A Chaotic Map and Data Communication

Willi-Hans Steeb and Yorick Hardy

International School for Scientific Computing, University of Johannesburg, Auckland Park 2006, South Africa
Reprint requests to W.-H.S.; E-mail: steebwilli@gmail.com

Z. Naturforsch. 65a, 613 – 614 (2010); received March 27, 2010 / revised May 19, 2010

We show how the symmetric tent map can be used for data communication.

Key words: Chaotic Maps; Communication Schemes; Bit Sequence.

Data communication utilizing chaotic maps has been studied by various authors (see [1-4] and references therein). Here we consider the symmetric tent map \( f : [-1, +1] \rightarrow [-1, +1] \)

\[
f(x) = \begin{cases} 
2x + 1, & \text{if } -1 \leq x \leq 0, \\
-2x + 1, & \text{if } 0 \leq x \leq 1. 
\end{cases}
\]

This is a fully chaotic map with Ljapunov exponent \( \lambda = \ln(2) \) and invariant density \( \rho = 1/2 \) [5, 6]. The fixed points are \( x^* = -1 \) and \( x^* = 1/3 \) which are unstable. The essential part for the data transmission is that \( f(x) = f(-x) \) and \( f : [-1, +1] \rightarrow [-1, +1] \). Thus we have the iteration \( x_{t+1} = f(x_t) \), where \( t = 0, 1, \ldots \) and \( x_0 \in [-1, +1] \) is the initial value.

We generate a sequence of length \( T \) from the map, i.e. \( x_0, x_1, \ldots, x_{T-1} \). This will be the transmitter. Given now the bitstring \( b = (b_0, b_1, \ldots, b_{T-1}) \) for the signal of length \( T \), where \( b_t \in \{-1, +1\} \). Next we form the transmitted signal \( s = (s_0, s_1, \ldots, s_{T-1}) \) via

\[
s_t = b_t x_t, \quad t = 0, 1, \ldots, T - 1.
\]

The receiver is now given by

\[
y_{t+1} = f(s_t), \quad t = 0, 1, \ldots, T - 2,
\]

where \( y_0 = x_0 \) and \( f \) is the symmetric tent map given above. The original bitsequence \( b \) can now found by forming the products

\[
s_t y_t, \quad t = 0, 1, \ldots, T - 1.
\]

If \( s_t y_t > 0 \), then \( b_t = 1 \) and if \( s_t y_t < 0 \), then \( b_t = -1 \).

The proof is as follows

\[
s_t y_t = s_t f(s_{t-1}) = s_t f(b_{t-1} x_{t-1}) = s_t f(x_{t-1}), \quad \text{since } f(x_t) = f(-x_t)
\]

\[
= b_t x_t f(x_{t-1}) = b_t x_t^2.
\]

Consequently,

\[
\text{sign}(s_t y_t) = \text{sign}(b_t x_t^2) = b_t.
\]

This scheme does not require a limit on the bit string length since divergence of the sequence only introduces local errors. The values \( s_t \) of the sequence are transmitted and operated on directly by the receiver. This scheme does not rely on the ability to find and control a specific orbit for communication [7].

Other chaotic maps \( g \) with the properties \( g : [-1, +1] \rightarrow [-1, +1] \) and \( g(x) = g(-x) \) can also be used such as the logistic map

\[
g(x) = 1 - 2x^2
\]

or the bungalow-tent map \( f_r : [-1, 1] \rightarrow [-1, 1] \) \((r \in (0, 1/2))\)

\[
f_r(x) = \begin{cases} 
\frac{1-r}{r}(1+x) - 1, & \text{if } x \in [-1, 2r - 1), \\
\frac{4r}{1-2r}(1+x) + \frac{1-r}{r}, & \text{if } x \in [2r - 1, 0), \\
\frac{4r}{1-2r}(1-x) + \frac{1-r}{r}, & \text{if } x \in [0, 1 - 2r), \\
\frac{1-r}{r}(1-x) - 1, & \text{if } x \in [1 - 2r, 1],
\end{cases}
\]

which contains a bifurcation parameter [5, 6]. For \( r = 1/3 \) we obtain the map (1). A SymbolicC++ implementation [8] of this communication scheme is available from the authors.