Heat Transfer Analysis on the Peristaltic Motion with Slip Effects

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The purpose of this paper is to highlight the combined effects of heat transfer and slip characteristics of magnetohydrodynamic (MHD) fluid with variable viscosity in a channel. The slip condition is imposed in terms of shear stress. An analysis is performed to derive the perturbation solution for long wavelength and small Reynolds number assumptions. Expressions of stream function, temperature and heat transfer coefficient are constructed and discussed.

Key words: Heat Transfer; Slip Conditions; Long Wavelength Approximation.

1. Introduction

The peristaltic flow in a channel or a tube is of considerable interest because it can occur in many engineering and physiological processes. In particular, the peristaltic motion occurs in urine transport from kidney to bladder, blood flow in arteries, spermatozoa transport in the ductus efferentes of the male reproductive tracts, roller and finger pumps, sanitary fluid transport etc. The peristaltic flows with magnetic field effects are important in biomagnetic fluid dynamics. Some useful existing contributions on the topic have been reported in the studies [1 – 10]. In [11] Ali et al. discussed the peristaltic flow of MHD viscous fluid with variable viscosity and slip effects.

The object of the present study is to extend the analysis of [11] for the heat transfer situation. Symmetric nature of flow is considered in a planar channel. Important flow quantities like temperature and heat transfer coefficient are analyzed.

2. Problem Development

We consider a Cartesian coordinate system with the $\bar{X}$-axis along the centreline of the channel and the $\bar{Y}$-axis normal to it. An incompressible viscous fluid of variable viscosity is considered in a channel of thickness $2a$. The fluid is electrically conducting whereas the channel walls are insulating. The fluid is conducting under the application of a constant magnetic field $\mathbf{B}_0$ in the $\bar{Y}$-direction. No electric field is applied. The induced magnetic field is also not taken into account. Furthermore, the channel walls have the same temperature. The motion in the flow system is induced by a small amplitude sinusoidal wave train moving with constant speed c. The shape of wall surface is

$$\bar{h}(\bar{X},\bar{t}) = a + b\cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right], \tag{1}$$

in which b is the wave amplitude, λ the wavelength, $\bar{t}$ the time, and $\bar{X}$ the direction in which the wave propagates. If $(\bar{U},\bar{V})$ and $(\bar{u},\bar{v})$ are the velocities in the laboratory $(\bar{X},\bar{Y})$ and wave $(\bar{x},\bar{y})$ frames, respectively, then

$$\begin{aligned} &\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \\ &\bar{v} = \bar{V}, \quad p(\bar{x}) = P(\bar{X},\bar{t}), \end{aligned} \tag{2}$$

where P and p are the corresponding pressures in laboratory and wave frames, respectively.

Now continuity, Navier-Stokes, and energy equations give

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{3}$$

$$\begin{aligned} \rho\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{u} &= -\frac{\partial \bar{p}}{\partial \bar{x}} + 2\frac{\partial}{\partial \bar{x}}\left(\mu(\bar{y})\frac{\partial \bar{u}}{\partial \bar{x}}\right) \\ &+ \frac{\partial}{\partial \bar{y}}\left[\bar{\mu}(\bar{y})\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] \\ &- \sigma B_0^2(\bar{u} + c), \end{aligned} \tag{4}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial\bar{x}}+\bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{v}=-\frac{\partial\bar{p}}{\partial\bar{y}}+2\frac{\partial}{\partial\bar{y}}\left(\bar{\mu}(\bar{y})\frac{\partial\bar{u}}{\partial\bar{y}}\right)+\frac{\partial}{\partial\bar{x}}\left[\bar{\mu}(\bar{y})\left(\frac{\partial\bar{v}}{\partial\bar{x}}+\frac{\partial\bar{u}}{\partial\bar{y}}\right)\right], \tag{5}$$

$$\zeta\left(\bar{u}\frac{\partial}{\partial\bar{x}}+\bar{v}\frac{\partial}{\partial\bar{y}}\right)T=\frac{\bar{\mu}(\bar{y})}{\rho}\left[2\left\{\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2+\left(\frac{\partial\bar{v}}{\partial\bar{y}}\right)^2\right\}+\left(\frac{\partial\bar{u}}{\partial\bar{y}}+\frac{\partial\bar{v}}{\partial\, bx}\right)^2\right]+\frac{k}{\rho}\left[\frac{\partial^2T}{\partial\bar{x}^2}+\frac{\partial^2T}{\partial\bar{y}^2}\right], \tag{6}$$

in which ρ indicates the density, $\bar{\mu}(\bar{y})$ is the viscosity function, ζ is the specific heat at constant volume, T is the temperature, and k is the thermal conductivity.

