Temperature Dependence of the Magnetoresistance Effect in Metals

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The paper examines a well-known experimental property of increase of the magnetoresistance effect in a metal observed with a decrease of the metal temperature. This property is explained by the fact that magnetoresistance is a quantity proportional to the relaxation time of the electric conduction of the metal sample which is a parameter observed in the absence of the magnetic field. Since the electric conduction, as well as the corresponding relaxation time, increase with the lowering of temperature, they provide us necessarily with an increase of magnetoresistance. The phenomenon is investigated quantitatively in this paper for numerous metal cases taken as examples.

Key words: Temperature Dependence; Magnetoresistance Effect; Metals.
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1. Introduction

Experimentally, it became evident since a long time that, in general, the magnetoresistance effect in a metal increases with the purity of a metal sample, as well as the lowering of the sample temperature; see e.g. [1]. More recently, an interest in the temperature dependence of magnetoresistance has been raised because of the measurements of this effect done in the planar heterostructures; see e.g. [2 – 5]. Here it is found rather systematically that magnetoresistance considered for a constant strength of the magnetic induction increases to higher values the lower is the sample temperature. This kind of behaviour was observed also for samples submitted to a high external pressure [6].

However, a theoretical approach to the temperature dependence of magnetoresistance seems to be not satisfactory for both three-dimensional as well as two-dimensional systems. This situation is tried to be improved in the present paper for three-dimensional metallic systems, before a similar problem in heterostructures can be approached.

Certainly, the magnetoresistance effect depends strongly on the band structure of metals, especially the details of that structure represented by the character of the Fermi surface of a metal (see e.g. [7]). In general, for crystals having closed Fermi surfaces like aluminium, indium, sodium, and lithium, or in crystals in which the number of directions pointing to a closed Fermi surface predominates heavily over those pointing to an open character of that surface, an increase of magnetoresistance due to the increase of the magnetic field induction B is found experimentally to exist continuously up to the highest fields; see e.g. [1]. The experimental methods show that the character of this magnetoresistance increase is mainly linear, especially for the metals which have an evident free-electron background [8, 9].

Theoretically, an unlimited linear increase of magnetoresistance with increase of B is demonstrated in [10]. But independentely of the behaviour defined by the change of B it is found that magnetoresistance is proportional also to the relaxation time τ of the electric resistance:

$$\frac{\Delta\rho}{\rho} \sim \tau(0)\,. \tag{1}$$

Here $\Delta\rho$ is the increment of the electric resistance $\rho = \rho(0)$ at $B = 0$ due to the magnetic field induction $B \neq 0$. The argument of zero in (1) makes reference to the fact that the relaxation time is exhibited by a metal at $B = 0$, so it is connected with the electron transport in the electric field alone. Since $\tau(0)$ in (1) systematically increases with the lowering of temperature, a general experimental increase of magnetoresistance mentioned above is confirmed also on a theo-

retical way [10]. A characteristic point is that this behaviour is represented not only in single crystals like sodium or silver, but also for the metal samples in a rough crystal state like zinc, cadmium, aluminium, and platinum. Simultaneously, an isotropic, or quasi-isotropic, character of the magnetoresistance becomes more or less perturbed.

A particular aim of the present paper is to point out that already a single-band approximation to the electron transport in the magnetic field [10, 11] can give a quantitative insight into the temperarature dependence of the magnetoresistance effect. In fact, we show that the ratios of the relaxation time (1) taken at different temperatures can give a realistic approach to the corresponding ratios of magnetoresistance. To this purpose we choose the experimental results for the metal samples which are submitted to the changes of temperature but, at the same time, the strength value of the magnetic field induction B acting on a given sample remains unchanged.

2. The Formalism

In case B does not change for the measurements performed in temperature T_1 and T_2, we should have

$$\left(\frac{\Delta\rho}{\rho}\right)_{T_1} : \left(\frac{\Delta\rho}{\rho}\right)_{T_2} = \frac{\tau_{T_1}(0)}{\tau_{T_2}(0)} \tag{2}$$

for any pair of temperatures T_1 and T_2. But, in the next step, no theoretical calculations have to be done on the ratio entering the right-hand side of (2) if we note that

$$\frac{\tau_{T_1}(0)}{\tau_{T_2}(0)} = \frac{\rho_{T_2}(0)}{\rho_{T_1}(0)}\,. \tag{3}$$

This formula is in fact the ratio of the electric resistance (at $B = 0$) which is measured for two different temperatures T_1 and T_2. Usually, both the numerator as well as the denominator on the right-hand side in (3) are represented by their ratios r_{T_1} and r_{T_2} to the electric resistance at some standard temperature which is chosen to be at zero Celsius degree. In effect, the ratio (3) is transformed into

$$\frac{\rho_{T_2}(0)}{\rho_{T_1}(0)} = \frac{r_{T_2}}{r_{T_1}}\,, \tag{4}$$

where r_{T_2}, r_{T_1} are reported, for example, in [1] for numerous metals and temperature cases. Regularly, we have $r_{T_2} < r_{T_1}$ for $T_2 < T_1$, therefore the ratio in (3) is smaller than unity in this case.

A reference of the present approach to the Kohler's rule [5, 13, 14] should be also done. The rule applies the ratio

$$x = \frac{B}{\rho_T(0)} \tag{5}$$

as an independent variable in calculating the magnetoresistance:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta\rho}{\rho(0)} = \frac{\rho_T^{\mathrm{tot}}(B) - \rho_T(0)}{\rho_T(0)} = f(x)\,. \tag{6}$$

This formula means that dependence of magnetoresistance on B and T is in fact incorporated in the variable (5).

