Effect of Heat Transfer on Magnetohydrodynamic Axisymmetric Flow Between Two Stretching Sheets

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This investigation describes the effects of heat transfer on magnetohydrodynamic (MHD) axisymmetric flow of a viscous fluid between two radially stretching sheets. Navier-Stokes equations are transformed into the ordinary differential equations by utilizing similarity variables. Solution computations are presented by using the homotopy analysis method. The convergence of obtained solutions is checked. Skin friction coefficient and Nusselt number are given in tabular form. The dimensionless velocities and temperature are also analyzed for the pertinent parameters entering into the problem.

Key words: Heat Transfer; Porous Medium; Skin Friction Coefficient; Nusselt Number.

1. Introduction

Investigation of heat transfer on stretching flows of viscous and non-Newtonian fluids is significant in engineering and industrial processes. For example, there is stretching of sheet when extrusion of polymer sheet from a die is considered. Such process in the presence of a magnetic field is helpful in controlling the cooling rate and properties of the final product. At present, the literature on the topic is quite extensive when viscous/non-Newtonian fluids are considered over a stretching surface. For instance Prasad and Vajravelu [1] studied heat transfer characteristics on the MHD flow of power law fluids over a non-isothermal stretching sheet. Liu [2] investigated the effect of heat transfer on a second-grade fluid flow driven by stretching sheet in the presence of a transverse magnetic field. Prasad et al. [3] examined the effect of variable viscosity on MHD viscoelastic fluid flow induced by a stretching sheet with heat transfer. Bataller [4] studied stretching flow taking various effects like heat source/sink, radiation, and work done by deformation into account. Three-dimensional stretching flow was considered by Hayat et al. [5]. Sajid et al. [6] constructed the non-similar solutions and heat transfer in the flow regime of a third-order fluid over a stretching sheet. Bataller [7] analyzed the effects of a non-uniform heat source, viscous dissipation, and thermal radiation on viscoelastic fluid flow due to a stretching sheet. Abbas et al. [8] investigated MHD flow driven by an oscillatory stretching sheet. Hayat et al. [9] communicated slip and heat transfer effects on the flow of a second-grade fluid over a stretching sheet immersed in a porous space. Hayat et al. [10] considered thin film flow of a second-grade fluid flow over an unsteady stretching sheet. Hayat et al. [11] conducted analysis regarding the effect of thermal radiation on the flow of a second-grade fluid. Hayat et al. [12] considered Soret and Dufour's effects on mixed convection boundary layer flow over a vertical stretching sheet in a porous medium filled with viscoelastic fluid. Hayat et al. [13] examined the influence of chemical reaction on unsteady flow of heat and mass transfer of a third-grade fluid over a stretching surface. Hayat et al. [14] studied MHD stagnation point flow of an upper-convected Maxwell fluid over a stretching surface. Hayat et al. [15] investigated the effect of thermal radiation on unsteady stretching flow. Salem and El-Aziz [16] reported the effects of Hall currents and chemical reaction on MHD flow of an internal heat generating/absorbing fluid over a vertical stretching surface. Abo-Eldahab and Salem [17] constructed numerical solutions for free convection flow of a power-law fluid over a stretching surface. Grukba and Bobba [18] reported the heat transfer characteristics on a continuous stretching surface. Ali [19] discussed thermal boundary layer flow of a power-law stretched surface with suction or injection.

It is noted from the existing literature that not much attention has been focused to the flow between radially stretching sheets. To the best of the author's knowledge even no such attempt is available for a viscous fluid. Therefore, this paper provides the MHD flow analysis of a viscous fluid between two radially stretching sheets. The fluid is electrically conducting and the sheets are not conducting. An incompressible fluid saturates the porous medium. In addition, heat transfer is considered. The nonlinear equations are solved by a homotopy analysis method [20 – 34]. Salient features of the constructed solutions are discussed.

