Reductions and Solutions of Two Types of Coupled Nonlinear Evolution Equations in Optical Fibers and Fluid Dynamics

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Under investigation in this paper are the coupled nonlinear Schrödinger equations (CNLSEs) and coupled Burgers-type equations (CBEs), which are, respectively, a model for certain birefringent optical fibers Raman-scattering, Kerr and gain/loss effects, and a generalized model in fluid dynamics. Special attention should be paid to the existing claim that the solitons for the CNLSEs do not exist. Through certain dependent-variable transformations, the CNLSEs are reduced to a Manakov system and the CBEs are linearized. In that way, some new solutions of the CNLSEs and CBEs are obtained via symbolic computation. Especially the one-dark-soliton-like solutions for the CNLSEs have been found, against the existing claim.

Key words: Coupled Nonlinear Schrödinger Equations; Coupled Burgers-Type Equations; Dependent-Variable Transformation; Symbolic Computation; Soliton.

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1. Introduction

Nonlinear evolution equations (NLEEs) have been seen in plasma physics, optical fibers, fluid mechanics, etc. [1-5]. Methods have been explored to study the NLEEs [6-10], e.g., the inverse scattering method [11], Bäcklund transformation method [12], and Hirota bilinear method [13-15]. However, sometimes, it may be difficult to address the soliton problems with the methods mentioned above [16]. With certain transformations, some NLEEs may be transformed into the simpler ones, even the linear ones [17].

In this paper, we will deal with two types of the coupled NLEEs as follows:

(i) Describing the wave propagation in birefringent optical fibers with Raman-scattering, Kerr and gain/loss effects, the coupled nonlinear Schrödinger equations (CNLSEs) appear as [18]

$$\mathbf{i}\varphi_t + \chi \varphi_{xx} \mp 2\mu \left(\frac{|\varphi|^2 + |\omega|^2}{|\varphi|^2 |\omega|^2}\right) \varphi = R_1, \qquad (1a)$$

$$i\omega_t + \chi \omega_{xx} \mp 2\mu \left(\frac{|\varphi|^2 + |\omega|^2}{|\varphi|^2 |\omega|^2}\right) \omega = R_2, \quad (1b)$$

where x is the scaled space and t is the scaled time, while the perturbative terms R_1 and R_2 , and the real parameters χ and μ are respectively defined as

$$R_{1} = \frac{2\chi\varphi_{x}^{2}}{\varphi}, R_{2} = \frac{2\chi\omega_{x}^{2}}{\omega},$$

$$\chi = \frac{\alpha + \delta}{\alpha(\alpha - \delta)}, \mu = \frac{\alpha^{2} - \delta^{2}}{\alpha^{2}\delta},$$
(2)

with α and δ as two real constants. Using the Zakharov–Shabat (ZS) dressing method [19], author of [18] has obtained the solutions of (1) as

$$\varphi(x,t) = -\frac{1 + ABC}{DB}, \quad \omega(x,t) = -\frac{1 + ABC}{EB}, \quad (3)$$

where

$$A = \frac{\alpha \mu (|D|^2 + |E|^2)}{(\alpha^2 - \delta^2)(\rho - \sigma)^2}, \ B = \mathrm{e}^{\rho(\delta - \alpha)x - \mathrm{i}\frac{\rho^2}{\alpha}(\delta^2 - \alpha^2)t},$$

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$$C = e^{-\sigma(\delta - \alpha)x + i\frac{\sigma^2}{\alpha}(\delta^2 - \alpha^2)t},$$
(4)

where *D* and *E* are complex constants, and ρ and σ are imaginary constants with $\rho^* = -\sigma$. Based on Solutions (3), $\varphi(x,t)$ and $\omega(x,t)$ satisfy $\omega(x,t) = \frac{D}{E}\varphi(x,t)$. The Lax pair of (1) has been listed as [18]

$$L = \begin{pmatrix} 0 & \frac{\alpha - \delta}{\varphi} & \frac{\alpha - \delta}{\omega} \\ \pm \frac{\delta - \alpha}{\varphi^*} & 0 & 0 \\ \pm \frac{\delta - \alpha}{\omega^*} & 0 & \delta \end{pmatrix} + \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \delta \end{pmatrix} \frac{\partial}{\partial x},$$
(5a)