Let us note next that the magnetoresistance ratio is given by the expression [10, 11]

$$\frac{\Delta\rho}{\rho(0)} = \frac{\tau_T(0)}{\tau(B)} = \frac{\Omega_0\tau_T(0)}{\xi}\,. \tag{7}$$

Here Ω_0 is the circulation frequency given in (8), $\tau_T(0)$ is the relaxation time of the electric resistance at $B = 0$ dependent also on temperature T, $\tau(B)$ is the relaxation time provided by the electron circulation in the magnetic field essentially independent of T, and ξ is a constant number; see [10, 11]. Since the electron circulation frequency in the field B is

$$\Omega_0 = \frac{eB}{mc} \tag{8}$$

and the electric resistance

$$\rho_T(0) = \frac{m}{\tau_T(0) n_s \mathrm{e}^2}\,, \tag{9}$$

together with $\tau_T(0)$, is a temperature-dependent quantity (n_s is the concentration of the electron carriers), we obtain for (6)

$$\frac{\Delta\rho}{\rho(0)} = \frac{1}{en_s c\xi}\,x\,. \tag{10}$$

Therefore, the magnetoresistance becomes a linear function of the variable x, since the coefficient multiplying x on the right-hand side of (10) is a constant term.

A linear behaviour of (10) has been checked experimentally a time ago [15] when a dependence of the magnetoresistance of magnesium metal on B has been examined for different temperatures. It is shown that the plots of lines versus B obtained for different temperatures are in fact straight-linear and tend to merge together into one line.

3. Results for the Transversal Magnetoresistance

The formalism of Section 2 is applied in Tables 1 – 5 to examine transversal magnetoresistance for different metal cases. The experimental data taken to calculations are from [1], but their pattern of the tables is made similar to that applied in [10]. In fact, one printing error has been detected in Table 7 of [10]: the second symbol Ga in that table should be replaced by In.

Regularly, for a given metal sample, the experimental data are concerning the same induction value B, but taken for different temperatures. In the first stage of calculations no reference to the directional character of the magnetic field, for example that considered with respect to the symmetry species of the metal crystal, is taken into account. Consequently, all theoretical calculations of the present section and results of [10, 11] are based on a single-band free-electron model of a metal (see e.g. [7]), and the only directional effect is coming from the geometry imposed by the magnetic field.

The calculations show that in fact for a constant temperature ratio the magnetoresistance ratio considered at some B gives an almost constant number dependent only slightly on B. But the next result is that the values of the magnetoresistance ratios in two different temperatures (C1) become rather close to the electrical resistance ra-

	B (in kG)	$\Delta\rho_{78\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{78\,K}$ (C2)	C1/C2
Be polycrystal	100	$0.4 : 0.1 = 4.0$	$1 : 0.33 = 3.0$	1.3
	200	$1.3 : 0.35 = 3.7$	3.0	1.3
	300	$2.3 : 0.66 = 3.5$	3.0	1.2
	B (in kG)	$\Delta\rho_{195\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{195\,K}$ (C2)	
Mg polycrystal	100	$0.05 : 0.02 = 2.5$	$1 : 0.68 = 1.5$	1.7
	200	$0.17 : 0.08 = 2.1$	1.5	1.4
	300	$0.31 : 0.17 = 1.8$	1.5	1.2
	B (in kG)	$\Delta\rho_{78\,K} : \Delta\rho_{195\,K}$ (C1)	$r_{195\,K} : r_{78\,K}$ (C2)	
Mg polycrystal	100	$0.5 : 0.05 = 10$	$0.68 : 0.17 = 4.0$	2.5
	200	$1.6 : 0.17 = 9.4$	4.0	2.4
	300	$2.8 : 0.31 = 9.0$	4.0	2.3
	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	
Ba polycrystal I	26.6 – 26.9	$0.66 : 0.28 = 2.4$	$0.0275 : 0.0152 = 1.8$	1.3
	33.2	$0.96 : 0.41 = 2.3$	1.8	1.3
	B (in kG)	$\Delta\rho_{1.85\,K} : \Delta\rho_{4.22\,K}$ (C1)	$r_{4.22\,K} : r_{1.85\,K}$ (C2)	
Ba polycrystal II	33.9	$3.09 : 3.02 = 1.0$	$0.0079 : 0.0071 = 1.1$	0.9

Table 1. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{T_1} : \Delta\rho_{T_2}$) at a constant B compared with the ratios of the electric resistance ($r_{T_2} : r_{T_1}$) at $B = 0$; different temperature pairs T_1 and T_2 are considered. Be, Mg, and two Ba metal samples are taken into account [1, 10].

	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20\,K}$ (C1)	$r_{20\,K} : r_{14\,K}$ (C2)	C1/C2
Al single crystal	5	$0.93 : 0.77 = 1.2$	$0.0018 : 0.0014 = 1.3$	0.9
	10	$1.48 : 1.22 = 1.2$	1.3	0.9
	20	$1.91 : 1.81 = 1.1$	1.3	0.8
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{77.2\,K}$ (C1)	$r_{77.2\,K} : r_{20.4\,K}$ (C2)	C1/C2
Al polycrystal	10.3	$0.017 : 0.0025 = 6.8$	$0.161 : 0.0675 = 2.4$	2.8
	14.6	$0.031 : 0.0048 = 6.5$	2.4	2.7
	20.1	$0.051 : 0.0087 = 5.9$	2.4	2.5
	23.5	$0.064 : 0.011 = 5.8$	2.4	2.4
	B (in kG)	$\Delta\rho_{14.2\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14.2\,K}$ (C2)	C1/C2
Al polycrystal	10.3	$0.0185 : 0.0172 = 1.1$	$00675 : 0.0665 = 1.0$	1.1
	14.6	$0.0326 : 0.0308 = 1.1$	1.0	1.1
	20.1	$0.0539 : 0.0509 = 1.1$	1.0	1.1
	23.5	$0.0678 : 0.0640 = 1.1$	1.0	1.1

Table 2. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{T_1} : \Delta\rho_{T_2}$) at a constant B compared with the ratios of the electric resistance ($r_{T_2} : r_{T_1}$) at $B = 0$; different temperature pairs T_1 and T_2 are considered, metal samples of Al, W, Re, Rh, and Pt are taken into account [1, 10].

Table 2. Continued.