To facilitate the subsequent analysis we introduce

$$x=\frac{\bar{x}}{\lambda},\quad y=\frac{\bar{y}}{a},\quad u=\frac{\bar{u}}{c},\quad v=\frac{\bar{v}}{c\delta},\quad t=\frac{c\bar{t}}{\lambda},\quad \delta=\frac{a}{\lambda},\quad p=\frac{a^2\bar{p}}{\mu_0c\lambda},\quad h=\frac{\bar{h}}{a},\quad \phi=\frac{b}{a},$$
$$\mu(y)=\frac{\bar{\mu}(\bar{y})}{\mu_0},\quad Re=\frac{\rho ca}{\mu_0},\quad S=\frac{a\bar{S}}{\mu_0 c}, \tag{7}$$
$$M=\sqrt{\frac{\sigma}{\mu_0}}aB_0,\quad P_r=\frac{\rho v_0\zeta}{k},\quad \theta=\frac{T-T_0}{T_0},\quad E=\frac{c^2}{\zeta T_0},\quad v_0=\frac{\mu_0}{\rho},$$

where M represents the Hartman number, Re the Reynolds number, σ the electric conductivity, δ the wave number, μ_0 the constant viscosity, P_r the Prandtl number, θ the dimensionless temperature, T_0 the value of temperature at the lower and upper walls, and E is the Eckert number.

Following the long wavelength analysis presented in [11] we have

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right)-M^2\left(\frac{\partial\psi}{\partial y}+1\right), \tag{8}$$

$$0=-\frac{\partial p}{\partial y}, \tag{9}$$

$$0=\frac{\partial^2\theta}{\partial y^2}+B_r\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2, \tag{10}$$

where $B_r=P_rE$. The subjected boundary conditions are

$$\psi=0,\quad \frac{\partial^2\psi}{\partial y^2}=0\ \text{at}\ y=0, \tag{11a}$$

$$\psi=F,\quad \frac{\partial\psi}{\partial y}+\beta\mu(y)\frac{\partial^2\psi}{\partial y^2}=-1\quad \text{at}\ y=h=1+\phi\cos[2\pi x], \tag{11b}$$

$$\frac{\partial\theta}{\partial y}=0\ \text{at}\ y=0, \tag{11c}$$

$$\theta=0\ \text{at}\ y=h=1+\phi\cos[2\pi x], \tag{11d}$$

$$\eta-1=F=\int_0^h\frac{\partial\psi}{\partial y}\mathrm{d}y=\psi(h)-\psi(0), \tag{12}$$

$$\mu(y)=\mathrm{e}^{-\alpha y}\ \text{or}\ \mu(y)=1-\alpha y\ \text{for}\ \alpha\ll 1, \tag{13}$$

in which ψ is the stream function, β the slip parameter, $\mu(y)$ the dimensionless viscosity function, $\phi(=\frac{b}{a}<1)$ the amplitude ratio, h the dimensionless form of the peristaltic wall, θ the dimensionless temperature, η and F the dimensionless mean flow rates in the laboratory and wave frames, respectively, and α the viscosity parameter.

3. Solution of the Problem

Case 1 ($M=0$)

The associated equations (8) – (10) in this case are

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right), \tag{14}$$

$$0=-\frac{\partial p}{\partial y}, \tag{15}$$

$$0=\frac{\partial^2\theta}{\partial y^2}+B_r\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2. \tag{16}$$

From (14) and (15) we can easily write

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right)=0. \tag{17}$$

The solutions of (16) and (17) satisfying the boundary conditions (11a) – (11d) are

$$\psi=\frac{\left[\begin{array}{c}y(2h^3+\alpha h^4-hy^2(2+\alpha y)\\ +F(6h^2+4\alpha h^3-y^2(2+\alpha y)+12h\beta))\end{array}\right]}{h^2(4h+3\alpha h^2+12\beta)}, \tag{18}$$

$$\theta=\frac{3B_r(F+h)^2(3h^5\alpha-y^4(5+3y\alpha)+5h^4)}{10h^4(h+3\beta)(h(2+3h\alpha)+6\beta)}. \tag{19}$$

Case 2 ($M \neq 0$)

With the help of (8)–(10) we obtain

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right)-M^2\frac{\partial^2\psi}{\partial y^2}=0, \tag{20}$$

$$\frac{\partial^2\theta}{\partial y^2}+B_r\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2=0. \tag{21}$$