$$M = 2 \begin{pmatrix} h_x^{(1)} & -\frac{\varphi_x}{\varphi^2} & -\frac{\omega_x}{\omega^2} \\ \mp \frac{\varphi_x^*}{\varphi^{*2}} & h_x^{(2)} & h_x^{(3)} \\ \mp \frac{\omega_x^*}{\omega^{*2}} & h_x^{(4)} & h_x^{(5)} \end{pmatrix} + i\frac{\partial^2}{\partial x^2},$$
(5b)

where

$$h_{x}^{(1)} = \mp \frac{\alpha - \delta}{\alpha} \left(\frac{|\varphi|^{2} + |\omega|^{2}}{|\varphi|^{2}|\omega|^{2}} \right),$$

$$h_{x}^{(2)} = \mp \frac{\delta - \alpha}{\delta |\varphi|^{2}}, \quad h_{x}^{(3)} = \mp \frac{\delta - \alpha}{\delta \varphi^{*}\omega},$$

$$h_{x}^{(4)} = \mp \frac{\delta - \alpha}{\delta \varphi \omega^{*}}, \quad h_{x}^{(5)} = \mp \frac{\delta - \alpha}{\delta |\omega|^{2}}.$$

(6)

Especially, author of [18] has claimed that System (1) has 'no soliton solution', on which we are going to present our discussion and disagreement.

(ii) As a generalized model in fluid dynamics, the coupled Burgers-type equations (CBEs) [17, 20, 21] may describe certain phenomena, e.g., the sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids under the effect of gravity. The CBEs can be written as [20]

$$u_{\tau} = \frac{1}{2}u_{\varepsilon\varepsilon} + \frac{1}{2}v_{\varepsilon\varepsilon} + 3uu_{\varepsilon} + 5u_{\varepsilon}v + 3uv_{\varepsilon} + 5vv_{\varepsilon} - 4u^{3}$$
$$-4u^{2}v + 4uv^{2} + 4v^{3}, \qquad (7a)$$

$$v_{\tau} = \frac{1}{2}u_{\varepsilon\varepsilon} + \frac{1}{2}v_{\varepsilon\varepsilon} + 5uu_{\varepsilon} + 3u_{\varepsilon}v + 5uv_{\varepsilon} + 3vv_{\varepsilon} + 4u^{3} + 4u^{2}v - 4uv^{2} - 4v^{3}, \tag{7b}$$

where ε is the scaled space and τ is the scaled time. Author of [20] has seen (7) to be integrable. In [21], another set of CBEs has been considered. Author of [17] has gained the multiple kink and singular kink solutions of (7) under the constraint $u(\varepsilon, \tau) = v(\varepsilon, \tau)$. The structure of this paper will be as follows. In Section 2, based on the symbolic computation [6, 7, 9, 10], we will demonstrate that (1) can be reduced into a Manakov system [22-27] and obtain some new analytic solutions. Especially, the one-dark-soliton-like solution will be given, against [18]'s claim. In Section 3, we will reduce (7) to a linear one and give their more general solutions than those in [17]. Section 4 will be our conclusions.

2. Reduction and New Solutions for (1)

On (1), we can give the following transformations:

$$\varphi(x,t) = \frac{1}{p}, \quad \omega(x,t) = \frac{1}{q}.$$
(8)

Substituting (8) into (1) yields

$$-ip_t - \chi p_{xx} \mp 2\mu \left(|p|^2 + |q|^2 \right) p = 0, \qquad (9a)$$

$$-iq_t - \chi q_{xx} \mp 2\mu \left(|p|^2 + |q|^2 \right) q = 0.$$
 (9b)

System (9) is a Manakov system, which is seen in the pulse propagation in two-mode optical fibres [22] and in the theory of soliton wavelength division multiplexing [23]. Furthermore, the bright and dark vector soliton solutions [24, 25], periodic solutions [26], and effects of an initial phase difference and interactions of two solitons with different amplitudes for System (9) [27] have also been investigated.

In order to get the solutions of (1), we can firstly obtain the solutions of (9), and then give the solutions of (1) through Transformations (8).

