Inner-Resonance for a Coupled Oscillator Arising in a Cubic Nonlinear Packaging System with Critical Component

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In this paper, a dynamic model was proposed for a cushioning packaging system. Conditions for resonance were obtained by applying the variational iteration method (VIM), which should be avoided in the cushioning packaging design.

Key words: Resonance; Cushion; Nonlinear Oscillator.

1. Introduction

Packaged products can be potentially damaged by dropping, and it is very important to investigate the oscillation process of the packaging system. In the past, great efforts have been made in this special field [1, 2]. In order to prevent any damage, a critical component and a cushioning packaging are always included in a packaging system, as shown in Figure 1 [3 – 5]. Here the coefficients m_1 and m_2 denote the mass of the critical component and the main part of product, respectively, while k_1 and k_2 are the coupling stiffness of the critical component and that of the cushioning pad, respectively.

It is very important to investigate the condition for resonance. However, the oscillation in the packaging system is of inherent nonlinearity [3 – 5], and it remains the problem to obtain the resonance condition for a nonlinear packaging system, especially for a multi-freedom degree nonlinear cushioning packaging system. The governing equations of the cushioning packaging system can be expressed as

$$\begin{aligned} & m_1\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+k_1(x-y)=0, \\ & x(0)=0,\ \dot{x}(0)=\sqrt{2gh}, \\ & m_2\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+k_2y+r_2y^3-k_1(x-y)=0, \\ & y(0)=0,\ \dot{y}(0)=\sqrt{2gh}. \end{aligned} \tag{1}$$

Here the coefficient r_2 denotes the incremental rate of the cushioning pad, and h is the dropping height. While x and y are the response displacement of the critical component and the product, respectively.

By introducing the parameters $T_0=\sqrt{m_2/k_2}$, $L=\sqrt{k_2/r_2}$, and setting $X=\frac{x}{L}$, $Y=\frac{y}{L}$, $T=\frac{t}{T_0}$, (1) can be equivalently written in the following form:

$$\begin{aligned} & \frac{\mathrm{d}^2X}{\mathrm{d}T^2}+\omega_{01}^2X-\omega_{01}^2Y, \\ & X(0)=0,\ X'(0)=\frac{T_0}{L}\sqrt{2gh}, \end{aligned} \tag{2}$$

Dr. Jun Wang and Fang Duan contributes equally to this paper as co-first authors.

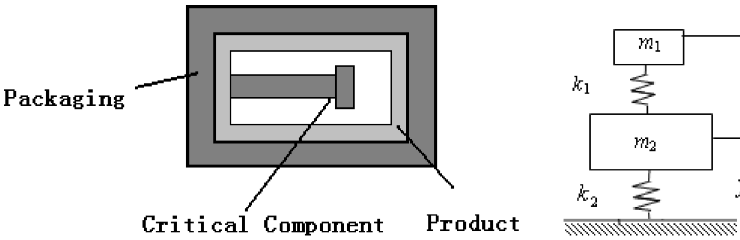

Fig. 1. Model of packaging system with critical component.

$$\frac{d^2Y}{dT^2}+\omega_{02}^2 Y+Y^3+\left(1-\omega_{02}^2\right),$$
$$X=0,\ Y(0)=0,\ Y'(0)=\frac{T_0}{L}\sqrt{2gh},$$

where

$$\omega_{01}=\lambda_1, \tag{3}$$
$$\omega_{02}=\sqrt{1+\lambda_1^2\lambda_2}, \tag{4}$$
$$\lambda_1=\omega_1/\omega_2, \tag{5}$$
$$\lambda_2=m_1/m_2, \tag{6}$$
$$\omega_1=\sqrt{k_1/m_1}, \tag{7}$$
$$\omega_2=\sqrt{k_2/m_2}. \tag{8}$$

2. Variational Iteration Method

There are various methods existing for solving nonlinear differential equations [6–22], including the famous variational iteration method (VIM). The variational iteration method, first proposed by He, has been widely applicated in solving many different kinds of nonlinear equations [14–20], and is especially effective in solving nonlinear vibration problems with approximations [21, 22]. Applying the variational iteration method [6, 23], the following iteration formulae can be constructed:

$$Y_{n+1}(t)=Y_n(t)+\int_0^t \lambda\left(LY_n(s)+N\breve{Y}_n(s)-g(s)\right)\mathrm{d}s. \tag{9}$$

Here the subscript n denotes the nth-order approximation, and λ is called a general Lagrange multiplier, which can be identified optimally via the variational theory. $\breve{Y}_n(s)$ is considered as a restricted variation, i.e., $\delta\breve{Y}_n(s)=0$. Recall the second part of (2), we have the following stationary conditions:

$$\begin{aligned}&\ddot{\lambda}(s)+\omega_{02}^2\lambda(s)=0,\\&\lambda(s)|_{s=t}=0,\\&1-\dot{\lambda}(s)|_{s=t}=0.\end{aligned} \tag{10}$$

Therefore, the multiplier, can be identified as

$$\lambda=\frac{1}{\omega_{02}}\sin\omega_{02}(s-t). \tag{11}$$

Then the following iteration formula can be constructed for the second part of (2):

$$\begin{aligned}Y_1=Y_0+\frac{1}{\omega_{02}}\int_0^t &\sin\omega_{02}(s-t)\\ &\cdot\left[Y_0''+\omega_{02}^2Y_0+Y_0^3+\left(1-\omega_{02}^2\right)X_0\right]\mathrm{d}s.\end{aligned} \tag{12}$$