We expand the flow quantities in term of the small parameter α as follows:

$$\psi=\psi_0+\alpha\psi_1+o(\alpha^2), \tag{22}$$

$$F=F_0+\alpha F_1+o(\alpha^2), \tag{23}$$

$$\theta=\theta_0+\alpha\theta_1+o(\alpha^2). \tag{24}$$

Invoking (22)–(24) into (20) and (21) and the boundary conditions and then adopting the same methodology as in [11], the resulting expressions for stream function and temperature are

$$\begin{aligned}\psi=&\frac{y\{FM\cosh[hM]+(1+FM^2\beta)\sinh[hM]\}-(F+h)\sinh[My]}{hM\cosh[hM]+(-1+hM^2\beta)\sinh[hM]}\\&+\frac{\alpha}{8(hM\cosh[hM]+(-1+hM^2\beta)\sinh[hM])^2}\Big\{(F+h)(y\cosh[2hM]+y(-1+2h^2M^2))\\&-2My\cosh[My](hM\cosh[hM]+(-1+hM^2\beta)\sinh[hM])-2hM\sinh[2hM]\\&+2(hM(y+h^2M^2\beta)cosh[hM]+(-y+hM^2(h^2-h\beta+y\beta)\sinh[hM])\sinh[My])\Big\},\end{aligned} \tag{25}$$

$$\begin{aligned}\theta=&\frac{1}{192(hM\cosh[hM]+K_5\sinh[hM])^3}\Big\{(F+h)^2MB_r((3\alpha+M^2(4hA_1+3hA_2\alpha\beta))\cosh[hM]\\&+3A_3\cosh[3hM]-3A_4\cosh[M(h-2y)]+3A_5\cosh[M(h+2y)]+A_6\sinh[hM]\\&-3MA_7\sinh[3hM]+3A_8M\sinh[M(h-2y)]+3MA_9\sinh[M(h+2y)]\Big\},\end{aligned} \tag{26}$$

hence

$$\begin{aligned}&A=h^2-y^2,\\&A_1=3+2M^2(-6h^2-h^3\alpha+y^2(6+y\alpha)),\\&A_2=-1+4h^2M^2(-1+2M^2A),\\&A_3=-\alpha+hM^2(4-4h\alpha+\alpha\beta),\\&A_4=4hM^2+\alpha-2hM^2y\alpha+2M^2y^2\alpha\\&\qquad-hM^2\alpha\beta-2h^3M^4\alpha\beta-2hM^4y^2\alpha\beta,\\&A_5=\alpha+M^2(-4h+2y\alpha(h+y)+h(-1+2M^2A)\alpha\beta),\\&A_6=(12M+48M^3A-8M^3y^3\alpha)(1-hM^2\beta)-3hM\alpha\\&\qquad+20h^3M^3\alpha+24h^2M^5\alpha(hA+y^2\beta)-32h^4M^5\alpha\beta,\\&A_7=4-3h\alpha-4hM^2\beta,\\&A_8=4+(h-2y)\alpha+2hM^2\alpha(h^2+y^2)\\&\qquad-2hM^2\beta(2+\alpha(1-y)),\\&A_9=4+(-2y+h(-1+2M^2A))\alpha\\&\qquad-2hM^2(2+(h-y)\alpha)\beta.\end{aligned} \tag{27}$$

The expression of the heat transfer coefficient (Z) at the upper wall is

$$Z=h_x\theta_y \tag{28}$$

which, after utilizing (26), yields

$$\begin{aligned}Z=&\frac{-\pi\phi\sin[2\pi x]}{4(hM\cosh[hM]+K_5\sinh[hM])^3}\{(F+h)^2M^3B_r(\sinh[hM](-My(4+2h^3M^2\alpha+y\alpha)\\&+hM^3y(4+2h\alpha+y\alpha)\beta+My^2\alpha(1-hM^2\beta)\cosh[2My]+(2+hM^2(h\alpha(h-\beta)-2\beta))\sinh[2My])\\&+hM\cosh[hM](4+y\alpha-2h^2M^2a\beta-y\alpha\cosh[2My])+(-2+h^2M^2\alpha\beta)\sinh[2My])\}.\end{aligned} \tag{29}$$