Case 1

In the following, we will construct the one-darksoliton-like solutions for (1), which is against [18]'s claim. Assuming that q = c p, where c is a complex constant, we can reduce (1) to the form of

$$ip_t + \chi p_{xx} + 2\mu (|c|^2 + 1)|p|^2 p = 0, \qquad (10)$$

which is a nonlinear Schrödinger equation [28]. Taking the transformation $p = \frac{g}{f}$, we can get the following bilinear form of (10):

$$(\mathbf{i}D_t + \chi D_x^2 - \lambda)g \cdot f = 0, \qquad (11a)$$

$$\left(D_x^2 - \frac{\lambda}{\chi}\right)f \cdot f - \frac{2\mu(|c|^2 + 1)}{\chi}gg^* = 0, \quad (11b)$$

where λ is a real constant. We assume that

$$f = 1 + \varepsilon f_1, \quad g = g_0(1 + \varepsilon g_1),$$
 (12)

where f_1 is a real function, g_0 and g_1 are complex functions, and ε is a parameter. Substituting (12) into (11), and collecting the coefficients of ε^0 , we obtain

$$ig_{0t} + \chi g_{xx} - \lambda g_0 = 0, \qquad (13a)$$

$$\lambda + 2\mu (|c|^2 + 1)g_0 g_0^* = 0.$$
(13b)

Solving (13), we get

$$g_0 = a e^{kx + i(\chi k^2 - \lambda)t}, \tag{14}$$

where *a* is an arbitrary complex constant, k = bi and $\lambda = -2|a|^2\mu(|c|^2 + 1)$. The coefficients of ε and ε^2 lead to the equations as follows:

$$(iD_t + \chi D_x^2 - \lambda)(g_0 \cdot f_1 + g_0 g_1 \cdot 1) = 0,$$
(15a)

$$\left(D_x^2 - \frac{\lambda}{\chi}\right)(1 \cdot f_1 + f_1 \cdot 1) = \frac{2\mu}{\chi}(|c|^2 + 1)|g_0|^2(g_1 + g_1^*),$$
(15b)

$$(iD_t + \chi D_x^2 - \lambda)(g_0g_1 \cdot f_1) = 0,$$
 (15c)

$$\left(D_x^2 - \frac{\lambda}{\chi}\right)(f_1 \cdot f_1) = \frac{2\mu}{\chi}(|c|^2 + 1)|g_0|^2|g_1|^2,$$
 (15d)

which can be solved to give

$$f_1 = d e^{\sqrt{\frac{2\lambda}{\chi}}x - 2b\sqrt{2\lambda\chi}},$$
(16a)

$$g_1 = -d \, \mathrm{e}^{\sqrt{\frac{2\lambda}{\chi}x - 2b\sqrt{2\lambda\chi}}},\tag{16b}$$

where *d* is a real constant. Without loss of generality, taking $\varepsilon = 1$, through the transformation $p = \frac{g}{f}$, we have a solution of *p* as

$$p = \frac{a e^{kx + i(\chi k^2 - \lambda)t} \left(1 - d e^{\sqrt{\frac{2\lambda}{\chi}}x - 2b\sqrt{2\lambda\chi}}\right)}{1 + d e^{\sqrt{\frac{2\lambda}{\chi}}x - 2b\sqrt{2\lambda\chi}}}.$$
 (17)

Based on Expressions (8), we can get

$$\varphi(x,t) = \frac{1+d e^{\sqrt{\frac{2\lambda}{\chi}x-2b\sqrt{2\lambda\chi}}}}{a e^{kx+i(\chi k^2-\lambda)t} \left(1-d e^{\sqrt{\frac{2\lambda}{\chi}x-2b\sqrt{2\lambda\chi}}}\right)},$$
(18a)
$$\omega(x,t) = \frac{1+d e^{\sqrt{\frac{2\lambda}{\chi}x-2b\sqrt{2\lambda\chi}}}}{c a e^{kx+i(\chi k^2-\lambda)t} \left(1-d e^{\sqrt{\frac{2\lambda}{\chi}x-2b\sqrt{2\lambda\chi}}}\right)},$$
(18b)

which are the one-dark-soliton-like solutions for (7), to be plotted in Figure 1.