2. Mathematical Formulation

We consider the MHD steady and axisymmetric flow of an incompressible viscous fluid between two radially stretching sheets at $z = \pm L$. An incompressible fluid fills the porous medium. Motion in fluid is due to the linear radial stretching of the both upper and lower sheets. We denote the velocity components (u, v, w) in the cylindrical coordinate system (r, θ, z). The axial symmetry is considered about $z = 0$ [35]. A constant magnetic field $\mathbf{B}_0$ is applied in the z-direction. The induced magnetic field is neglected under the assumption of small magnetic Reynolds number. There is no external electric field. Both the sheets have constant temperature T_w. The effects of viscous dissipation and Joule heating are taken into account. In view of the axial symmetry, the velocity field is defined as

$$\mathbf{V} = [u(r,z), 0, w(r,z)]. \tag{1}$$

With above form of velocity field, the continuity, Navier-Stokes, and energy equations are

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right] - \left(\frac{\sigma B_\mathrm{o}^2}{\rho} + \frac{\upsilon \varphi_\mathrm{m}}{k}\right)u, \tag{3}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{\nu\varphi_\mathrm{m}}{k}w, \tag{4}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{K}{\rho c_\mathrm{p}}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\sigma B_\mathrm{o}^2}{\rho c_\mathrm{p}}u^2 + \frac{\mu}{\rho c_\mathrm{p}}\left[2\frac{u^2}{r^2} + 2\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + 2\frac{\partial w}{\partial r}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + 2\left(\frac{\partial w}{\partial z}\right)^2\right]. \tag{5}$$

Here u and w are the velocity components in radial (r) and axial (z) directions, respectively, ρ is the density, ν the kinematic viscosity, σ the electrical conductivity of the fluid, p the pressure, μ the dynamic viscosity, c_p the specific heat, φ_m the porosity of the medium, T the temperature, K the thermal conductivity, and k the permeability of the porous medium.

The prescribed boundary conditions are

$$\frac{\partial u}{\partial z} = 0, \quad w = 0, \quad \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \\ u = ar, \quad w = 0, \quad T = T_\mathrm{w} \text{ at } z = L, \quad a > 0. \tag{6}$$

From (2) – (6), after employing the transformations

$$u = raF'(\eta), w = -2aLF(\eta), \theta = \frac{T}{T_\mathrm{w}}, \eta = \frac{z}{L}, \tag{7}$$

and eliminating the pressure terms, one obtains

$$F^{(iv)}(\eta) - \mathrm{Re}(M+\phi)F''(\eta) + 2\,\mathrm{Re}\,F(\eta)F'''(\eta) = 0, \\ F(0) = 0,\ F(1) = 0, F'(1) = 1,\ F''(0) = 0, \tag{8}$$

$$\theta''(\eta) + 2\,\mathrm{Re}\,\mathrm{Pr}\,F(\eta)\theta'(\eta) + \mathrm{Re}\,\mathrm{M}\,\mathrm{Pr}\,\mathrm{Ec}[F'(\eta)]^2 + \mathrm{Pr}\,\mathrm{Ec}\left[(F''(\eta))^2 + \frac{12}{\delta}(F'(\eta))^2\right] = 0, \tag{9}$$

$$\theta'(0) = 0, \quad \theta(1) = 1,$$

in which

$$\mathrm{Re} = \frac{aL^2}{\nu}, \quad \mathrm{M} = \frac{\sigma B_0^2}{\rho a}, \quad \mathrm{Pr} = \frac{\mu c_\mathrm{p}}{K}, \\ \mathrm{Ec} = \frac{a^2 r^2}{c_\mathrm{p} T_\mathrm{w}}, \quad \phi = \frac{\nu \varphi_\mathrm{m}}{ka}, \quad \delta = \frac{r^2}{L^2}$$

denote, respectively, the Reynolds number, Hartman number, Prandtl number, local Eckert number, and the porosity parameter.

The skin friction coefficient C_F and Nusselt number Nu are defined as follows:

$$C_\mathrm{F} = \frac{\tau_{rz}|_{z=0}}{\rho(ar)^2} = \frac{\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\Big|_{z=L}}{\rho(ar)^2} = \frac{1}{\mathrm{Re_r}}F''(1), \\ \mathrm{Nu} = \frac{Lq_\mathrm{w}}{KT_\mathrm{w}} = -\frac{LK\,\frac{\partial T}{\partial z}\Big|_{z=L}}{KT_\mathrm{w}} = -\theta'(1), \tag{10}$$

where $\mathrm{Re_r}$ $(= ar^2/\nu)$ indicates the local Reynolds number.