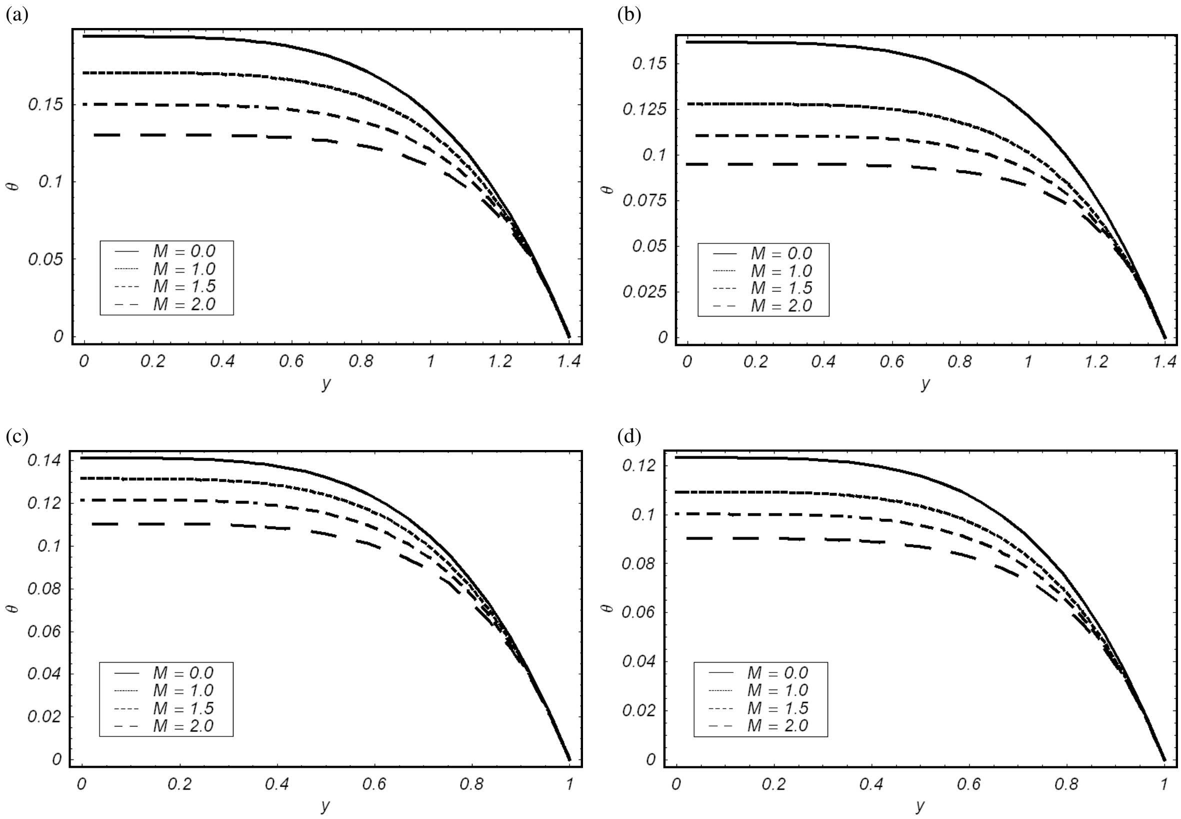

Fig. 1. Effects of M on temperature for $\eta = 0.65$, $\phi = 0.4$, $\beta = 0.02$, $B_r = 0.5$, where in (a), (b) $x = 0$ and in (c), (d) $x = 0.25$.

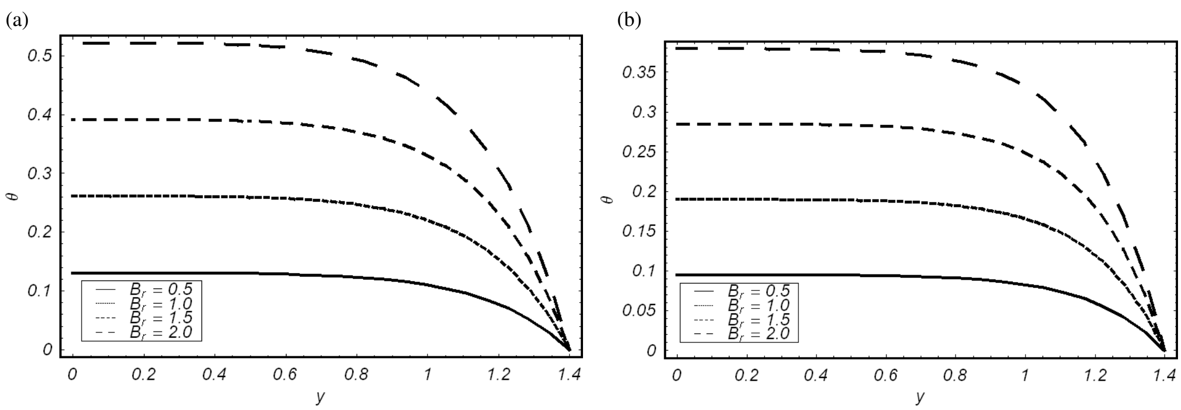

Fig. 2. Effects of B_r on temperature for $\eta = 0.65$, $\phi = 0.4$, $\beta = 0.02$, $M = 2$, $x = 0$.