As seen in Figure 1, the amplitude of the wave keeps invariant in the propagation, so they are the one-dark-soliton-like solutions for (1), against [18]'s claim that System (1) has 'no soliton solution'.

In the following, we will derive some other types of solutions for (1).

Case 2

As the procedure of deriving the solution of (9) in [25], we take

$$p = \frac{a\chi e^{\theta}}{\chi (k+k^*)^2 + \mu (|a|^2 + |b|^2) e^{(\theta+\theta^*)}},$$
(19a)

$$q = \frac{b\chi e^{s}}{\chi (k+k^{*})^{2} + \mu (|a|^{2} + |b|^{2}) e^{\theta + \theta^{*})}},$$
(19b)

where $\theta = kx + i\chi k^2 t$ and k is a complex constant. Through Transformations (8), we can give solutions of (1) as

$$\varphi(x,t) = \frac{1}{a} \Big[(k+k^*)^2 e^{-\theta} + \mu(|a|^2+|b|^2) e^{\theta^*} \Big], \quad (20a)$$

$$\omega(x,t) = \frac{1}{b} \Big[(k+k^*)^2 e^{-\theta} + \mu(|a|^2+|b|^2) e^{\theta^*} \Big]. \quad (20b)$$

If we assume that

$$k = \rho(\delta - \alpha), \ |D|^2 + |E|^2 = \chi(|a|^2 + |b|^2),$$
(21)

Solutions (20) are equivalent to Solutions (3) in [18].

Case 3

As the procedure of deriving the two-soliton solutions of (9) in [25], we take

$$p = \frac{a_1 e^{\theta_1} + a_2 e^{\theta_2} + e^{\theta_1 + \theta_2 + \eta_1} + e^{\theta_1 + \theta_2 + \theta_1^* + \gamma_1} + e^{\theta_1 + \theta_2 + \theta_2^* + \gamma_2}}{1 + e^{\theta_1 + \theta_1^* + \eta_1} + e^{\theta_1 + \theta_2^* + \eta_0} + e^{\theta_1^* + \theta_2 + \eta_0^*} + e^{\theta_2 + \theta_2^* + \eta_2} + \beta e^{\theta_1^* + \theta_1^* + \theta_2^* + \theta_2^*}},$$
(22a)

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$$q = \frac{b_1 e^{\theta_1} + b_2 e^{\theta_2} + e^{\theta_1 + \theta_2 + \theta_1^* + \gamma_3} + e^{\theta_1 + \theta_2 + \theta_2^* + \gamma_4}}{1 + e^{\theta_1 + \theta_1^* + \eta_1} + e^{\theta_1 + \theta_2^* + \eta_0} + e^{\theta_1^* + \theta_2 + \eta_0^*} + e^{\theta_2 + \theta_2^* + \eta_2} + \beta e^{\theta_1^1 + \theta_1^* + \theta_2^2 + \theta_2^*}},$$
(22b)



Fig. 1 (colour online). Onedark-soliton-like solutions via (18). Parameters in (a_i) (i = 1,2) are $\chi = 0.1$, $\mu = -0.1$, c = 1.2, b = -0.03, a = 0.1 + 0.1i, d = -0.1. Parameters in (b_i) , (c_i) and (d_i) are the same as those in (a_i) except for $\chi = 1$, $\mu = -1$ in (b_i) , b = -0.3 in (c_i) and $\mu = -1$ and b = -0.3 in (d_i) (i = 1,2).

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$$\begin{aligned} \theta_{1} &= k_{1}x + i\chi k_{1}^{2}, \quad \theta_{2} &= k_{2}x + i\chi k_{2}^{2}, \\ e^{\eta_{1}} &= \frac{\mu(|a_{1}|^{2} + |b_{1}|^{2})}{\chi(k_{1} + k_{1}^{*})^{2}}, \quad e^{\eta_{2}} &= \frac{\mu(|a_{2}|^{2} + |b_{2}|^{2})}{\chi(k_{2} + k_{2}^{*})^{2}}, \\ e^{\eta_{0}} &= \frac{\mu(a_{1}a_{2}^{*} + b_{1}b_{2}^{*})}{\chi(k_{1} + k_{2}^{*})^{2}}, \quad e^{\eta_{0}^{*}} &= \frac{\mu(a_{1}^{*}a_{2} + b_{1}^{*}b_{2})}{\chi(k_{1}^{*} + k_{2})^{2}}, \\ e^{\eta} &= \lambda_{1}a_{2}e^{\eta_{1}} + \lambda_{2}a_{1}e^{\eta_{0}^{*}}, \quad e^{\eta_{2}} &= \lambda_{3}a_{2}e^{\eta_{0}} + \lambda_{4}a_{1}e^{\eta_{2}}, \\ e^{\eta_{3}} &= \lambda_{1}b_{2}e^{\eta_{1}} + \lambda_{2}b_{1}e^{\eta_{0}^{*}}, \quad e^{\eta_{4}} &= \lambda_{3}b_{2}e^{\eta_{0}} + \lambda_{4}b_{1}e^{\eta_{2}}, \end{aligned}$$