3. Solutions of the Problems

In order to obtain the homotopy solutions, $F(\eta)$ and $\theta(\eta)$ in the form of base functions

$$\{\eta^{2n+1}, \quad n \geq 0\}, \quad \{\eta^{2n}, \quad n \geq 0\} \tag{11}$$

are written as

$$F(\eta) = \sum_{n=0}^{\infty} a_n \eta^{2n+1}, \quad \theta(\eta) = \sum_{n=0}^{\infty} b_n \eta^{2n}, \tag{12}$$

in which a_n and b_n are coefficients to be determined. Further, initial guesses $F_0(\eta)$, $\theta_0(\eta)$, and linear operators $\mathcal{L}_i$ $(i = 1,2)$ are selected of the forms given below

$$F_0(\eta) = \frac{1}{2}\eta(\eta^2 - 1), \quad \theta_0(\eta) = \eta^2, \tag{13}$$

$$\mathcal{L}_1[F(\eta)] = \frac{\mathrm{d}^4 F}{\mathrm{d}\eta^4}, \quad \mathcal{L}_2[\theta(\eta)] = \frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}. \tag{14}$$

We note that the operators $\mathcal{L}_i$ $(i = 1,2)$ satisfy the following properties:

$$\begin{aligned} &\mathcal{L}_1[C_1 + C_2\eta + C_3\eta^2 + C_4\eta^3] = 0, \\ &\mathcal{L}_2[C_5 + C_6\eta] = 0, \end{aligned} \tag{15}$$

where C_i $(i = 1-6)$ are the constants.

3.1. Zeroth-Order Deformation Problems

The deformation problems at the zeroth order are

$$\begin{aligned} &(1-q)\mathcal{L}_1[\Phi(\eta;q) - F_0(\eta)] = q\hbar_1\mathcal{N}_1[\Phi(\eta;q)], \\ &(1-q)\mathcal{L}_2[\Psi(\eta;q) - \theta_0(\eta)] = q\hbar_2\mathcal{N}_2[\Psi(\eta;q)], \end{aligned} \tag{16}$$

$$\begin{aligned} &\Phi(0;q) = 0, \quad \Phi(1;q) = 0 \\ &\left.\frac{\partial\Phi(\eta;q)}{\partial\eta}\right|_{\eta=1} = 1, \quad \left.\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right|_{\eta=0} = 0, \\ &\Psi(0;q) = 1, \quad \left.\frac{\partial\Psi(\eta;q)}{\partial\eta}\right|_{\eta=0} = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} \mathcal{N}_1[\Phi(\eta;q)] = &\frac{\partial^4\Phi(\eta;q)}{\partial\eta^4} - \mathrm{Re}(\mathrm{M}+\phi)\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2} \\ &+ 2\,\mathrm{Re}\,\Phi(\eta;q)\frac{\partial^3\Phi(\eta;q)}{\partial\eta^3} \end{aligned} \tag{18}$$

$$\begin{aligned} \mathcal{N}_2[\Psi(\eta;q)] = &\frac{\partial^2\Psi}{\partial\eta^2} + 2\,\mathrm{Re}\,\mathrm{Pr}\frac{\partial\Psi(\eta;q)}{\partial\eta}\Phi(\eta;q) \\ &+ \mathrm{Pr}\,\mathrm{Re}\,\mathrm{Ec}\,\mathrm{M}\left(\frac{\partial\Phi(\eta;q)}{\partial\eta}\right)^2 \\ &+ \mathrm{Pr}\,\mathrm{Ec}\left[\left(\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right)^2 + \frac{12}{\delta}\left(\frac{\partial\Phi(\eta;q)}{\partial\eta}\right)^2\right]. \end{aligned} \tag{19}$$

In the above expressions $q \in [0,1]$ and $\hbar_i \neq 0$ $(i = 1,2)$ are, respectively, the embedding and auxiliary parameters and $\Phi(\eta;0) = F_0(\eta)$, $\Psi(\eta;0) = \theta_0(\eta)$ and $\Phi(\eta;1) = F(\eta)$, $\Psi(\eta;1) = \theta(\eta)$. When q varies from 0 to 1, then $\Phi(\eta;q)$ varies from the initial guess $F_0(\eta)$ to $F(\eta)$, $\Psi(\eta;q)$ varies from the initial guess $\theta_0(\eta)$ to $\theta(\eta)$ and $\mathcal{N}_i$ $(i = 1,2)$ are nonlinear operators.