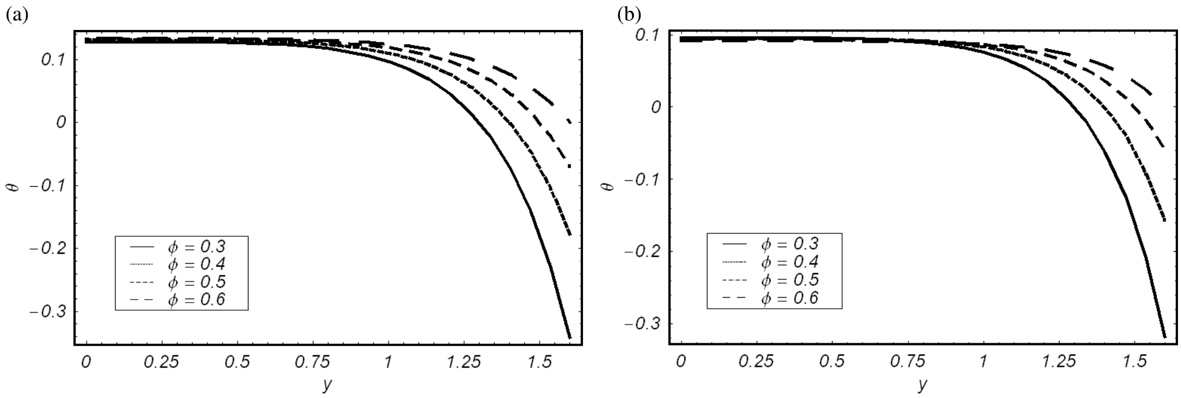

Fig. 3. Effects of ϕ on temperature for $\eta = 0.65$, $\beta = 0.02$, $B_r = 0.5$, $M = 2$, $x = 0$.

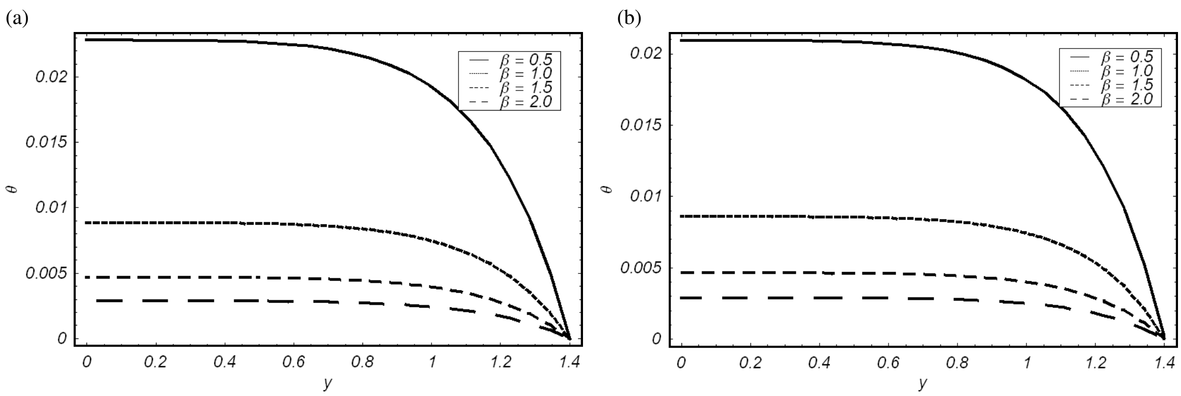

Fig. 4. Effects of β on temperature for $\eta = 0.65$, $\phi = 0.4$, $M = 2$, $B_r = 0.5$, $x = 0$.