$$\begin{aligned} \lambda_{1} &= \frac{(k_{2} - k_{1})(k_{1} + k_{1}^{*})}{(k_{1} + k_{1}^{*})(k_{2} + k_{1}^{*})}, \quad \lambda_{1} &= \frac{(k_{1} - k_{2})(k_{2} + k_{1}^{*})}{(k_{1} + k_{1}^{*})(k_{2} + k_{1}^{*})}, \\ \lambda_{3} &= \frac{(k_{2} - k_{1})(k_{1} + k_{2}^{*})}{(k_{1} + k_{2}^{*})(k_{2} + k_{2}^{*})}, \quad \lambda_{4} &= \frac{(k_{1} - k_{2})(k_{2} + k_{2}^{*})}{(k_{1} + k_{2}^{*})(k_{2} + k_{2}^{*})}, \\ \beta &= \frac{e^{\gamma_{1} + \gamma_{1}^{*}} + e^{\gamma_{3} + \gamma_{3}^{*}}}{e^{\eta_{1}}(k_{2} + k_{2}^{*})^{2}}, \end{aligned}$$
(23)

while k_1 , k_2 , a_1 , a_2 , b_1 , and b_2 are complex constants. Through Transformations (8), we can obtain new solutions of (1) as

$$\varphi(x,t) = \frac{1 + e^{\theta_1 + \theta_1^* + \eta_1} + e^{\theta_1 + \theta_2^* + \eta_0} + e^{\theta_1^* + \theta_2 + \eta_0^*} + e^{\theta_2 + \theta_2^* + \eta_2} + \beta e^{\theta_1^1 + \theta_1^* + \theta_2^2 + \theta_2^*}}{a_1 e^{\theta_1} + a_2 e^{\theta_2} + e^{\theta_1 + \theta_2 + \theta_1^* + \eta_1} + e^{\theta_1 + \theta_2 + \theta_2^* + \eta_2}},$$
(24a)

$$\omega(x,t) = \frac{1 + e^{\theta_1 + \theta_1^* + \eta_1} + e^{\theta_1 + \theta_2^* + \eta_0} + e^{\theta_1^* + \theta_2 + \eta_0^*} + e^{\theta_2 + \theta_2^* + \eta_2} + \beta e^{\theta_1^1 + \theta_1^* + \theta_2^2 + \theta_2^*}}{b_1 e^{\theta_1} + b_2 e^{\theta_2} + e^{\theta_1 + \theta_2 + \theta_1^* + \gamma_3} + e^{\theta_1 + \theta_2 + \theta_2^* + \gamma_4}}.$$
(24b)

3. Reduction and New Solutions of (7)

The addition of (7a) and (7b) leads to

$$(u+v)_{\tau} = (u+v)_{\varepsilon\varepsilon} + 8(u+v)(u+v)_{\varepsilon}.$$
(25)

On the other hand, the substraction of (7a) and (7b) gives

$$(u-v)_{\tau} = -2(u+v)_{\varepsilon}(u-v) - 8(u+v)^2(u-v).$$
 (26)

We assume that

$$u + v = f, \ u - v = g.$$
 (27)