Beginning with the initial solutions

$$\begin{aligned}X_0&=A_1\sin(\Omega_1 t),\\ Y_0&=A_2\sin(\Omega_2 t),\end{aligned} \tag{13}$$

we have

$$\begin{aligned}Y_1&=\frac{A_1\left(1-\omega_{02}^2\right)}{\Omega_1^2-\omega_{02}^2}\sin(\Omega_1 t)+A_2\sin(\Omega_2 t)\\&-\frac{A_2^3}{4\left(\Omega_2^2-\omega_{02}^2\right)}\sin(3\Omega_2 t)\\&+\left[\frac{A_1\Omega_1\left(1-\omega_{02}^2\right)}{\omega_{02}\left(\Omega_1^2-\omega_{02}^2\right)}-\frac{3A_2^3\Omega_2}{4\omega_{02}\left(9\Omega_2^2-\omega_{02}^2\right)}\right]\sin(\omega_{02}t).\end{aligned} \tag{14}$$

Substituting (14) into (2), yields

$$\begin{aligned}X_1&=\frac{A_1\omega_{01}^2\left(\omega_{02}^2-1\right)}{\left(\Omega_1^2-\omega_{01}^2\right)\left(\Omega_1^2-\omega_{02}^2\right)}\sin(\Omega_1 t)\\&+\frac{A_2\omega_{01}^2}{\Omega_2^2-\omega_{01}^2}\sin(\Omega_2 t)+\frac{A_2^3\omega_{01}^2}{4\left(9\Omega_2^2-\omega_{01}^2\right)\left(\Omega_2^2-\omega_{02}^2\right)}\\&\cdot\sin(3\Omega_2 t)+\frac{\omega_{01}^2}{\omega_{01}^2-\omega_{02}^2}\left[\frac{A_1\Omega_1\left(1-\omega_{02}^2\right)}{\omega_{02}\left(\Omega_1^2-\omega_{02}^2\right)}\right.\\&\left.-\frac{3A_2^3\Omega_2}{4\omega_{02}\left(9\Omega_2^2-\omega_{02}^2\right)}\right]\sin(\omega_{02}t),\end{aligned} \tag{15}$$

where

$$A_1 = \frac{T_0}{\Omega_1 L}\sqrt{2gh}, \quad A_2 = \frac{T_0}{\Omega_1 L}\sqrt{2gh}. \tag{16}$$

3. Resonance

The resonance can be expected when one of the following conditions meet:

$$\Omega_1 = \sqrt{\frac{1}{2}\left(\omega_{02}^2 + \omega_{02}\sqrt{\omega_{02}^2 - 4\omega_{01}^2}\right)}, \tag{17}$$

$$\Omega_1 = \sqrt{\frac{1}{2}\left(\omega_{02}^2 - \omega_{02}\sqrt{\omega_{02}^2 - 4\omega_{01}^2}\right)}, \tag{18}$$

$$\Omega_1 = \omega_{01}, \tag{19}$$

$$\Omega_1 = \omega_{02}, \tag{20}$$

$$\Omega_2 = \sqrt{\omega_{02}^2 + \frac{3}{4}A_2^2}, \tag{21}$$

$$\Omega_2 = \omega_{02}, \tag{22}$$

$$\Omega_2 = \frac{1}{3}\omega_{01}, \tag{23}$$

$$\Omega_2 = \frac{1}{3}\omega_{02}, \tag{24}$$

$$\omega_{01} = \omega_{02}. \tag{25}$$

These conditions should be avoided during the cushioning packaging design procedure.

4. Results

To verify the proposed method, the dropping shock response acceleration of the critical component for a typical cushioning packaging system was calculated and compared with the numerical integration solutions using a built-in ODE-solver in MATLAB, as illustrated in Figure 2, showing good agreement. The results are obtained for the amounts $\lambda_1 = 0.5$ and $\lambda_2 = 0.1$.

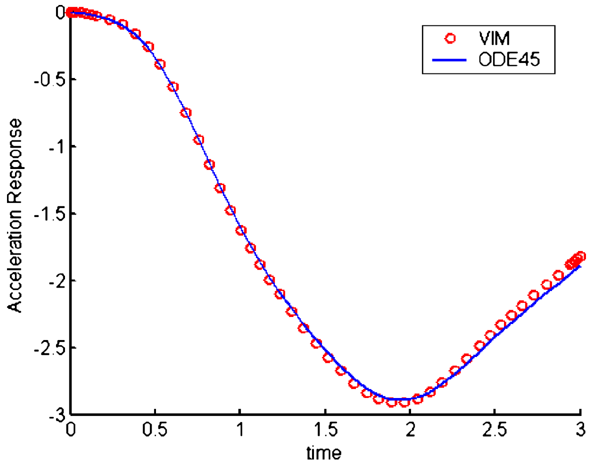

Fig. 2 (colour online). Comparison between the VIM solution and the solution obtained with the ODE-solver in MATLAB.

5. Conclusions

In order to prevent any damage, a critical component and a cushioning packaging are always included in a cushioning packaging system. The dropping damage evaluation parameters for a cushioning packaging system can be predicted by VIM. The conditions for resonance, which should be avoided in the cushioning packaging design procedure, were obtained using the variational iteration method.

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