Interplay Between Dispersion and Nonlinearity in Femtosecond Soliton Management

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In this paper, we investigate the inhomogeneous higher-order nonlinear Schrödinger (NLS) equation governing the femtosecond optical pulse propagation in inhomogeneous fibers using gauge transformation and generate bright soliton solutions from the associated linear eigenvalue problem. We observe that the amplitude of the bright solitons depends on the group velocity dispersion (GVD) and the self-phase modulation (SPM) while its velocity is dictated by the third-order dispersion (TOD) and GVD. We have shown how the interplay between GVD, SPM, and TOD can be profitably exploited to change soliton width, amplitude (intensity), shape, phase, velocity, and energy for an effective femtosecond soliton management. The highlight of our paper is the identification of 'optical similaritons' arising by virtue of higher-order effects in the femtosecond regime.

Key words: Inhomogeneous Nonlinear Schrödinger (NLS) Equation; Bright Femtosecond Solitons; Gauge Transformation.

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1. Introduction

The identification of optical solitons in a dielectric fiber by Hasegawa and Tappert [1] and the subsequent experimental verification by Mollenauer et al. [2] have made a huge turnaround in the field of communications. It is known that the propagation of these temporal solitons whose pulsewidth is of the order of picoseconds is governed by a nonlinear Schrödinger (NLS) equation involving group velocity dispersion (GVD) and self-phase modulation (SPM) [3 – 12]. These picosecond pulses which are regarded as the natural data bit constitute the mainstay of soliton based transmission systems delivering information in excess of 1 Tb/s [13]. There has been a relentless demand for increasing the bandwidth of these transmission systems. This can be realized by shortening the width of the picosecond pulses which subsequently takes them into the femtosecond regime.

For such pulses of shorter duration, in addition to self-phase modulation associated with the standard NLS equation, several higher-order nonlinear effects like self-steepening and delayed nonlinear response will have to be included in the evolution equation to shrink the pulse width. To counterbalance these higher-order nonlinear effects, the third-order dispersion in addition to GVD will also have to be considered. Accordingly, the dynamics of such shorter pulses whose width lies in the femtosecond region is now governed by a higher-order NLS equation. However, it should also be mentioned that in realistic transmission lines no fiber is homogeneous due to long distance communication and manufacturing defects. Thus, the propagation of femtosecond pulses in inhomogeneous fibers is governed by the following inhomogeneous higher-order NLS equation of the form [14]

$$q_z = \mathrm{i}\alpha_1(z)q_{tt} + \mathrm{i}\alpha_2(z)|q|^2q + \alpha_3(z)q_{ttt} + \alpha_4(z)(|q|^2q)_t + \alpha_5(z)q(|q|^2)_t + \Gamma(z)q, \quad (1)$$

where $q(z,t)$ represents the complex envelope of the electrical field, z is the normalized propagation distance, t is the normalized retarded time. In the above equation, $\alpha_1(z)$, $\alpha_2(z)$, $\alpha_3(z)$, $\alpha_4(z)$, and $\alpha_5(z)$ are the fiber parameters which are functions of the propagation distance and are related to group velocity dispersion (GVD), self-phase modulation (SPM), third-order dispersion (TOD), self-steepening, and the delayed nonlinear response effect, respectively. $\Gamma(z)$ represents the amplification or absorption coefficient. It would be interesting to investigate (1) to understand the effect of decreased pulsewidth on the propagation of information through optical fibers. It should be mentioned that (1) is integrable under the following parametric restriction:

$$3\alpha_2\alpha_3 = \alpha_4\alpha_1, \qquad \alpha_4 + \alpha_5 = 0, \tag{2a}$$

$$\Gamma(z) = \frac{\alpha_{1z}\alpha_2 - \alpha_1\alpha_{2z}}{2\alpha_1\alpha_2}. \tag{2b}$$

For the above parametric choice (1) has been already investigated using the Hirota method [15, 16], Darboux transformation [17], and ansatz method [18], and bright solitons have been obtained while dark solitons have also been derived using ansatz approach [19]. Recently, (1) has also been analysed numerically [20] and combined solitary waves (both darklike and brightlike solitary waves) were obtained. In this paper, we investigate (1) using gauge transformation approach and generate bright soliton solution from the associated linear eigenvalue problem starting from a trivial input solution. We show that one can control the shape, velocity, amplitude, exchange of energy, and trajectories by suitably manipulating the fiber parameters.