Then, (25) and (26) can be rewritten as

$$f_{\tau} = f_{\varepsilon\varepsilon} + 8ff_{\varepsilon}, \tag{28a}$$

$$g_{\tau} = -2f_{\varepsilon}g - 8f^2g. \tag{28b}$$

Solving (28b), we have

$$g = \mathrm{e}^{\int (-2f_{\varepsilon} - 8f^2) \,\mathrm{d}\tau}.$$

To this stage, if we can get a solution of f from (28a), the solution of g can be calculated through (28b). Equation (28a) is a Burgers equation [17], which can be linearized as

$$p_{\tau} - p_{\varepsilon\varepsilon} = 0 \tag{30}$$

through the following Cole–Hopf transformation [17]:

$$f = \frac{1}{4} \frac{p_{\varepsilon}}{p}.$$
(31)

Based on (29), we can get

$$g = \frac{1}{\sqrt{p}}.$$
(32)

With the known solutions for (30) [17], the solutions for *f* can be given from (31), and then those for *g* from (29). Finally, the solutions for $u(\varepsilon, \tau)$ and $v(\varepsilon, \tau)$ can be respectively expressed as

$$u(\varepsilon,\tau) = \frac{f+g}{2}, \ v(\varepsilon,\tau) = \frac{f-g}{2}.$$
 (33)

Solving (30), we get

$$p = 1 + \sum_{i=1}^{n} c_i \, e^{k_i \varepsilon + k_i^2 \tau}, \tag{34}$$

where c_i and k_i $(i = 1, 2, \dots, n)$ are constants. Therefore, we can derive f and g as

$$f = \frac{k_i \sum_{i=1}^{n} c_i e^{k_i \varepsilon + k_i^2 \tau}}{4\left(1 + \sum_{i=1}^{n} c_i e^{k_i \varepsilon + k_i^2 \tau}\right)},$$
(35a)

$$g = \frac{1}{\sqrt{1 + \sum_{i=1}^{n} c_i \, e^{k_i \varepsilon + k_i^2 \tau}}}.$$
 (35b)

Thus, $u(\varepsilon, \tau)$ and $v(\varepsilon, \tau)$ can be shown as

$$u(\varepsilon,\tau) =$$
(36a)
$$\frac{1}{2} \left[\frac{k_i \sum_{i=1}^n c_i e^{k_i \varepsilon + k_i^2 \tau}}{4 \left(1 + \sum_{i=1}^n c_i e^{k_i \varepsilon + k_i^2 \tau} \right)} + \frac{1}{\sqrt{1 + \sum_{i=1}^n c_i e^{k_i \varepsilon + k_i^2 \tau}}} \right],$$
$$v(\varepsilon,\tau) =$$
(36b)

$$\frac{1}{2} \left[\frac{k_i \sum_{i=1}^n c_i e^{k_i \varepsilon + k_i^2 \tau}}{4 \left(1 + \sum_{i=1}^n c_i e^{k_i \varepsilon + k_i^2 \tau} \right)} - \frac{1}{\sqrt{1 + \sum_{i=1}^n c_i e^{k_i \varepsilon + k_i^2 \tau}}} \right].$$

With Transformations (27), we have converted (7) into (28a) and (29) without any extra restrictions. Compared with [17], in which there is a restriction $u(\varepsilon, \tau) = v(\varepsilon, \tau)$ (g = 0 assumed), the solutions obtained here are more general. Actually, corresponding to Solutions (72) and (78) in [17], which are called the general kink and singular kink solutions, we only need to adopt $c_i = 1$ and $c_i = -1$ ($i = 1, 2, \dots, n$), respectively. Our results are more general. Relevant issues can bee seen in [29, 30].

4. Conclusions

The CNLSEs [i.e., (1)] and CBEs [i.e., (7)] are, respectively, a model for certain birefringent optical fibers with Raman-scattering, Kerr and gain/loss effects, and a generalized model in fluid dynamics. In

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this paper, with dependent-variable transformations (8) and (27), we have converted (1) to the Manakov system [i.e., (9)], and (7) to a linear equation [i.e., (30)] together with Expression (29). Moreover, we have derived some new solutions for (1) and (7) based on Transformations (27) and (8), seen as Solutions (18), (24), and (36), respectively. Especially, we have illustrated one-dark-soliton-like Solutions (18) for (1) in Figure 1, against [18]'s claim that the solution of (1) has 'no soliton solution'. Through our work, (1) and (7) are reduced to simpler ones, seen as Expressions (9), (28a), and (29), respectively.

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