# Is the Phase of Plane Waves an Invariant?

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Based on the invariance of the phase of waves, plane waves were shown to propagate with *negative frequencies* in a medium which moves at 'superluminal' speed opposite to the propagation direction of the plane waves. The validity of the invariance of the phase of plane waves was then called into question. A radical change of the conventional concept of plane waves is recently proposed to solve the problem of negative frequency of waves. We will explicitly point out flaws in that proposal. Thus, the validity of the invariance of the phase of plane waves remains questionable.

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The invariance of the phase of plane waves is an important concept in physics. Is the phase of plane waves a frame-independent quantity? To almost all physicists, the answer is yes. Nevertheless, the invariance of the phase of plane waves has never been proved; it is only postulated or argued to be valid. From the invariance of the phase of plane waves, the 4-vector $(\omega/c, \boldsymbol{k})$ of plane waves is shown to be Lorentz-covariant. Surprisingly, based on the Lorentz-covariance of the 4-vector $(\omega/c, \boldsymbol{k})$, plane waves are shown to propagate with *negative frequencies* in a medium which moves at superluminal speed opposite to the propagation direction of the plane waves [1]. Most of physicists are unfamiliar with this newly-found problem of negative frequency of waves; hence, we first recapitulate this problem.

Referring to Figure 1, consider monochromatic waves propagating in a medium at rest in the frame $S'$. This frame $S'$ moves with a constant velocity $\boldsymbol{V}$ along the negative $x$-axis direction with respect to the frame $S$. For simplicity, the wave vector $\boldsymbol{k}'$ of the waves is chosen to be parallel to the $(x', y')$ coordinate plane and it makes an angle $\theta'$ with the $x'$-axis. Based on the invariance of the phase of waves, the transformation for the 4-vector $(\omega/c, \boldsymbol{k})$ of wave motion is the usual transformation [2, 3]

$$\frac{\omega}{c} = \gamma\Big(\frac{\omega'}{c} - \beta\, k'_x\Big), \qquad (1)$$
$$k_x = \gamma\Big(k'_x - \beta\,\frac{\omega'}{c}\Big), \quad k_y = k'_y, \quad k_z = k'_z,$$

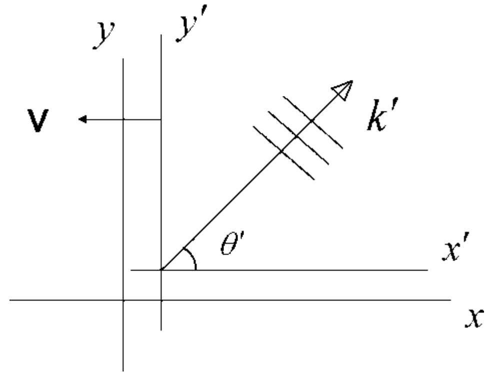


Fig. 1. Frame $S'$ moves with a constant velocity **V** along the negative $x$-axis direction, with respect to the frame $S$. The frame $S$ is schematically represented by the $(x, y)$ coordinate system. Monochromatic waves propagate with the wave vector $\boldsymbol{k}'$ relative to the frame $S'$. The direction of the wave vector $\boldsymbol{k}'$ makes an angle $\theta'$ with the $x'$-axis of the frame $S'$.

where $\gamma = (1-\beta^2)^{-1/2}$ with $\beta = V/c$. Consider the particular case that light waves in the medium propagate with a frequency $\omega'$ in the positive $x$-axis direction. Thus, we have $k'_x = k'$, and $\omega'/k' = v'$, where $v'$ is the speed of the light waves in the medium. Also, assume that the speed of the medium (the frame $S'$) relative to the frame $S$ is larger than the speed of light waves in the medium relative to the frame $S'$, that is $v' < V$. By the above transformation (1), we have

$$\frac{\omega}{c} = \gamma\frac{\omega'}{c}\left(1 - \frac{V}{v'}\right) < 0. \qquad (2)$$

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This shows that the light waves propagate with *negative frequency* with respect to the frame $S$. However, that light waves propagate with negative frequency is physically unreasonable.