	B (in kG)	$\Delta\rho_{14.2\,K} : \Delta\rho_{20.5\,K}$ (C1)	$r_{20.5\,K} : r_{14.2\,K}$ (C2)	C1/C2
W single crystal	5.4–5.5	$41.3 : 14.8 = 2.8$	$0.00086 : 0.00048 = 1.8$	1.4
	11.2–11.3	$164 : 53 = 3.1$	1.8	1.7
	16.6	$345 : 107 = 3.2$	1.8	1.8
	21.9	$587 : 182 = 3.2$	1.8	1.8
	26.3	$805 : 248 = 3.2$	1.8	1.8
	B (in kG)	$\Delta\rho_{4.2\,K} : \Delta\rho_{14.2\,K}$ (C1)	$r_{14.2\,K} : r_{4.2\,K}$ (C2)	C1/C2
W single crystal	5.4–5.6	$81 : 41 = 2.0$	$0.0048 : 0.0035 = 1.4$	1.4
	10.7–11.2	$279 : 164 = 1.7$	1.4	1.2
	15.8–16.6	$583 : 345 = 1.7$	1.4	1.2
	21.4–21.9	$1040 : 587 = 1.8$	1.4	1.3
	24.6–26.3	$1360 : 805 = 1.7$	1.4	1.2
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{80\,K}$ (C1)	$r_{80\,K} : r_{20.4\,K}$ (C2)	C1/C2
Re polycrystal	23.4–25.1	$0.175 : 0.015 = 11.7$	$0.166 : 0.0179 = 9.3$	1.3
	34.3	$0.266 : 0.0196 = 13.6$	9.3	1.5
	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	C1/C2
Rh polycrystal	6.2	$0.337 : 0.300 = 1.1$	$0.0036 : 0.0033 = 1.1$	1.0
	13.0	$0.700 : 0.628 = 1.1$	1.1	1.0
	20.2	$1.111 : 0.981 = 1.1$	1.1	1.0
	25.0–25.3	$1.324 : 1.232 = 1.1$	1.1	1.0
	31.3–31.6	$1.547 : 1.435 = 1.1$	1.1	1.0
	36.2–36.3	$1.652 : 1.546 = 1.1$	1.1	1.0
	B (in kG)	$\Delta\rho_{4.22\,K} : \Delta\rho_{14\,K}$ (C1)	$r_{14\,K} : r_{4.22\,K}$ (C2)	C1/C2
Rh polycrystal	6.2	$0.430 : 0.337 = 1.3$	$0.0033 : 0.0029 = 1.1$	1.2
	12.2–13.0	$0.788 : 0.700 = 1.1$	1.1	1.0
	20.2–20.7	$1.295 : 1.111 = 1.2$	1.1	1.1
	25.3–25.8	$1.517 : 1.324 = 1.1$	1.1	1.0
	31.1–31.6	$1.709 : 1.547 = 1.1$	1.1	1.0
	36.2–37.6	$1.867 : 1.652 = 1.1$	1.1	1.0
	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	C1/C2
Pt polycrystal	8	$0.1193 : 0.0427 = 2.8$	$0.0067 : 0.0034 = 2.0$	1.4
	19.5	$0.3977 : 0.1538 = 2.6$	2.0	1.3
	26.4	$0.5909 : 0.2336 = 2.5$	2.0	1.3
	30.7	$0.7159 : 0.2849 = 2.5$	2.0	1.3
	35.8	$0.8920 : 0.3533 = 2.5$	2.0	1.3
	40.1	$1.0682 : 0.4216 = 2.5$	2.0	1.3
	B (in kG)	$\Delta\rho_{4.22\,K} : \Delta\rho_{14\,K}$ (C1)	$r_{14\,K} : r_{4.22\,K}$ (C2)	C1/C2
Pt polycrystal	8	$0.2284 : 0.1193 = 1.9$	$0.0034 : 0.0022 = 1.5$	1.3
	19.5	$0.7474 : 0.3977 = 1.9$	1.5	1.3
	26.4	$1.1038 : 0.5909 = 1.9$	1.5	1.3
	30.7	$1.3379 : 0.7159 = 1.9$	1.5	1.3
	35.8	$1.6655 : 0.8920 = 1.9$	1.5	1.3
	40.1	$1.9896 : 1.0682 = 1.9$	1.5	1.3

tios measured for the same temperatures (C2) on condition a reversed sequence of temperatures is taken into account; see (2) and (3). A rather evident exception is here the metal of lead for which the magnetoresistance ratio is about ten times larger than the ratio obtained from the electric resistance; see Table 5. A similar behaviour is obtained for a cadmium sample at low B (Table 4).

However, some careful measurements concerning the magnetoresistance in the Be metal were done already a time ago [12] in which the dependence of

	B (in kG)	$\Delta\rho_{63.8\,K} : \Delta\rho_{77.2\,K}$ (C1)	$r_{77.2\,K} : r_{63.8\,K}$ (C2)	C1/C2
Cu polycrystal	23.6	$0.02 : 0.01 = 2.0$	$0.148 : 0.094 = 1.6$	1.3
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{63.8\,K}$ (C1)	$r_{63.8\,K} : r_{20.4\,K}$ (C2)	C1/C2
Cu polycrystal	23.6	$0.19 : 0.02 = 9.5$	$0.094 : 0.02 = 4.7$	2.0
	B (in kG)	$\Delta\rho_{14.2\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14.2\,K}$ (C2)	C1/C2
Cu polycrystal	23.6	$0.19 : 0.19 = 1.0$	$0.02 : 0.02 = 1.0$	1.0
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{79\,K}$ (C1)	$r_{79\,K} : r_{20.4\,K}$ (C2)	C1/C2
Au polycrystal	33.9–35.8	$1.42 : 0.0177 = 80.2$	$0.219 : 0.0071 = 30.8$	2.6
	39.8–40.1	$1.71 : 0.018 = 95.0$	30.8	3.1
	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	C1/C2
Au polycrystal	7.8	$0.91 : 0.25 = 3.6$	$0.0071 : 0.0023 = 3.1$	1.2
	15.3	$1.91 : 0.59 = 3.2$	3.1	1.0
	26.0	$3.24 : 1.07 = 3.0$	3.1	1.0
	33.9	$4.15 : 1.42 = 2.9$	3.1	0.9
	39.8	$4.95 : 1.71 = 2.9$	3.1	0.9
	B (in kG)	$\Delta\rho_{4.2\,K} : \Delta\rho_{14\,K}$ (C1)	$r_{14\,K} : r_{4.2\,K}$ (C2)	C1/C2
Au polycrystal	7.8–8.0	$2.2 : 0.9 = 2.4$	$0.0230 : 0.00085 = 2.7$	0.9
	15.3–15.8	$4.5 : 1.9 = 2.4$	2.7	0.9
	26–26.5	$7.6 : 3.2 = 2.4$	2.7	0.9
	33.9–34.2	$9.7 : 4.2 = 2.3$	2.7	0.9
	39.8–40.1	$11.2 : 5.0 = 2.2$	2.7	0.8
	B (in kG)	$\Delta\rho_{14.2\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14.2\,K}$ (C2)	C1/C2
Zn polycrystal	5.2	$0.4 : 0.21 = 1.9$	$0.0125 : 0.07 = 1.8$	1.1
	10.3	$1.0 : 0.49 = 2.0$	1.8	1.1
	14.6	$1.6 : 0.75 = 2.1$	1.8	1.2
	20.1	$2.5 : 1.12 = 2.2$	1.8	1.2
	23.6	$3.1 : 1.38 = 2.2$	1.8	1.2
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{77.2\,K}$ (C1)	$r_{77.2\,K} : r_{20.4\,K}$ (C2)	C1/C2
Zn polycrystal	23.6	$1.377 : 0.018 = 76.5$	$0.202 : 0.0125 = 16.2$	4.7
	B (in kG)	$\Delta\rho_{78\,K} : \Delta\rho_{195\,K}$ (C1)	$r_{195\,K} : r_{78\,K}$ (C2)	C1/C2
Zn polycrystal	300	$0.93 : 0.12 = 7.8$	$0.68 : 0.19 = 3.6$	2.2
	B (in kG)	$\Delta\rho_{195\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{195\,K}$ (C2)	C1/C2
Zn polycrystal	300	$0.12 : 0.06 = 2.0$	$1 : 0.68 = 1.5$	1.3