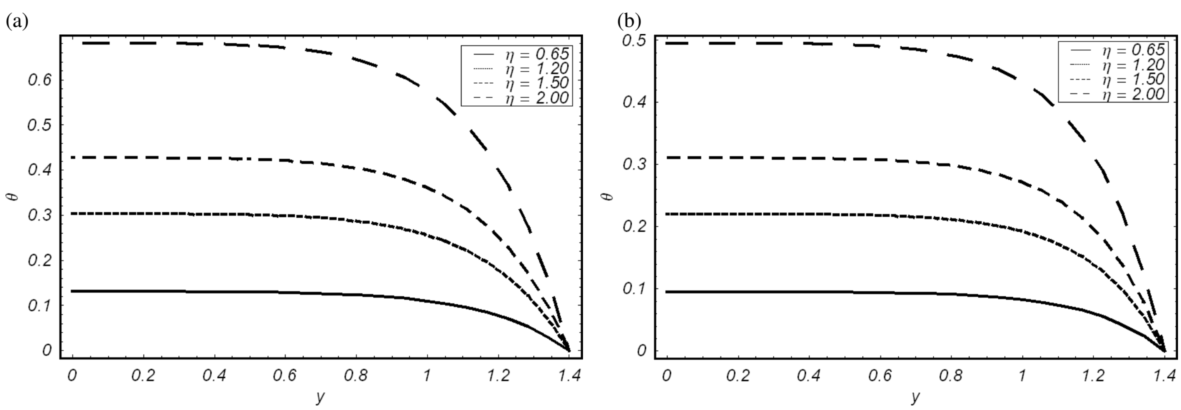

Fig. 5. Effects of η on temperature for $\phi = 0.4$, $\beta = 0.02$, $B_r = 0.5$, $M = 2$, $x = 0$.

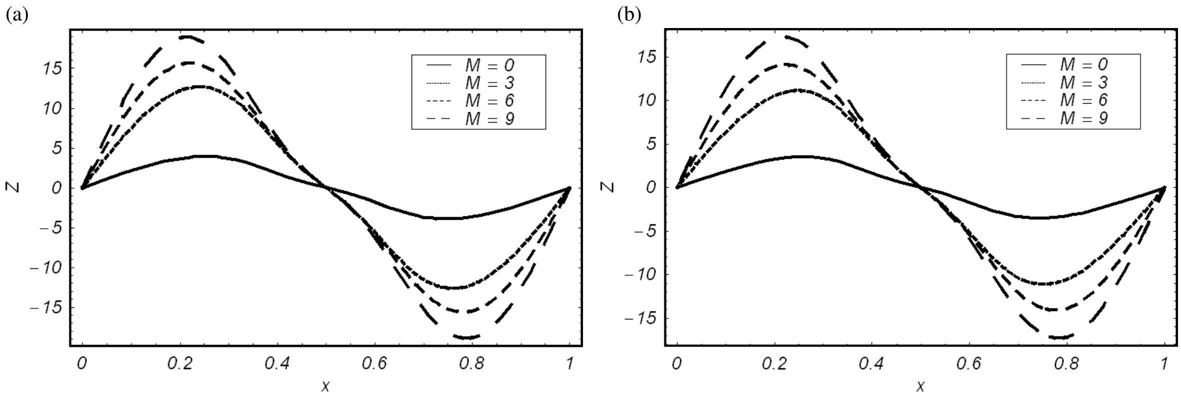

Fig. 6. Effects of M on heat transfer coefficient when $\eta = 0.7$, $\phi = 0.5$, $\beta = 0.02$, $B_r = 3$, $y = h$.