Recently, a new concept of plane waves is proposed to resolve the problem of negative frequency of waves [4]. New forms of the 'phase' $\Phi = \boldsymbol{k}\cdot\boldsymbol{r} - \boldsymbol{k}\cdot\boldsymbol{u}t$ and of the frequency $\omega = |\boldsymbol{k}\cdot\boldsymbol{u}|$ for plane waves are introduced to replace the conventional one $\phi = \boldsymbol{k}\cdot\boldsymbol{r} - \omega t$. According to the proposal, with the invariance of the new 'phase' of plane waves and the Lorentz-covariant 4-vector $(ct, \boldsymbol{r})$, the relativistic transformations of $\boldsymbol{u}$, $\boldsymbol{k}$, and $\omega$ are obtained there as [4]

$$k_x = \gamma(k'_x + \frac{V}{c^2}\boldsymbol{k'}\cdot\boldsymbol{u'}), \quad k_y = k'_y, \quad k_z = k'_z, \tag{3}$$

$$\frac{\boldsymbol{k}\cdot\boldsymbol{u}}{c} = \gamma\left(\frac{k'_x V}{c} + \frac{\boldsymbol{k'}\cdot\boldsymbol{u'}}{c}\right), \tag{4}$$

$$u_x = \frac{u'_x + V}{1 + \frac{u'_x V}{c^2}}, \quad u_y = \frac{u'_y/\gamma}{1 + \frac{u'_x V}{c^2}}, \quad u_z = \frac{u'_z/\gamma}{1 + \frac{u'_x V}{c^2}}, \tag{5}$$

$$\omega' = |\boldsymbol{k'}\cdot\boldsymbol{u'}|, \quad \omega = |\boldsymbol{k}\cdot\boldsymbol{u}|, \tag{6}$$

where $\boldsymbol{V}$ is the relative velocity along the $x$-axis between the frames. For the particular case that the medium moves at superluminal speed opposite to the propagation direction of plane waves, the negative frequency problem is resolved in accordance with these new relativistic transformations.

In the proposal, it is claimed that the appearance of the negative frequency of waves is due to an ignorance of the effect of relativistically-induced *optical* anisotropy because of the motion of the medium. Usually, only the Lorentz transformation is used in studying electromagnetic waves in moving media – not is this claimed effect taken into account [5–8]. Similarly, only the Lorentz transformation is considered in investigating the relativistic *acoustic* Doppler effect [9–11]. By the same arguments above, one will find that acoustic waves propagate with negative frequencies in a medium which moves, against the propagation of the acoustic waves, with a speed larger than the speed of the acoustic waves. This means that the negative frequency problem is not limited to the *optical* case. Suppose that the effect of relativistically-induced anisotropy is not just a red herring, then this effect is not limited solely to the *optical* case, as claimed by the proposal. Also, there is no *dynamical* effect due to the relative motion. The problem of negative frequency is a byproduct of the Lorentz transformation, irrespective of optical and mechanical properties of the medium, as long as the medium is assumed uniform and isotropic. Then, the crucial question is how to resolve the negative frequency problem. To resolve the negative frequency problem, the proposal alters the conventional concept of waves, but preserves the invariance of the phase of waves. As a consequence, new relativistic transformations, (3)–(6), are introduced to replace the usual Lorentz transformation of the 4-vector $(\omega/c, \boldsymbol{k})$. To the contrary, we have resolved this problem by introducing an additional relativistic transformation – the dual Lorentz transformation [1]. The dual Lorentz transformation can not be derived from the invariance of the phase of plane waves. Furthermore, the Doppler effect is formulated without assuming the invariance of the phase of plane waves [12]. The invariance of the phase of plane waves has never been theoretically or experimentally verified. Thus, the validity of the invariance of the phase of plane waves is called into question.

In the following, we will show that the proposal not only casts conceptual problems on the characterization of plane waves, but also has a consequence that contradicts with the relativistic transverse Doppler effect. Conventionally, the wavefront of plane waves is clearly described by the wave vector $\boldsymbol{k}$, and the *phase velocity* is defined precisely as $\boldsymbol{v} = (\omega/k)\hat{\boldsymbol{k}}$. Based on the definition of the phase velocity, one can certainly determine the phase velocity of waves propagating in the medium which is either at rest or in motion. The new concept of plane waves by the proposal substantially alters the conventional concept of plane waves. The new velocity $\boldsymbol{u}$ of plane waves is simply described as the velocity of the profile of plane waves – its precise definition is not given. This kind of description of the velocity $\boldsymbol{u}$ is only applicable to the case that plane waves propagate in the medium-rest frame. Based on this kind of description, one could determine the velocity $\boldsymbol{u}$ of plane waves propagating in the medium which is in motion?

