation of the Rayleigh number with the wave number for different values of the Dufour parameter; it has been found that the Rayleigh number decreases with an increase in the value of Dufour parameter, thus the Dufour parameter has a destabilizing effect on stationary convection. It is the same result as obtained by Motsa [18]

Figure 3 shows the variation of the Rayleigh number with the wave number for different values of the Soret parameter; it has been found that the Rayleigh number first increases then decreases and finally increases again with an increase in the value of Soret parameter, thus the Soret parameter has both stabilizing and destabilizing effect on the stationary convection. It is the same result as obtained by Motsa [18]

Figure 4 shows the variation of the Rayleigh number with the wave number for different values of the Lewis number; it has been found that the Rayleigh number first increases then decreases and finally increases again with an increase in the value of Lewis number, thus the Lewis number has both stabilizing and destabilizing effect on the stationary convection.

Figure 5 shows the variation of the Rayleigh number with the wave number for different values of the solutal Rayleigh number; it has been found that the thermal Rayleigh number increases with an increase

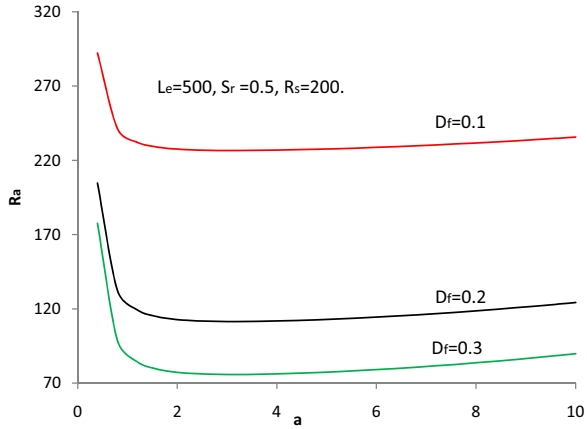

Fig. 2 (colour online). Variation of Rayleigh number R_a with wave number a for different values of Dufour parameter.

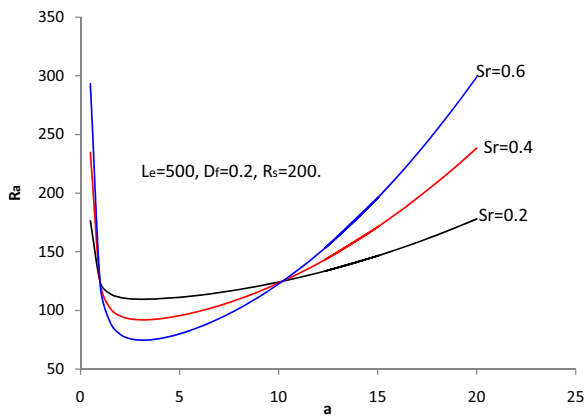

Fig. 3 (colour online). Variation of Rayleigh number R_a with wave number a for different values of Soret parameter.

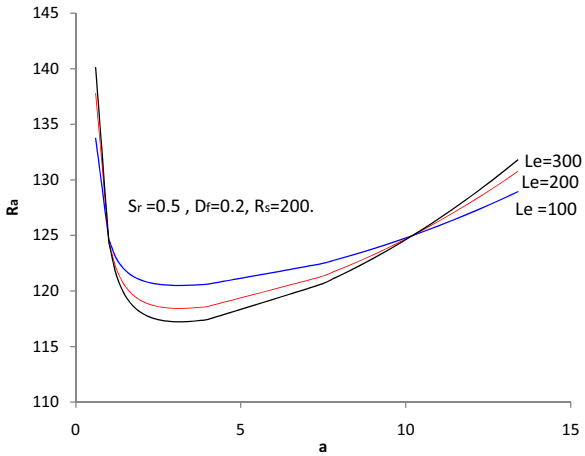

Fig. 4 (colour online). Variation of Rayleigh number R_a with wave number a for different values Lewis number.