By Taylors' series, one obtains

$$\begin{aligned} &\Phi(\eta;q) = F_0(\eta) + \sum_{m=1}^{\infty} F_{\mathrm{m}}(\eta)q^m, \\ &\Psi(\eta;q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_{\mathrm{m}}(\eta)q^m, \end{aligned} \tag{20}$$

$$\begin{aligned} &F_{\mathrm{m}}(\eta) = \left.\frac{1}{m!}\frac{\partial^m\Phi(\eta;q)}{\partial\eta^m}\right|_{q=0}, \\ &\theta_{\mathrm{m}}(\eta) = \left.\frac{1}{m!}\frac{\partial^m\Psi(\eta;q)}{\partial\eta^m}\right|_{q=0}. \end{aligned} \tag{21}$$

3.2. Higher-Order Deformation Problems

Writing

$$\begin{aligned} &F_{\mathrm{m}}(\eta) = \{F_0(\eta), F_1(\eta), F_2(\eta), F_3(\eta), \ldots, F_{\mathrm{m}}(\eta)\}, \\ &\theta_{\mathrm{m}}(\eta) = \{\theta_0(\eta), \theta_1(\eta), \theta_2(\eta), \theta_3(\eta), \ldots, \theta_{\mathrm{m}}(\eta)\}, \end{aligned} \tag{22}$$

the deformation problems at the mth order are

$$\begin{aligned} &\mathcal{L}_1[F_{\mathrm{m}}(\eta) - \chi_{\mathrm{m}}F_{m-1}(\eta)] = \hbar_1\mathcal{R}_{1m}(F_{m-1}(\eta)), \\ &F_{\mathrm{m}}(0) = 0,\ F_{\mathrm{m}}(1) = 0,\ F'_{\mathrm{m}}(1) = 0,\ F''_{\mathrm{m}}(0) = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} &\mathcal{L}_2[\theta_{\mathrm{m}}(\eta) - \chi_{\mathrm{m}}\theta_{m-1}(\eta)] = \hbar_2\mathcal{R}_{2m}(\theta_{m-1}(\eta)), \\ &\theta'_{\mathrm{m}}(0) = 0, \quad \theta_{\mathrm{m}}(1) = 0, \quad \chi_{\mathrm{m}} = \begin{cases} 0, m \leq 1, \\ 1, m > 1, \end{cases} \end{aligned} \tag{24}$$

$$\begin{aligned} \mathcal{R}_{1m}(F_{m-1}(\eta)) = &F_{m-1}^{(iv)}(\eta) - \mathrm{Re}(\mathrm{M}+\phi)F''(\eta) \\ &+ 2\,\mathrm{Re}\sum_{n=0}^{m-1} F_{\mathrm{n}}(\eta)F'''_{m-1-n}(\eta), \end{aligned} \tag{25}$$

$$\mathcal{R}_{2m}(G_{m-1}(\eta)) = G''_{m-1}(\eta) + 2\,\mathrm{Re}\,\mathrm{Pr}\sum_{n=0}^{m-1} F_n(\eta)\theta'_{m-1-n}(\eta) + \mathrm{M}\,\mathrm{Pr}\,\mathrm{Re}\,\mathrm{Ec}\sum_{n=0}^{m-1} F'_n(\eta)F'_{m-1-n}(\eta) + \mathrm{Pr}\,\mathrm{Ec}\sum_{n=0}^{m-1}\left[F''_n(\eta)F''_{m-1-n}(\eta) + \frac{12}{\delta}F'_n(\eta)F'_{m-1-n}(\eta)\right], \quad (26)$$

Table 1. Convergence of HAM solutions for different order of approximations when M = 2, Re = 2, $\phi = 2$, Pr = Ec = 0.5, $\delta = 12$, and $\hbar_1 = \hbar_2 = -0.9$.