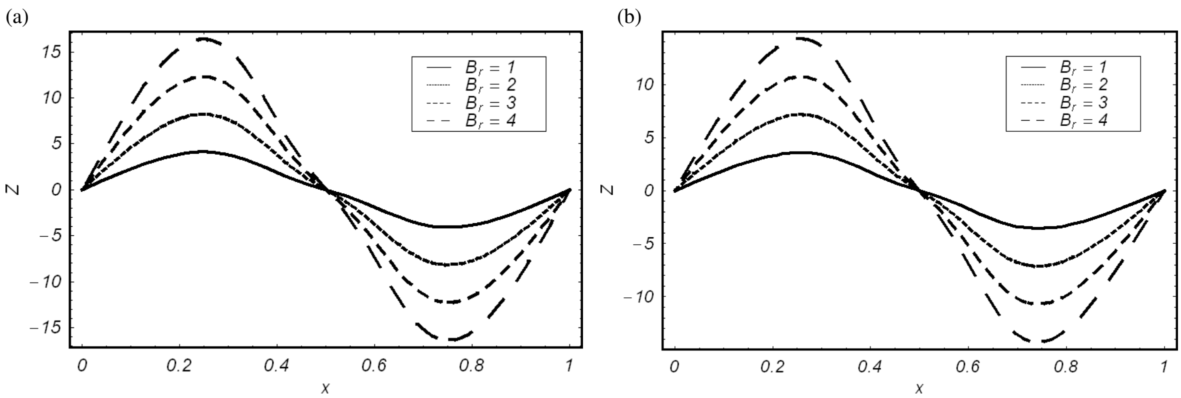

Fig. 7. Effects of B_r on heat transfer coefficient when $\eta = 0.7$, $\phi = 0.5$, $\beta = 0.02$, $M = 2$, $y = h$.

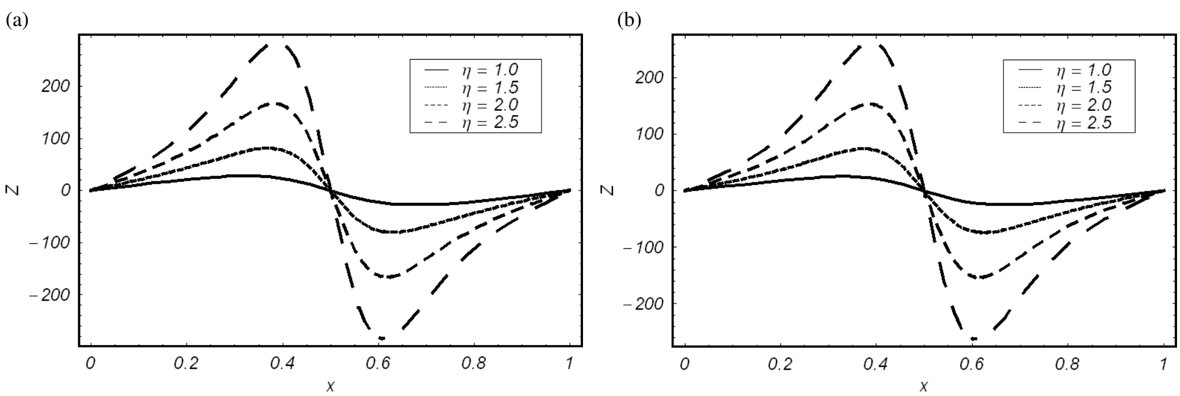

Fig. 8. Effects of η on heat transfer coefficient when $\phi = 0.5$, $\beta = 0.02$, $B_r = 3$, $M = 2$, $y = h$.

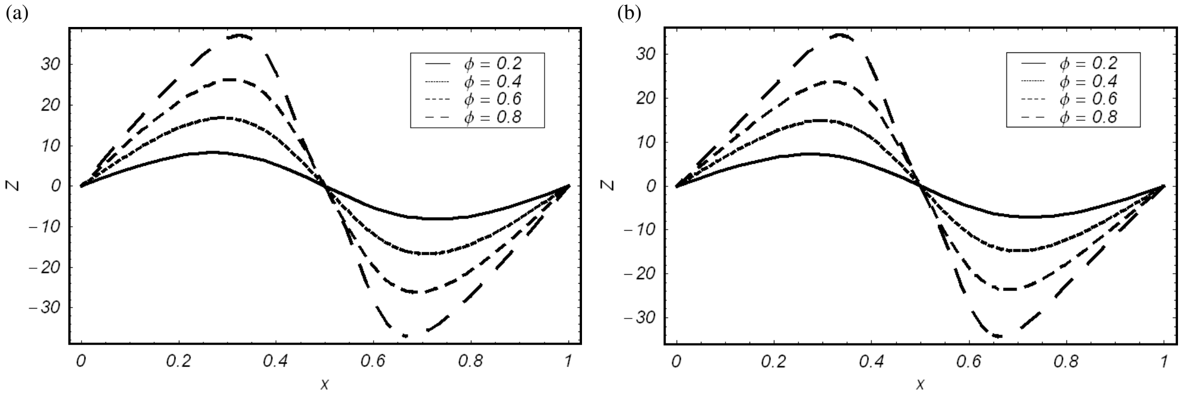

Fig. 9. Effects of ϕ on heat transfer coefficient when $\eta = 0.9$, $B_r = 3$, $\beta = 0.02$, $M = 2$, $y = h$.