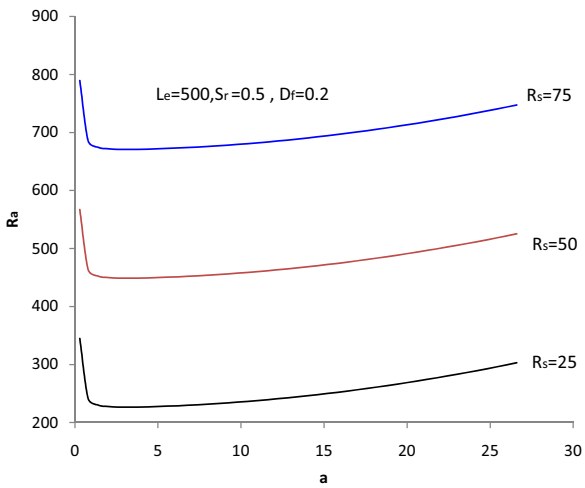

Fig. 5 (colour online). Variation of Rayleigh number R_a with wave number a for different values solutal Rayleigh number.

in the value of solutal Rayleigh number, thus the solutal Rayleigh number stabilizes the stationary convection.

9. Conclusions

We used linear instability analysis to study Soret and Dufour effects in double diffusive convection of a Rivlin–Erickson elastico–viscous fluid in a porous medium. An expression for the Rayleigh number for stationary and oscillatory convection is obtained. We draw following conclusions:

(i) In stationary convection, the Rivlin–Ericksen elastico–viscous fluid behaves like an ordinary Newtonian fluid.

(ii) The Dufour parameter destabilizes the stationary convection.

(iii) The Soret parameter and the Lewis number have both stabilizing and destabilizing effect on the stationary convection.

(iv) The solutal Rayleigh number stabilizes the stationary convection.

(v) In the limiting case when $R_s = S_r = D_f = 0$ the critical thermal Rayleigh number obtained is the same as reported by Nield and Bejan [5].

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Appendix

Δ_1 and Δ_2 appearing in (24) are derived as

$$\Delta_1 = \frac{1}{a^2}\left[J^4\left(\frac{1}{L_e}-D_f\right)^2 - J^4(S_r-1)D_f\left(\frac{1}{L_e}-D_f\right)\right.$$
$$-\omega^2\left\{\frac{\varepsilon}{\sigma}J^2\left(\frac{1}{L_e}-D_f\right)+J^3F\left(\frac{1}{L_e}-D_f\right)^2\right.$$
$$+J^3FD_f\left(\frac{1}{L_e}-D_f\right)+\omega^2JF\frac{\varepsilon^2}{\sigma^2}-\frac{\varepsilon}{\sigma}J\left(\frac{1}{L_e}-D_f\right)$$
$$\left.\left.-J\frac{\varepsilon^2}{\sigma}\right\}+a^2\left(J^2(S_r-1)_f\left(\frac{1}{L_e}-D_f\right)-\omega^2\frac{\varepsilon}{\sigma}\right)R_s\right]$$
$$\cdot\left[J^2\left(\frac{1}{L_e}-D_f\right)^2+\omega^2\frac{\varepsilon^2}{\sigma^2}\right]^{-1},$$

$$\Delta_2 = \frac{1}{a^2}\left[J^4F\left(\frac{1}{L_e}-D_f\right)^2+J^2F\left(\frac{1}{L_e}-D_f\right)^2\right.$$
$$+\frac{\varepsilon}{\sigma}J^2F\left(\frac{1}{L_e}-D_f\right)-J^3\frac{\varepsilon}{\sigma}\left(\frac{1}{L_e}-D_f\right)$$
$$+J^3(S_r-1)D_f\frac{\varepsilon}{\sigma}-a^2\left(J\left(\frac{1}{L_e}-D_f\right)\right.$$
$$\left.+J\frac{\varepsilon}{\sigma}(S_r-1)\right)R_s-\omega^2\left\{\frac{\varepsilon}{\sigma}J^2F\left(\frac{1}{L_e}-D_f\right)\right.$$
$$\left.\left.-J^2F\left(\frac{1}{L_e}-D_f\right)\frac{\varepsilon}{\sigma}-\frac{\varepsilon^2}{\sigma^2}J(1+JF)-J^2D_f\frac{\varepsilon}{\sigma}\right\}\right]$$
$$\cdot\left[J^2\left(\frac{1}{L_e}-D_f\right)^2+\omega^2\frac{\varepsilon^2}{\sigma^2}\right]^{-1}$$

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