Order of approximations	$F''(1)$	$\theta'(1)$
5	4.543714562	1.015725364
10	4.545135472	1.015550040
15	4.545143321	1.015552361
30	4.545143377	1.015552407
35	4.545143377	1.015552407
40	4.545143377	1.015552407
45	4.545143377	1.015552407

Table 2. Variation of skin friction coefficient C_F for different values of involved parameters.

Re	M	ϕ	$\mathrm{Re_r}\, C_F$
0.5	2.0	2.0	3.436286042
1.0			3.835401237
2.0			4.545143377
3.0			5.165328420
2.0	0.0	2.0	3.951059167
	1.0		4.259675434
	2.0		4.545143377
	3.0		4.811550468
2.0	2.0	0.0	3.951059167
		1.0	4.259675434
		2.0	4.545143377
		3.0	4.811550472

The general solutions of the problems (23) and (24) are

$$F(\eta) = F^*(\eta) + C_1^m + C_2^m\eta + C_3^m\eta^2 + C_4^m\eta^3, \quad (27)$$
$$\theta(\eta) = \theta^*(\eta) + C_5^m + C_6^m\eta,$$

where $F^*(\eta)$ and $\theta^*(\eta)$ are particular solutions of problems (23) and (24).

4. Results and Discussion

Note that the analytical solutions (27) strongly depend upon the non-zero auxiliary parameters $\hbar_i$ ($i = 1,2$). These auxiliary parameters play an important role in the convergence of the derived series solutions. Therefore, for this objective $\hbar_i$-curves are plotted (Fig. 1 and 2). The admissible ranges of $\hbar_1$ and $\hbar_2$ are $-1.2 \le \hbar_1 \le -0.8$ and $-1.6 \le \hbar_2 \le -0.4$. The whole analysis is carried out when $\hbar_1 = \hbar_2 = -0.9$.

Figures 3 – 5 show the plots of dimensionless radial velocity $F'(\eta)$ for the influence of Reynolds number Re, Hartman number M, and porosity parameter ϕ. Figures 6 – 8 present the variation of the dimensionless axial velocity $F(\eta)$ for various values of Reynolds number Re, Hartman number M, and porosity parameter ϕ. In order to describe the influence of Ec, Pr, M, Re, and ϕ on dimensionless temperature $\theta(\eta)$, Figures 9 – 13 have been displayed.

From Figure 3 it is noted that the dimensionless radial velocity $F'(\eta)$ decreases by increasing Re. Fig-

Table 3. Variation Nusselt number Nu for different values of involved parameters.

Re	M	ϕ	Pr	Ec	δ	−Nu
0.5	2.0	2.0	0.2	0.2	12	0.1358446222
1.0						0.1438008008
1.5						0.1516863551
2.0						0.1594056036
2.0	0.0	2.0	0.2	0.2	12	0.1326583328
	1.0					0.1466251342
	2.0					0.1594056039
	3.0					0.1712231305
2.0	2.0	0.0	0.2	0.2	12	0.1579486489
		1.0				0.1583096885
		2.0				0.1594056039
		3.0				0.1609928560
2.0	2.0	2.0	0.12	0.2	12	0.0951691998
			0.32			0.2569850981
			0.52			0.4230222358
			0.72			0.5935531440
2.0	0.0	2.0	0.2	0.0	12	0.7970280195
				1.0		0.7970280195
				2.0		1.594056039
				3.0		2.391084059

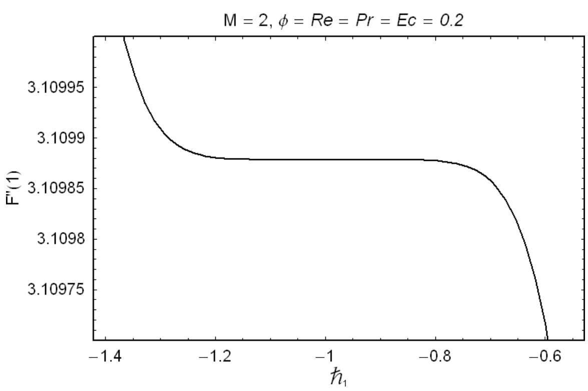

Fig. 1. $\hbar_1$-curve for 15th-order approximations of $F'(\eta)$.