Table 3. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{T_1} : \Delta\rho_{T_2}$) at a constant B compared with the ratios of the electric resistance ($r_{T_2} : r_{T_1}$) at $B = 0$; different temperature pairs T_1 and T_2 are considered. Metal samples of Cu, Au, and Zn are taken into account [1, 10].

the effect of the field direction with respect to the crystallographic axes has been considered. These results are examined, from the point of view of the present theory, in Section 5.

4. Temperature Dependence of the Longitudinal Magnetoresistance

In calculations quoted in Section 3 only the temperature data for the transversal magnetoresistance were examined. In fact, the longitudinal magnetoresistance in metals may become also of interest [11]. The data for this kind of magnetoresistance can be approached on the same theoretical footing as in the transversal case. In Table 6, we present the ratios of the longitudinal magnetoresistance obtained at different temperatures when the magnetic field has its strength unchanged. Because the formulae (3) and (4) apply equally for the longitudinal magnetoresistance, the ratios (3) are compared with the experimental

	B (in kG)	$\Delta\rho_{195\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{195\,K}$ (C2)	C1/C2
Cd polycrystal	300	$0.19 : 0.08 = 2.4$	$1 : 0.68 = 1.5$	1.6
	B (in kG)	$\Delta\rho_{78\,K} : \Delta\rho_{195\,K}$ (C1)	$r_{195\,K} : r_{78\,K}$ (C2)	C1/C2
Cd polycrystal	300	$0.92 : 0.19 = 4.8$	$0.68 : 0.22 = 3.1$	1.5
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{77.2\,K}$ (C1)	$r_{77.2\,K} : r_{20.4\,K}$ (C2)	C1/C2
Cd polycrystal	2.1	$0.042 : 0.0004 = 105$	$0.2513 : 0.023 = 10.9$	9.6
	4.1	$0.122 : 0.00184 = 66$	10.9	6.1
	10.3	$0.430 : 0.00981 = 44$	10.9	4.0
	14.6	$0.663 : 0.0178 = 37$	10.9	3.4
	23.6	$1.163 : 0.0378 = 31$	10.9	2.8
	B (in kG)	$\Delta\rho_{14.2\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14.2\,K}$ (C2)	C1/C2
Cd polycrystal	2.1	$0.2 : 0.042 = 4.8$	$0.023 : 0.0081 = 2.8$	1.7
	4.1	$0.5 : 0.12 = 4.2$	2.8	1.5
	10.3	$1.5 : 0.43 = 3.5$	2.8	1.3
	14.6	$2.2 : 0.66 = 3.3$	2.8	1.2
	23.6	$4.1 : 1.16 = 3.5$	2.8	1.3
	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	C1/C2
Cd single crystal	5.7	$0.92 : 0.38 = 2.4$	$0.0193 : 0.0063 = 3.1$	0.8
	11.2	$2.01 : 0.85 = 2.4$	3.1	0.8
	18.4	$3.7 : 1.5 = 2.5$	3.1	0.8
	24.3	$5.3 : 2.0 = 2.7$	3.1	0.9
	30.4	$7.5 : 2.5 = 3.0$	3.1	1.0
	36.6	$9.4 : 2.9 = 3.2$	3.1	1.0
	B (in kG)	$\Delta\rho_{80\,K} : \Delta\rho_{195\,K}$ (C1)	$r_{195\,K} : r_{80\,K}$ (C2)	C1/C2
Hg polycrystal	300	$0.05 : 0.02 = 2.5$	$0.67 : 0.25 = 2.7$	0.9
	B (in kG)	$\Delta\rho_{14.2\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14.2\,K}$ (C2)	C1/C2
Hg polycrystal	5.2	$0.0023 : 0.0004 = 5.8$	$0.0147 : 0.0082 = 1.8$	3.2
	10.6	$0.0057 : 0.0020 = 2.9$	1.8	1.6
	15.9	$0.0125 : 0.0036 = 3.5$	1.8	1.9
	23.1	$0.0256 : 0.0078 = 3.3$	1.8	1.8
	29.8	$0.0398 : 0.0124 = 3.2$	1.8	1.8
	32.3	$0.0459 : 0.0145 = 3.2$	1.8	1.8
	B (in kG)	$\Delta\rho_{80\,K} : \Delta\rho_{195\,K}$ (C1)	$r_{195\,K} : r_{80\,K}$ (C2)	C1/C2
Ga polycrystal	100	$0.28 : 0.033 = 8.5$	$0.65 : 0.21 = 3.1$	2.7
	200	$0.58 : 0.12 = 4.8$	3.1	1.5
	300	$0.87 : 0.17 = 5.1$	3.1	1.6
	B (in kG)	$\Delta\rho_{80\,K} : \Delta\rho_{165\,K}$ (C1)	$r_{165\,K} : r_{80\,K}$ (C2)	C1/C2
In polycrystal	300	$0.14 : 0.03 = 4.7$	$0.63 : 0.22 = 2.9$	1.6
	B (in kG)	$\Delta\rho_{20.4\,K} : \Delta\rho_{78\,K}$ (C1)	$r_{78\,K} : r_{20.4\,K}$ (C2)	C1/C2
In polycrystal	10	$0.018 : 0.0006 = 30$	$0.2171 : 0.0230 = 9.4$	3.2
	15	$0.037 : 0.0013 = 28$	9.4	3.0
	20	$0.058 : 0.0020 = 29$	9.4	3.1
	25	$0.083 : 0.0029 = 29$	9.4	3.1
	30	$0.111 : 0.0040 = 28$	9.4	3.0
	35	$0.143 : 0.0056 = 26$	9.4	2.8
	40	$0.173 : 0.0084 = 21$	9.4	2.2