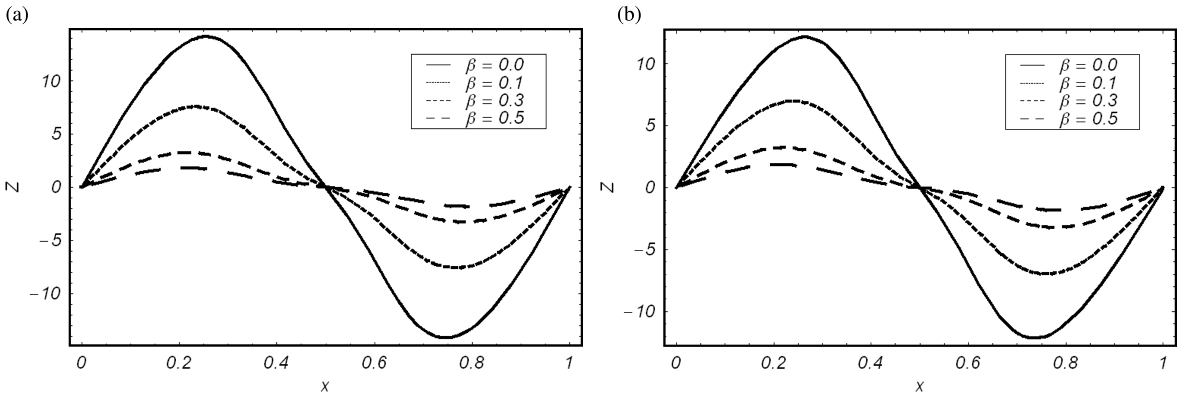

Fig. 10. Effects of β on heat transfer coefficient when $\eta = 0.7$, $\phi = 0.5$, $B_r = 3$, $M = 2$, $y = h$.

4. Discussion

Our main purpose here is to analyze the temperature and heat transfer coefficient for several values of Hartman number (M), Brinkman number (B_r), amplitude ratio (ϕ), slip parameter (β), and mean flow rate in the laboratory frame (η). Such variations have been displayed in Figures 1–5. In all figures, the used values of α for left and right panels are $\alpha = 0$ and $\alpha = 0.2$, respectively. The Figures 1, 2, 4, and 5 depict that a significant variation in θ occurs at the centre of the channel. It is noticed that there is no significant variation in θ near the upper wall. Under symmetric flow situation, there is also no significant change near the lower wall. However, Figure 3 indicates that the significant variation in temperature occurs near the upper wall.

Figure 1a–d shows the variation of the Hartman number (M) on temperature for both cases of constant and variable viscosities. It is found that an increase in M shows a decrease in temperature. The temperature decelerates for the fluid with variable viscosity. This observation is also true for the rest of the figures. We have observed that there is no change in the behaviour of temperature distribution when x is increased. Figure 2 shows that θ is an increasing function of B_r. Figure 3 displays the effects of ϕ (the amplitude ratio) on θ. Here we note that with an increase in ϕ, the temperature increases. The amplitude of temperature increases when channel width is increased. Figure 4 represents the effects of slip parameter β on θ. There is a decrease in θ when β increases. The effects of the mean flow rate (η) on the temperature distribution (θ) are plotted in Figure 5. We found that θ increases by increasing η. Through Figures 1 – 5, we observe that the parameters M and β have the similar effects on the temperature distribution. Also the effects of other parameters B_r, ϕ, and η on θ are qualitatively similar. However, it is noted that the effects of parameters M, β, and B_r, ϕ, η on the temperature are opposite.

Figures 6 – 10 are prepared for the variations of M, B_r, ϕ, η and β on the heat transfer coefficient at the upper wall. All these figures depict that the heat transfer coefficient Z has an oscillatory behaviour for the variation in one parameter and fixed values of the other parameters. Like temperature distribution, heat transfer coefficient decelerates the fluid with variable viscosity. This observation is true in all figures for Z. We also note that the absolute value of heat transfer coefficient increases by increasing M, B_r, η, and ϕ, but it decreases when the slip parameter β is increased.

5. Concluding Remarks

This study theoretically discusses the problem of heat transfer in peristaltic transport of a MHD viscous fluid with variable viscosity. Analysis is given in the presence of a slip condition. The expression of the stream function, the temperature, and the heat transfer coefficient is derived analytically. The effects of pertinent parameters on the temperature and heat transfer coefficient have been investigated. The temperature is found to decrease by increasing M and β and it increases by increasing B_r, ϕ, and η. The heat transfer coefficient has an oscillatory behaviour which may be due to the peristalsis. There is no qualitative change in the behaviour of the absolute value of the heat transfer coefficient.

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