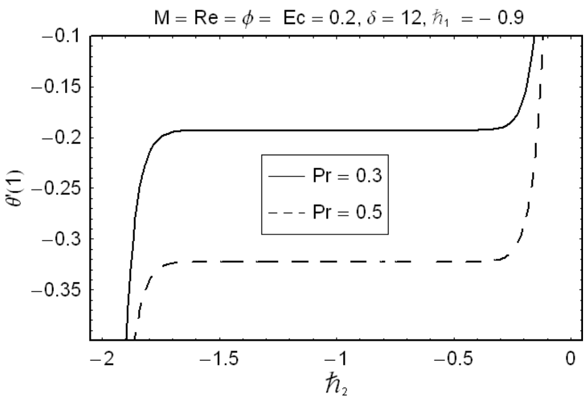

Fig. 2. $\hbar_2$-curve for 20th-order approximations of $\theta(\eta)$.

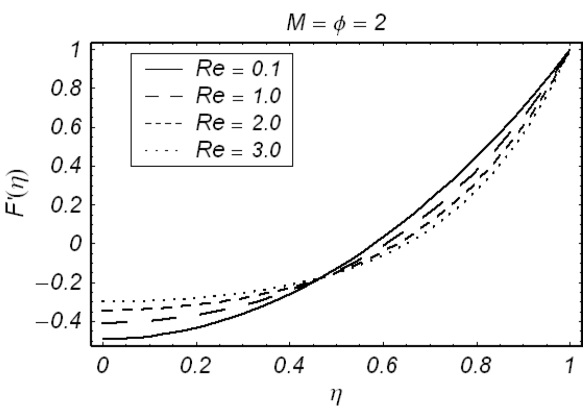

Fig. 3. Influence of Re on $F'(\eta)$.

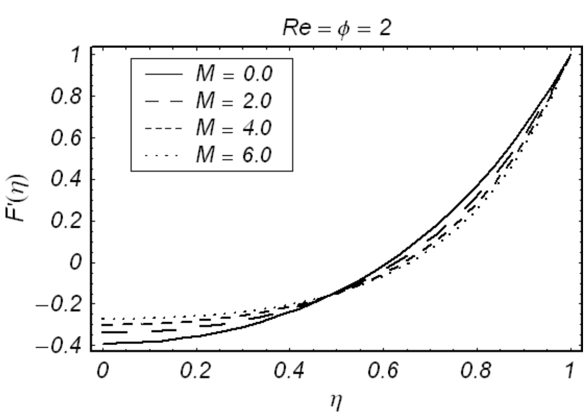

Fig. 4. Influence of M on $F'(\eta)$.

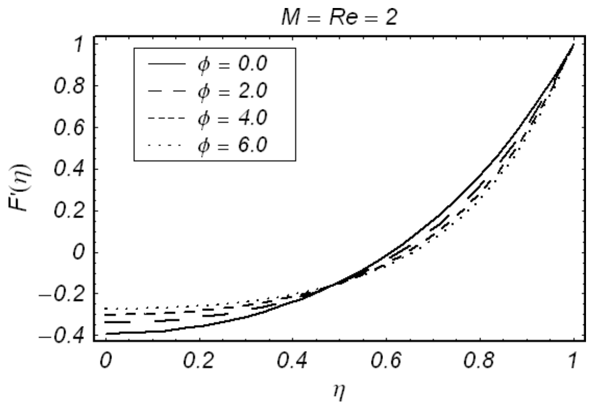

Fig. 5. Influence of ϕ on $F'(\eta)$.

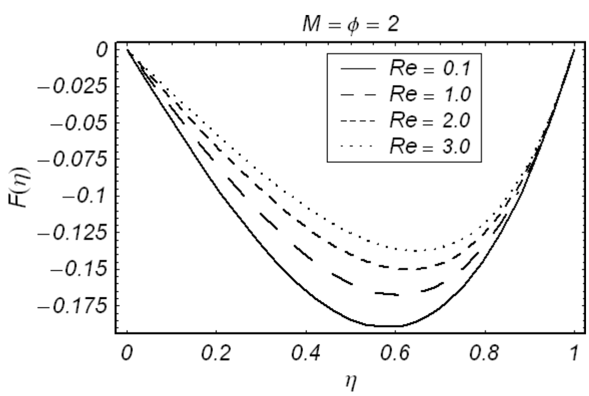

Fig. 6. Influence of Re on $F(\eta)$.