Table 4. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{T_1} : \Delta\rho_{T_2}$) at a constant B compared with the ratios of the electric resistance ($r_{T_2} : r_{T_1}$) at $B = 0$; different temperature pairs T_1 and T_2 are considered. Metal samples of Cd, Hg, Ga, In, and Sn are taken into account [1, 10].

Table 4. Continued.

	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	C1/C2
In polycrystal	10	$0.090 : 0.018 = 5.0$	$0.0230 : 0.00855 = 2.7$	1.9
	15	$0.157 : 0.037 = 4.2$	2.7	1.6
	20	$0.23 : 0.058 = 4.0$	2.7	1.5
	25	$0.30 : 0.083 = 3.6$	2.7	1.3
	30	$0.38 : 0.111 = 3.4$	2.7	1.3
	35	$0.45 : 0.143 = 3.1$	2.7	1.1
	40	$0.52 : 0.173 = 3.0$	2.7	1.1
	B (in kG)	$\Delta\rho_{4.2\,K} : \Delta\rho_{14\,K}$ (C1)	$r_{14\,K} : r_{4.2\,K}$ (C2)	C1/C2
In polycrystal	10	$0.84 : 0.09 = 9.3$	$0.00855 : 0.00119 = 7.2$	1.3
	15	$1.02 : 0.16 = 6.4$	7.2	0.9
	20	$1.12 : 0.23 = 4.9$	7.2	0.7
	25	$1.19 : 0.30 = 4.0$	7.2	0.6
	30	$1.25 : 0.38 = 3.3$	7.2	0.5
	35	$1.45 : 0.45 = 2.8$	7.2	0.4
	B (in kG)	$\Delta\rho_{80\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{80\,K}$ (C2)	C1/C2
Sn polycrystal	300	$0.23 : 0.02 = 11$	$1 : 0.22 = 4.5$	2.4

Table 5. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{T_1} : \Delta\rho_{T_2}$) at a constant B compared with the ratios of the electric resistance ($r_{T_2} : r_{T_1}$) at $B = 0$; different temperature pairs T_1 and T_2 are considered. Metal samples of Pb and Sb are taken into account [1, 10].

	B (in kG)	$\Delta\rho_{80\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{80\,K}$ (C2)	C1/C2
Pb polycrystal	300	$0.05 : 0.01 = 5.0$	$1 : 0.25 = 4.0$	1.3
	B (in kG)	$\Delta\rho_{14\,K} : \Delta\rho_{20.4\,K}$ (C1)	$r_{20.4\,K} : r_{14\,K}$ (C2)	C1/C2
Pb polycrystal	8	$0.017 : 0.0026 = 6.5$	$0.02965 : 0.01052 = 2.8$	2.3
	15.8	$0.060 : 0.0078 = 7.7$	2.8	2.8
	24.4	$0.12 : 0.018 = 6.7$	2.8	2.4
	31.9	$0.19 : 0.031 = 6.1$	2.8	2.2
	39.8	$0.27 : 0.047 = 5.7$	2.8	2.0
	B (in kG)	$\Delta\rho_{4.2\,K} : \Delta\rho_{19\,K}$ (C1)	$r_{14\,K} : r_{4.2\,K}$ (C2)	C1/C2
Pb polycrystal	8	$12 : 0.017 = 706$	$0.01052 : 0.00175 = 60.1$	11.7
	15.8	$37 : 0.060 = 617$	60.1	10.3
	24.4	$74 : 0.12 = 612$	60.1	10.2
	31.9	$116 : 0.19 = 610$	60.1	10.1
	39.8–40	$174 : 0.27 = 644$	60.1	10.7
	B (in kG)	$\Delta\rho_{195\,K} : \Delta\rho_{291\,K}$ (C1)	$r_{291\,K} : r_{195\,K}$ (C2)	C1/C2
Sb polycrystal	100	$1.7 : 0.8 = 2.1$	$1 : 0.67 = 1.5$	1.4
	200	$4.8 : 2.2 = 2.2$	1.5	1.5
	300	$8.0 : 3.5 = 2.3$	1.5	1.5
	B (in kG)	$\Delta\rho_{80\,K} : \Delta\rho_{195\,K}$ (C1)	$r_{195\,K} : r_{80\,K}$ (C2)	C1/C2
Sb polycrystal	100	$7.0 : 1.7 = 4.1$	$0.67 : 0.3 = 2.2$	1.9
	200	$23 : 4.8 = 4.8$	2.2	2.2
	300	$40 : 8.0 = 5.0$	2.2	2.3

ratios of the r factors taken for the same temperatures, in analogy with the transversal magnetoresistance case.

Table 6 indicates that the rule

$$\mathrm{C1} \approx \mathrm{C2} \tag{11}$$

is especially well satisfied in that table. It can be noticed that for the longitudinal magnetoresistance the behaviour predicted by the rule (11) applies to very low temperatures, too. It should be noted that calculations were performed solely with the aid of the experimental data listed in [1].