We explicitly point out flaws in the new concept of plane waves. With respect to the medium-rest frame $S'$, the wave vector $\boldsymbol{k'}$ and the velocity $\boldsymbol{u'}$ of the plane waves are in the same direction, and $u' = \omega'/k'$. From (3)–(6), we have

$$\begin{aligned} k_x &= \gamma\left(k'\cos\theta' - V\,\omega'/c^2\right), \\ k_y &= k'\sin\theta', \\ \boldsymbol{k}\cdot\boldsymbol{u} &= \gamma\omega'\left(1 - V\cos\theta'/u'\right), \end{aligned} \tag{7}$$

and

$$u_x = \frac{u' \cos\theta' - V}{1 - u' \cos\theta' V/c^2},$$
$$u_y = \frac{u' \sin\theta'}{\gamma\,(1 - u' \cos\theta' V/c^2)}. \tag{8}$$

Then, from (7) and (8), we obtain

$$\frac{k_y}{k_x} = \frac{\sin\theta'}{\gamma\,(\cos\theta' - V\,u'/c^2)} \tag{9}$$

and

$$\frac{u_y}{u_x} = \frac{\sin\theta'}{\gamma\,(\cos\theta' - V/u')}. \tag{10}$$

According to (9) and (10), with respect to the frame $S$, the wave vector $\boldsymbol{k}$ and the velocity $\boldsymbol{u}$ of the plane waves are in general not along the same direction, except in the case that light waves propagate in vacuum, since $u' = c$. Without first knowing the wave vector $\boldsymbol{k'}$ and the velocity $\boldsymbol{u'}$ for the waves in the medium-rest frame $S'$, we can determine the phase velocity $\boldsymbol{v}$ of the plane waves with respect to the frame $S$, relative to which the medium is in motion. Contrarily, the given description of the velocity $\boldsymbol{u}$ by the proposal is incapable of enabling us to determine the velocity $\boldsymbol{u}$ of the plane waves with respect to the frame $S$, without first knowing the wave vector $\boldsymbol{k'}$ and the velocity $\boldsymbol{u'}$ for those waves in the medium-rest frame $S'$. The given description of the velocity $\boldsymbol{u}$ of plane waves is applicable only to the case that plane waves propagate in the medium-rest frame.

Now, let us examine the transverse Doppler effect in accordance with the new concept of plane waves. To find the frequency shift of plane waves propagating transversely with respect to the frame $S$, which criterion do we apply to judge whether or not plane waves propagate transversely: plane waves propagate along the direction of $\boldsymbol{u}$, or along the direction of $\boldsymbol{k}$? According to the new concept of plane waves, plane waves propagate with the velocity $\boldsymbol{u}$, not with the phase velocity $\boldsymbol{v} = (\omega/k)\hat{\boldsymbol{k}}$. In this sense, we let $u_x = 0$. Then, from (7) and (8), the frequency of the transverse plane waves with respect to the frame $S$ is obtained as $\omega = |\boldsymbol{k} \cdot \boldsymbol{u}| = \gamma|1 - (V/u')^2|\,\omega'$, where $\omega'$ is the frequency of those plane waves as observed in the medium-rest frame $S'$. Instead, suppose that we choose the other way, that plane waves propagate along the direction of the wave vector $\boldsymbol{k}$. Then, let $k_x = 0$. Similarly, from (7) and (8) the frequency of the transverse plane waves with respect to the frame $S$ is $\omega = |\boldsymbol{k} \cdot \boldsymbol{u}| = \omega'/\gamma$. This result gives the same transverse Doppler shift as that predicted by special relativity, and has been confirmed experimentally. By taking that plane waves propagate with the velocity $\boldsymbol{u}$ by the proposal, the transverse frequency shift as predicted is different from the experimentally-established one.

The new concept of plane waves of the proposal not only has conceptual problems on the characterization of plane waves, but also deduces a consequence that contradicts with the well-known relativistic transverse Doppler effect. Hence, the problem of negative frequency of waves is not resolved by the proposal. The validity of the invariance of the phase of plane waves remains questionable.

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[1] Y.-S. Huang, Europhys. Lett. **79**, 10006 (2007).
[2] J. D. Jackson, Classical Electrodynamics, third edition, John Wiley & Sons Inc., New York 1999, Sect. 11.3.
[3] W. Rindler, Introduction to Special Relativity, Clarendon Press, New York 1982, p. 48.
[4] A. Gjurchinovski, Europhys. Lett. **83**, 10001 (2008).
[5] P. Penfield, Jr., and H. A. Haus, Electrodynamics of moving medium, MIT Press, Cambridge, MA, 1967.
[6] J. Van Bladel, Relativity and Engineering, Springer-Verlag, New York 1984.
[7] U. Leonhardt and P. Piwnicki, Phys. Rev. Lett. **84**, 822 (2000).
[8] K. K. Nandi, Y.-Z. Zhang, P.M. Alsing, J.C. Evans, and A. Bhadra, Phys. Rev. D **67**, 025002 (2003).
[9] R. A. Bachman, Am. J. Phys. **50**, 816 (1982).
[10] R. E. Reynolds, Am. J. Phys. **58**, 390 (1990).
[11] G. Cook and T. Lesoing, Am. J. Phys. **59**, 218 (1991).
[12] Y.-S. Huang and K.-H. Lu, Can. J. Phys. **82**, 957 (2004).