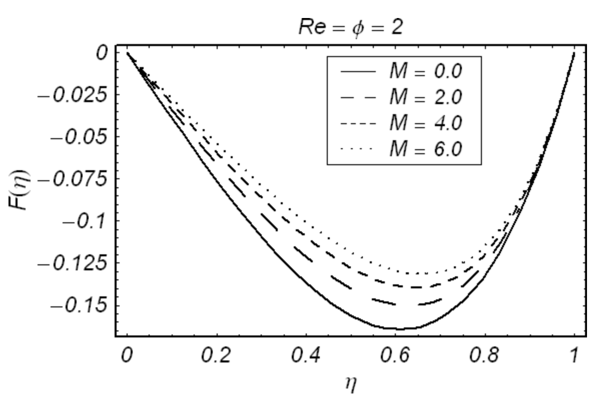

Fig. 7. Influence of M on $F(\eta)$.

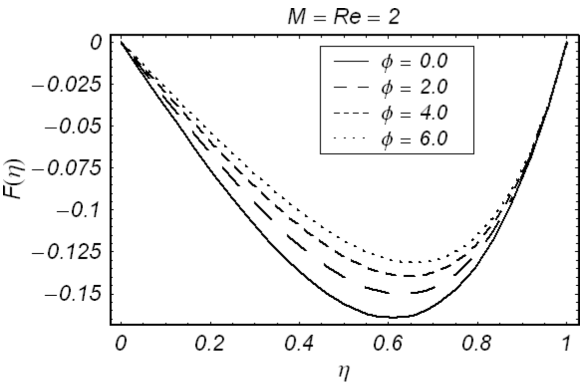

Fig. 8. Influence of ϕ on $F(\eta)$.

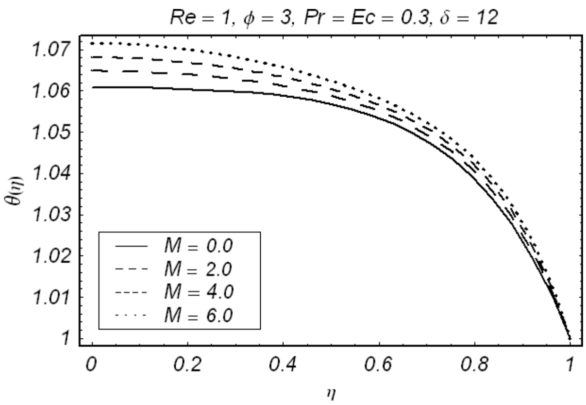

Fig. 11. Influence of M on $\theta(\eta)$.

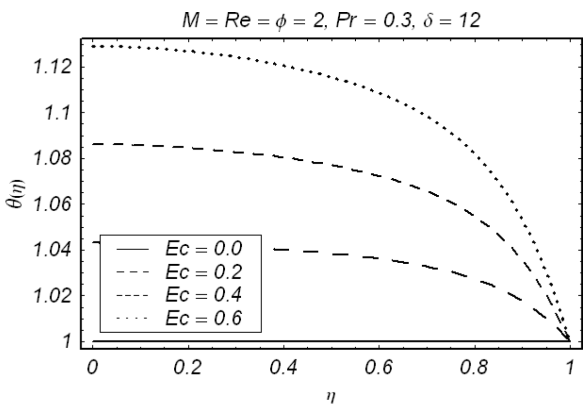

Fig. 9. Influence of Ec on $\theta(\eta)$.

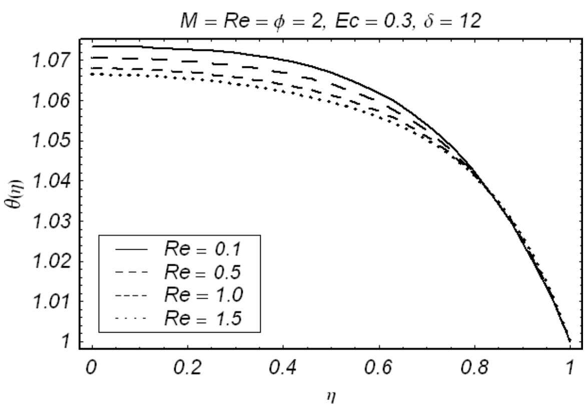

Fig. 12. Influence of Re on $\theta(\eta)$.