Table 6. Experimental ratios of longitudinal magnetoresistance ($\Delta\rho_{T_1} : \Delta\rho_{T_2}$) at a constant B (labelled by C1) compared with the ratios of the electric resistance ($r_{T_2} : r_{T_1}$) at $B = 0$ (labelled by C2). Different temperature pairs T_1 and T_2 are considered for the metal samples of Na, Al, Pt, Ag, and In [10, 11].

	B (in kG)	C1	C2	C1/C2
		$\Delta\rho_{20.4\,\mathrm{K}} : \Delta\rho_{78\,\mathrm{K}}$	$r_{78\,\mathrm{K}} : r_{20.4\,\mathrm{K}}$	
Na	16.5	$0.029 : 0.0006 = 48$	$0.189 : 0.0043 = 44$	1.09
		$\Delta\rho_{20.4\,\mathrm{K}} : \Delta\rho_{77.2\,\mathrm{K}}$	$r_{77.2\,\mathrm{K}} : r_{20.4\,\mathrm{K}}$	
Al	23.5	$0.0155 : 0.0067 = 2.3$	$0.161 : 0.0676 = 2.4$	0.96
		$\Delta\rho_{14.2\,\mathrm{K}} : \Delta\rho_{20.4\,\mathrm{K}}$	$r_{20.4\,\mathrm{K}} : r_{14.2\,\mathrm{K}}$	
Al	23.5	$0.0316 : 0.0155 = 2.0$	$0.0676 : 0.0668 = 1.0$	2.0
		$\Delta\rho_{14.0\,\mathrm{K}} : \Delta\rho_{20.4\,\mathrm{K}}$	$r_{20.4\,\mathrm{K}} : r_{14.0\,\mathrm{K}}$	
Pt	33.4	$0.333 : 1.146 = 2.3$	$0.0066 : 0.0034 = 1.9$	1.2
		$\Delta\rho_{4.22\,\mathrm{K}} : \Delta\rho_{14.0\,\mathrm{K}}$	$r_{14.0\,\mathrm{K}} : r_{4.22\,\mathrm{K}}$	
Pt	33.4	$0.405 : 0.333 = 1.2$	$0.0034 : 0.0024 = 1.4$	0.86
		$\Delta\rho_{10\,\mathrm{K}} : \Delta\rho_{20\,\mathrm{K}}$	$r_{20\,\mathrm{K}} : r_{10\,\mathrm{K}}$	
Ag	20	$0.288 : 0.190 = 1.5$	$0.0096 : 0.0067 = 1.4$	1.1
		$\Delta\rho_{5\,\mathrm{K}} : \Delta\rho_{10\,\mathrm{K}}$	$r_{10\,\mathrm{K}} : r_{5\,\mathrm{K}}$	
Ag	20	$0.302 : 0.288 = 1.05$	$0.0067 : 0.0064 = 1.05$	1.0
		$\Delta\rho_{14\,\mathrm{K}} : \Delta\rho_{20.4\,\mathrm{K}}$	$r_{20.4\,\mathrm{K}} : r_{10\,\mathrm{K}}$	
In	25.74	$0.101 : 0.045 = 2.2$	$0.023 : 0.0105 = 2.2$	1.0

5. Temperature Properties of Magnetoresistance Examined in Dependence on Direction of the Magnetic Field

Three crystal samples of the metallic beryllium (Be 3, Be 4 and Be 8) were examined experimentally in [15]. This has been done for a set of three temperatures:

$$t_{(1)} \cong -183\,^{\circ}\mathrm{C}, \tag{12}$$

$$t_{(2)} \cong -195\,^{\circ}\mathrm{C}, \tag{13}$$

$$t_{(3)} \cong -253\,^{\circ}\mathrm{C}, \tag{14}$$

which were approximately the same for each sample.

The ratios of the electric resistance measured at temperatures listed in (12)–(14) were as follows [12]:

$$\frac{r_{t_{(1)}}}{r_{t_{(2)}}} = \frac{0.02422}{0.01443} = 1.68, \tag{15}$$

$$\frac{r_{t_{(2)}}}{r_{t_{(3)}}} = \frac{0.01443}{0.00251} = 5.7 \tag{16}$$

for the sample Be 3,

$$\frac{r_{t_{(1)}}}{r_{t_{(2)}}} = \frac{0.02775}{0.01717} = 1.62, \tag{17}$$

$$\frac{r_{t_{(2)}}}{r_{t_{(3)}}} = \frac{0.01717}{0.00896} = 4.3 \tag{18}$$

for the sample Be 4, and

$$\frac{r_{t_{(1)}}}{r_{t_{(2)}}} = \frac{0.02467}{0.01515} = 1.63, \tag{19}$$

$$\frac{r_{t_{(2)}}}{r_{t_{(3)}}} = \frac{0.01515}{0.00243} = 6.2 \tag{20}$$

for the sample Be 8.

In Tables 7–9, we calculated the ratios of magnetoresistance measured at different temperature pairs for three directions of the magnetic field in each sample case. According to (2), these data are compared with the electric resistance ratios given in (15)–(20) in Table 11.

6. Summary

The aim of the paper was to present and check a rule concerning the temperature dependence of the magnetoresistance $\Delta\rho/\rho$ in metals. This rule is based on the result that any ratio $\Delta\rho/\rho$ is proportional both to the

B (in kG)	$\Delta\rho_{\parallel}^{(t_b)} : \Delta\rho_{\parallel}^{(t_a)}$	$\Delta\rho_{\perp}^{(t_b)} : \Delta\rho_{\perp}^{(t_a)}$	$\Delta\rho_{\max}^{(t_b)} : \Delta\rho_{\max}^{(t_a)}$
	(i) $t_a = t_1$, $t_b = t_2$; $t_1 > t_2$ see Section 5		
3.4	$4.94 : 3.59 = 1.38$	$3.37 : 2.47 = 1.36$	$5.69 : 3.94 = 1.44$
6.8	$12.88 : 10.01 = 1.29$	$10.13 : 6.88 = 1.47$	$17.4 : 11.85 = 1.47$
10.1	$22.55 : 17.78 = 1.27$	$19.7 : 13.19 = 1.49$	$34.16 : 22.74 = 1.50$
11.7	$27.58 : 22.13 = 1.25$	$25.4 : 16.8 = 1.51$	$44.8 : 29.44 = 1.50$
	(ii) $t_a = t_2$, $t_b = t_3$; $t_2 > t_3$ see Section 5		
3.4	$12.71 : 4.94 = 2.57$	$7.57 : 3.37 = 2.25$	$19.56 : 5.69 = 3.4$
6.8	$32.89 : 12.88 = 2.55$	$23.59 : 10.13 = 2.33$	$58.7 : 17.4 = 3.4$
10.1	$60.59 : 22.55 = 2.69$	$47.51 : 19.7 = 2.41$	$113.2 : 34.16 = 3.3$
11.7	$76.77 : 27.58 = 2.78$	$61.93 : 25.4 = 2.44$	$145.9 : 44.18 = 3.3$