Equation (1) admits the following linear eigenvalue problem [17, 18]:

$$\Psi_t = U\Psi = \begin{pmatrix} \lambda & \sqrt{\frac{\alpha_2}{2\mu\alpha_1}}q \\ -\mu\sqrt{\frac{\alpha_2}{2\mu\alpha_1}}q^* & -\lambda \end{pmatrix}, \tag{3a}$$

$$\Psi_z = V\Psi = \begin{pmatrix} A & B \\ C & -A \end{pmatrix}, \tag{3b}$$

where

$$\begin{aligned} A &= 4\alpha_3\lambda^3 + 2\mathrm{i}\alpha_1\lambda^2 + \frac{\alpha_2\alpha_3}{\alpha_1}|q|^2\lambda \\ &\quad + \frac{\alpha_2\alpha_3}{2\alpha_1}(q^*q_t - qq_t^*) + \mathrm{i}\frac{\alpha_2}{2}|q|^2, \\ B &= \sqrt{\frac{\alpha_2}{2\mu\alpha_1}}\Big[4\alpha_3 q\lambda^2 + 2(\alpha_3 q_t + \mathrm{i}\alpha_1 q)\lambda \\ &\quad + \alpha_3\left(q_{tt} + \frac{\alpha_2}{\alpha_1}q|q|^2\right) + \mathrm{i}\alpha_1 q_t\Big], \\ C &= \sqrt{\frac{\alpha_2}{2\mu\alpha_1}}\Big[-4\mu\alpha_3 q^*\lambda^2 + 2\mu(\alpha_3 q_t^* - \mathrm{i}\alpha_1 q^*)\lambda \\ &\quad - \mu\alpha_3\left(q_{tt}^* + \frac{\alpha_2}{\alpha_1}q^*|q|^2\right) + \mathrm{i}\mu\alpha_1 q_t^*\Big]. \end{aligned}$$

The compatibility condition $U_z - V_t + [U,V] = 0$ (derived from $(\Psi_t)_z = (\Psi_z)_t$) generates (1) with the amplification/absorption coefficient $\Gamma(z)$ (gain/loss) given by (2b). It should be mentioned that eventhough variable coefficient NLS type equations are in general associated with the non-isospectral parameter λ, the constraints given by (2a) and (2b), which represent the integrability condition, convert the non-isospectral λ to a constant spectral parameter.

2. Construction of Femtosecond Solitons and their Interaction

To generate bright solitons, we consider a vacuum solution $q^{(0)} = 0$ to obtain the following vacuum linear systems:

$$\Psi_t^{(0)} = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda \end{pmatrix}\Psi^0 = U^{(0)}\Psi^{(0)}, \tag{4a}$$

$$\begin{aligned} \Psi_z^{(0)} &= \begin{pmatrix} 4\alpha_3\lambda^3 + 2\mathrm{i}\alpha_1\lambda^2 & 0 \\ 0 & -4\alpha_3\lambda^3 - 2\mathrm{i}\alpha_1\lambda^2 \end{pmatrix}\Psi^{(0)} \\ &= V^{(0)}\Psi^{(0)}. \end{aligned} \tag{4b}$$

Solving the above vacuum linear systems, we have the vacuum eigenfunction given by

$$\Psi^{(0)}(t,z) = \begin{pmatrix} \mathrm{e}^{\lambda t + 4\lambda^3\int\alpha_3\mathrm{d}z + 2\mathrm{i}\lambda^2\int\alpha_1\mathrm{d}z} & 0 \\ 0 & \mathrm{e}^{-\lambda t - 4\lambda^3\int\alpha_3\mathrm{d}z - 2\mathrm{i}\lambda^2\int\alpha_1\mathrm{d}z} \end{pmatrix}. \tag{5}$$

We now gauge transform the vacuum eigenfunction $\Psi^{(0)}(t,z)$ by a meromorphic function χ_1 to obtain a new eigenfunction $\Psi^{(1)}(t,z)$ given by

$$\Psi^{(1)}(t,z) = \chi_1\Psi^{(0)}(t,z). \tag{6}$$

Hence, the new eigenvalue problem takes the following form:

$$\Psi_t^{(1)} = U^{(1)}\Psi^{(1)}, \tag{7a}$$

$$\Psi_z^{(1)} = V^{(1)}\Psi^{(1)}, \tag{7b}$$

with

$$U^{(1)} = \chi_1 U^{(0)}\chi_1^{-1} + \chi_{1t}\chi_1^{-1}, \tag{8a}$$

$$V^{(1)} = \chi_1 V^{(0)}\chi_1^{-1} + \chi_{1z}\chi_1^{-1}, \tag{8b}$$

where χ_1 remains to be specified. Although $U^{(1)}$ and $V^{(1)}$ already satisfy the compatibility condition, it represents a valid solution $q^{(1)}$ only if $U^{(1)}$ and $V^{(1)}$ possess exactly the correct λ structure just as $U^{(0)}$ and $V^{(0)}$.