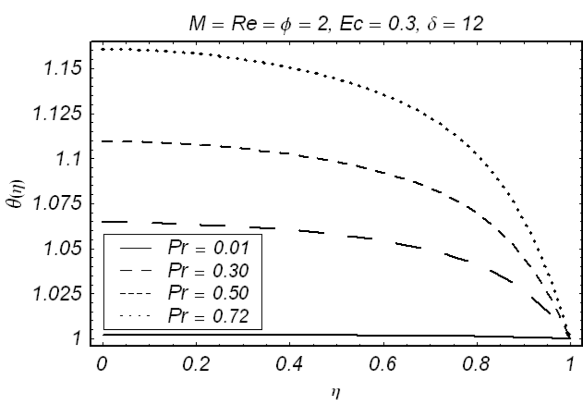

Fig. 10. Influence of Pr on $\theta(\eta)$.

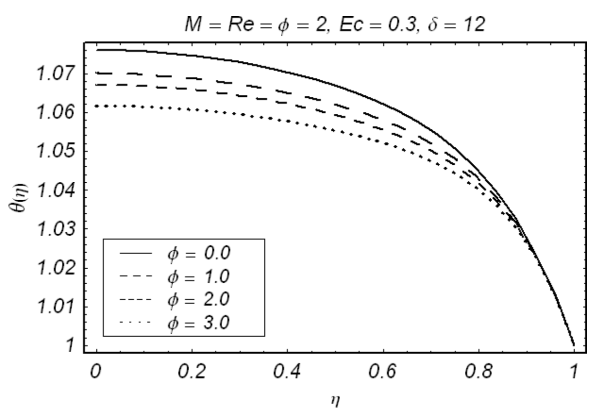

Fig. 13. Influence of ϕ on $\theta(\eta)$.

ure 4 depicts that $F'(\eta)$ is a decreasing function of M. This is in view of the fact that magnetic field slows down the motion of the fluid. Figure 5 indicates that the dimensionless radial velocity $F'(\eta)$ decreases when the porosity parameter ϕ increases. This indicates that the porous medium offers resistance to the flow of the fluid which makes sense. From Figures 6 – 8 one can conclude that the axial velocity $F(\eta)$ increases when M, Re, and ϕ are increased. It is obvious from Figures 3 – 5 that the boundary layer thickness for the radial velocity $F'(\eta)$ decreases by increasing M, Re, and ϕ but increases in case of axial velocity (Figs. 6 – 8). Figures 9 – 11 indicate that the dimensionless temperature $\theta(\eta)$ has increasing trend when Ec, Pr, and M increase. However, $\theta(\eta)$ decreases with the increase of Reynolds number Re and porosity parameter ϕ (Figs. 12 and 13). Table 1 is given to ensure the convergence of homotopology analysis method (HAM) solutions (27). From this table one can see that the convergence is achieved at 35th order of approximation. Table 2 shows the variation of skin friction coefficient C_F for different values of Re, M, and ϕ. This table depicts that the skin friction coefficient C_F increases when Re, M, and ϕ are increased. Variation of Nusselt number Nu for different values of physical parameters is shown in Table 3. It indicates that the Nusselt number Nu is an increasing function of Re, M, ϕ, Pr, and Ec.

5. Closing Remarks

This study analyses the flow and heat transfer characteristics on the MHD axisymmetric flow of an electrically conducting viscous fluid in the presence of viscous dissipation and Joule heating. Analytic solutions are developed. The main results can be summarized as follows:

- Effects of Re, M, and ϕ on $F'(\eta)$ and $F(\eta)$ are similar in a qualitative sense.
- Influence of ϕ on $\theta(\eta)$ is similar to that of $F'(\eta)$ and $F(\eta)$.
- Qualitatively, the behaviour of Ec and Pr on $\theta(\eta)$ is similar.
- Variations of Mand ϕ on $\theta(\eta)$, $F'(\eta)$, and $F(\eta)$ show opposite behaviour.
- The behaviour of Re, M, ϕ, Pr, and Ec on the skin friction coefficient and the Nusselt number are similar.

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