Table 7. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{t_2} : \Delta\rho_{t_1}$ and $\Delta\rho_{t_3} : \Delta\rho_{t_2}$) at a constant B considered for three directions of the magnetic field [parallel to the hexagonal symmetry axis ($\parallel$), perpendicular to the same axis ($\perp$), and in direction of a maximal magnetoresistance (max)] in the Be 3 sample [15]. These ratios are compared with those of the electric resistance at $B = 0$ in Table 11.

B (in kG)	$\Delta\rho_{\parallel}^{(t_b)} : \Delta\rho_{\parallel}^{(t_a)}$	$\Delta\rho_{\perp}^{(t_b)} : \Delta\rho_{\perp}^{(t_a)}$	$\Delta\rho_{\max}^{(t_b)} : \Delta\rho_{\max}^{(t_a)}$
	(i) $t_a = t_1,\ t_b = t_2,\ t_1 > t_2$; see Section 5		
3.4	5.01 : 3.72 = 1.34	3.28 : 2.46 = 1.33	5.57 : 4.00 = 1.39
10.1	23.37 : 18.95 = 1.26	19.00 : 13.16 = 1.44	33.6 : 23.18 = 1.45
11.7	29.1 : 23.60 = 1.23	24.35 : 16.8 = 1.45	43.28 : 29.76 = 1.45
	(ii) $t_a = t_2,\ t_b = t_3,\ t_2 > t_3$; see Section 5		
3.4	11.02 : 5.01 = 2.20	6.22 : 3.28 = 1.90	15.77 : 5.57 = 2.83
6.8	27.23 : 13.49 = 2.02	18.65 : 19.71 = 1.92	46.88 : 17.01 = 2.76
10.1	48.15 : 23.97 = 2.01	36.2 : 19 = 1.91	90.7 : 33.61 = 2.70
11.7	63.9 : 29.1 = 2.20	46.8 : 24.35 = 1.92	116.7 : 43.28 = 2.70

Table 8. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{t_2} : \Delta\rho_{t_1}$ and $\Delta\rho_{t_3} : \Delta\rho_{t_2}$) at a constant B considered for three directions of the magnetic field [parallel to the hexagonal symmetry axis ($\parallel$), perpendicular to the same axis ($\perp$), and in direction of a maximal magnetoresistance (max)] in the Be 4 sample [15]. These ratios are compared with those of the electric resistance at $B = 0$ in Table 11.

B (in kG)	$\Delta\rho_{\parallel}^{(t_b)} : \Delta\rho_{\parallel}^{(t_a)}$	$\Delta\rho_{\perp}^{(t_b)} : \Delta\rho_{\perp}^{(t_a)}$	$\Delta\rho_{\max}^{(t_b)} : \Delta\rho_{\max}^{(t_a)}$
	(i) $t_a = t_1,\ t_b = t_2;\ t_1 > t_2$; see Section 5		
3.4	4.83 : 3.49 = 1.40	3.65 : 2.58 = 1.60	5.69 : 3.91 = 1.46
6.8	13.17 : 9.97 = 1.32	10.93 : 7.37 = 1.48	17.46 : 11.75 = 1.49
10.1	23.08 : 18.02 = 1.28	21.23 : 14.17 = 1.50	34.33 : 23.07 = 1.49
11.7	28.39 : 22.32 = 1.27	27.07 : 18.13 = 1.49	43.52 : 29.6 = 1.47
12.2	30.25 : 23.83 = 1.27	29.15 : 19.46 = 1.50	47.1 : 31.93 = 1.48
	(ii) $t_a = t_2,\ t_b = t_3;\ t_2 > t_3$; see Section 5		
2.3	7.30 : 3.49 = 2.09	4.68 : 1.99 = 2.35	10.74 : 3 = 3.58
3.4	12.38 : 4.89 = 2.53	8.94 : 3.65 = 2.45	20.18 : 5.69 = 3.55
6.8	31.59 : 13.17 = 2.40	26.83 : 10.93 = 2.45	58.92 : 17.46 = 3.37
10.1	59.78 : 23.08 = 2.59	54.28 : 21.23 = 2.56	117.2 : 34.33 = 3.41
11.7	75.59 : 28.39 = 2.66	69.94 : 27.07 = 2.58	150.5 : 43.52 = 3.46
12.2	80.91 : 30.25 = 2.67	75.19 : 29.15 = 2.58	161.6 : 47.1 = 3.43

Table 9. Experimental ratios of transversal magnetoresistance ($\Delta\rho_{t_2} : \Delta\rho_{t_1}$ and $\Delta\rho_{t_3} : \Delta\rho_{t_2}$) at a constant B considered for three directions of the magnetic field [parallel to the hexagonal symmetry axis ($\parallel$), perpendicular to the same axis ($\perp$), and in direction of a maximal magnetoresistance (max)] in Be 8 sample [15]. These ratios are compared with those of the electric resistance at $B = 0$ in Table 11.

Table 10. Average value of the magnetoresistance ratio calculated in a metal for a given B at two different temperatures (C1) divided by the reciprocal value of the electric resistance ratio at $B = 0$ for the same temperature (C2). This average ratio is labelled by $\mathrm{C1/C2_{av}}$. Only the data listed in Tables 1 – 5 are taken into consideration.

