A Chaotic Map and Data Communication

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We show how the symmetric tent map can be used for data communication.

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Data communication utilizing chaotic maps has been studied by various authors (see [1-4] and references therein). Here we consider the symmetric tent map $f : [-1,+1] \to [-1,+1]$

$$f(x) = \begin{cases} 2x+1, & \text{if} \quad -1 \le x \le 0, \\ -2x+1, & \text{if} \quad 0 \le x \le 1. \end{cases}$$

This is a fully chaotic map with Ljapunov exponent $\lambda = \ln(2)$ and invariant density $\rho = 1/2$ [5,6]. The fixed points are $x^* = -1$ and $x^* = 1/3$ which are unstable. The essential part for the data transmission is that $f(x) = f(-x)$ and $f : [-1,+1] \to [-1,+1]$. Thus we have the iteration $x_{t+1} = f(x_t)$, where $t = 0,1,\ldots$ and $x_0 \in [-1,+1]$ is the initial value.

We generate a sequence of length T from the map, i. e. $x_0, x_1, \ldots, x_{T-1}$. This will be the transmitter. Given now the bitstring $\mathbf{b} = (b_0, b_1, \ldots, b_{T-1})$ for the signal of length T, where $b_t \in \{-1,+1\}$. Next we form the transmitted signal $\mathbf{s} = (s_0, s_1, \ldots, s_{T-1})$ via

$$s_t = b_t x_t, \qquad t = 0,1,\ldots,T-1\,.$$

The receiver is now given by

$$y_{t+1} = f(s_t), \qquad t = 0,1,\ldots,T-2,$$

where $y_0 = x_0$ and f is the symmetric tent map given above. The original bitsequence $\mathbf{b}$ can now found by forming the products

$$s_t y_t, \qquad t = 0,1,\ldots,T-1.$$

If $s_t y_t > 0$, then $b_t = 1$ and if $s_t y_t < 0$, then $b_t = -1$. The proof is as follows

$$\begin{aligned} s_t y_t &= s_t f(s_{t-1}) \\ &= s_t f(b_{t-1} x_{t-1}) \\ &= s_t f(x_{t-1}), \quad \text{since } f(x_t) = f(-x_t) \\ &= b_t x_t f(x_{t-1}) \\ &= b_t x_t^2\,. \end{aligned}$$

Consequently,

$$\text{sign}(s_t y_t) = \text{sign}(b_t x_t^2) = b_t\,.$$

This scheme does not require a limit on the bit string length since divergence of the sequence only introduces local errors. The values s_t of the sequence are transmitted and operated on directly by the receiver. This scheme does not rely on the ability to find and control a specific orbit for communication [7].

Other chaotic maps g with the properties $g : [-1,+1] \to [-1,+1]$ and $g(x) = g(-x)$ can also be used such as the logistic map

$$g(x) = 1 - 2x^2$$

or the bungalow-tent map $f_r : [-1,1] \to [-1,1]$ ($r \in (0,1/2)$)

$$f_r(x) = \begin{cases} \frac{1-r}{r}(1+x) - 1, & \text{if} \quad x \in [-1, 2r-1), \\ \frac{2r}{1-2r}(1+x) + \frac{1-4r}{1-2r}, & \text{if} \quad x \in [2r-1, 0), \\ \frac{2r}{1-2r}(1-x) + \frac{1-4r}{1-2r}, & \text{if} \quad x \in [0, 1-2r), \\ \frac{1-r}{r}(1-x) - 1, & \text{if} \quad x \in [1-2r, 1], \end{cases}$$

which contains a bifurcation parameter [5,6]. For $r = 1/3$ we obtain the map (1). A SymbolicC++ implementation [8] of this communication scheme is available from the authors.

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