Metal	$\mathrm{C1/C2_{av}}$	Metal	$\mathrm{C1/C2_{av}}$	Metal	$\mathrm{C1/C2_{av}}$
Be	1.2	Cu	1.4	Sn	2.4
Mg	2.2	Au	1.3		
Ba	1.2	Zn	1.8	Pb	6.0
Al	1.6	Cd	2.2	Sb	1.8
W	1.5	Hg	1.4		
Re	1.4				
Rh	1.0	Ga	1.9		
Pt	1.3	In	1.7		

magnetic induction B, as well as the relaxation time $\tau(0)$ possessed by a metal in the absence of the magnetic field [10]. In consequence, the ratio of two $\Delta\rho/\rho$ corresponding to the same B but considered at two different temperatures is defined by the ratio of two relaxation times $\tau(0)$ considered at $B = 0$ for the same temperatures pair; see (2). If the ratio of the relaxation times is known experimentally, we obtain immediately an approach to the dependence of the magnetoresistance ratio of (2) on the temperature ratio T_1/T_2.

Since both the relaxation time and the magnetoresistance are accessible experimentally in many cases, the numerical calculations which illustrate a referencee between the electric resistance and magnetoresistance can be done on the basis of the experimental data alone.

A general view on the rule of equal magnetoresistance ratios and those of the electric resistance is summarized in Table 10 for different metal cases. The rule, which seems to hold better for not very low temperatures, does work rather perfectly for such metals like rhodium, platinium, beryllium, and barium.

A supplementary examination is performed for the case of the directional effects of magnetoresistance for three crystal samples of beryllium; see Tables 7 – 9. We demonstrate that also in this case the ratios of magnetoresistance measured for different temperatures can approach the reciprocal ratios of the electric resistance obtained for the same temperature pairs; see Section 5 and Table 11 where a thorough comparison of the magnetoresistance data with those obtained from the electric resistance at $B = 0$ is presented.

Table 11. Magnetoresistance ratios of Tables 7–9 compared with the corresponding ratios of the electric resistance at $B = 0$; see (15)–(20). The data entering the calculations refer to a given temperature pair.

Be 3; temperature pair (i) considered in Table 7. The data taken from Table 7 are labelled by C1, those taken from (15) are C2 = 1.68.

B (in kG)	C1$\parallel$: C2	C1$\perp$: C2	$C1_{max}$: C2
3.4	1.38 : C2 = 0.82	1.36 : C2 = 0.81	1.44 : C2 = 0.86
6.8	1.29 : C2 = 0.77	1.47 : C2 = 0.88	1.47 : C2 = 0.88
10.1	1.27 : C2 = 0.76	1.49 : C2 = 0.89	1.50 : C2 = 0.89
11.7	1.25 : C2 = 0.74	1.51 : C2 = 0.90	1.50 : C2 = 0.89

Be 3; temperature pair (ii) considered in Table 7. The data taken from Table 7 are labelled by C1, those taken from (16) are C2 = 5.7.

B (in kG)	C1$\parallel$: C2	C1$\perp$: C2	$C1_{max}$: C2
3.4	2.57 : C2 = 0.45	2.25 : C2 = 0.39	3.4 : C2 = 0.60
6.8	2.55 : C2 = 0.45	2.33 : C2 = 0.41	3.4 : C2 = 0.60
10.1	2.69 : C2 = 0.48	2.41 : C2 = 0.42	3.3 : C2 = 0.58
11.7	2.78 : C2 = 0.49	2.44 : C2 = 0.43	3.3 : C2 = 0.58

Be 5; temperature pair (i) in Table 8. The data taken from Table 8 are labelled by C1, those taken from (17) are C2 = 1.62.

B (in kG)	C1$\parallel$: C2	C1$\perp$: C2	$C1_{max}$: C2
3.4	1.34 : C2 = 0.83	1.33 : C2 = 0.82	1.39 : C2 = 0.86
10.1	1.26 : C2 = 0.78	1.44 : C2 = 0.89	1.45 : C2 = 0.90
11.7	1.23 : C2 = 0.76	1.45 : C2 = 0.90	1.45 : C2 = 0.90

Be 5, temperature pair (ii) in Table 8. The data taken from Table 8 are labelled by C1, those taken from (18) are C2 = 4.3.

B (in kG)	C1$\parallel$: C2	C1$\perp$: C2	$C1_{max}$: C2
3.4	2.20 : C2 = 0.51	1.90 : C2 = 0.44	2.83 : C2 = 0.66
6.8	2.02 : C2 = 0.47	1.92 : C2 = 0.45	2.76 : C2 = 0.64
10.1	2.01 : C2 = 047	1.91 : C2 = 0.44	2.70 : C2 = 0.63
11.7	2.20 : C2 = 0.51	1.92 : C2 = 0.45	2.70 : C2 = 0.63

Be 8; temperature pair (i) in Table 9. The ratios of magnetoresistance taken from Table 9 are labelled by C1, the resistance ratio of (19) is C2 = 1.63.

B (in kG)	C1$\parallel$: C2	C1$\perp$: C2	$C1_{max}$: C2
3.4	1.40 : C2 = 0.86	1.60 : C2 = 0.98	1.49 : C2 = 0.91
6.8	1.32 : C2 = 0.81	1.48 : C2 = 0.90	1.49 : C2 = 0.91
10.1	1.28 : C2 = 0.79	1.50 : C2 = 0.92	1.49 : C2 = 0.91
11.7	1.27 : C2 = 0.78	1.49 : C2 = 0.91	1.47 : C2 = 0.90
12.2	1.27 : C2 = 0.78	1.50 : C2 = 0.92	1.48 : C2 = 0.90

Be 8; temperature pair (ii) in Table 9. The ratios of magnetoresistance from Table 9 are labelled by C1, the resistance ratio of (20) is C2 = 6.2.

B (in kG)	C1$\parallel$: C2	C1$\perp$: C2	$C1_{max}$: C2
2.3	2.09 : C2 = 0.34	2.35 : C2 = 0.38	3.58 : C2 = 0.58
3.4	2.53 : C2 = 0.41	2.45 : C2 = 0.40	3.55 : C2 = 0.57
6.8	2.40 : C2 = 0.39	2.45 : C2 = 0.40	3.37 : C2 = 0.54
10.1	2.59 : C2 = 0.42	2.56 : C2 = 0.41	3.41 : C2 = 0.55
11.7	2.66 : C2 = 0.43	2.58 : C2 = 0.42	3.46 : C2 = 0.56
12.2	2.67 : C2 = 0.43	2.58 : C2 = 0.42	3.43 : C2 = 0.55

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