To ensure that $U^{(1)}$ and $V^{(1)}$ have the correct λ structure, the transformation function χ_1 must be adopted from the solution of a certain Riemann problem and it should be a meromorphic function in the complex λ plane taking the following simple form:

$$\chi_1 = \left(1 + \frac{\lambda_1 - \mu_1}{\lambda - \lambda_1} P_1(t,z)\right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (9)$$

where λ_1 and μ_1 are arbitrary complex parameters and P_1 is a projection matrix ($P_1^2 = P_1$). The inverse of the meromorphic function takes the form

$$\chi_1^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \left(1 - \frac{\lambda_1 - \mu_1}{\lambda - \mu_1} P_1(t,z)\right). \quad (10)$$

The fact that $U^{(1)}$ and $V^{(1)}$ do not develop singularities around $\lambda = \lambda_1$ and $\lambda = \mu_1$ restrains the choice of the projection matrix to be the solution of the following set of partial differential equations:

$$\begin{aligned} P_{1t} = &(1 - P_1)\sigma_3 U^{(0)}(\mu_1)\sigma_3 P_1 \\ &- P_1 \sigma_3 U^{(0)}(\lambda_1)\sigma_3(1 - P_1), \end{aligned} \quad (11a)$$

$$\begin{aligned} P_{1z} = &(1 - P_1)\sigma_3 V^{(0)}(\mu_1)\sigma_3 P_1 \\ &- P_1 \sigma_3 V^{(0)}(\lambda_1)\sigma_3(1 - P_1), \end{aligned} \quad (11b)$$

where

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

From the above, it is obvious that one can generate the projection matrix $P_1(t,z)$ using the vacuum eigenfunction $\Psi^{(0)}(t,z)$ as

$$P_1(t,z) = \sigma_3 \frac{M^{(1)}}{[\operatorname{trace} M^{(1)}]} \sigma_3, \quad (12a)$$

$$M^{(1)} = \Psi^{(0)}(t,z,\mu_1) \begin{pmatrix} m_1 & \frac{1}{n_1} \\ n_1 & \frac{1}{m_1} \end{pmatrix} \Psi^{(0)}(t,z,\zeta_1)^{-1}, \quad (12b)$$

where m_1 and n_1 are arbitrary complex constants. Thus, choosing $\lambda_1 = a_1 + \mathrm{i}b_1$ and $\mu_1 = \lambda_1^*$, the projection matrix can be explicitly written as

$$P_1(t,z) = \begin{pmatrix} \frac{1}{2}\operatorname{sech}(\theta_1)\mathrm{e}^{\theta_1} & -\frac{1}{2}\operatorname{sech}(\theta_1)\mathrm{e}^{\mathrm{i}\xi_1} \\ -\frac{1}{2}\operatorname{sech}(\theta_1)\mathrm{e}^{-\mathrm{i}\xi_1} & \frac{1}{2}\operatorname{sech}(\theta_1)\mathrm{e}^{-\theta_1} \end{pmatrix}, \quad (13)$$

where

$$\begin{aligned} \theta_1 = 2b_1\Big(t - 12a_1^2 \int \alpha_3(z)\mathrm{d}z \\ + 4b_1^2 \int \alpha_3(z)\mathrm{d}z - 4a_1 \int \alpha_1(z)\mathrm{d}z\Big) + 2\delta_1, \end{aligned} \quad (14a)$$

$$\begin{aligned} \xi_1 = 2a_1 t + 24a_1 b_1^2 \int \alpha_3(z)\mathrm{d}z - 8a_1^3 \int \alpha_3(z)\mathrm{d}z \\ - 4\left(a_1^2 - b_1^2\right) \int \alpha_1(z)\mathrm{d}z - 2\phi_1. \end{aligned} \quad (14b)$$

Now, substituting (11a), (9) in (8a), we get

$$\begin{aligned} U^1(\lambda) = &\begin{pmatrix} \lambda & \sqrt{\frac{\alpha_2}{2\mu\alpha_1}} q^{(0)} \\ -\mu\sqrt{\frac{\alpha_2}{2\mu\alpha_1}} q^{*(0)} & -\lambda \end{pmatrix} \\ &- 2\mathrm{i}(\lambda_1 - \mu_1) \begin{pmatrix} 0 & P_{1(12)} \\ -P_{1(21)} & 0 \end{pmatrix}. \end{aligned} \quad (15)$$

Hence, at $\mu = 1$, the bright femto soliton can be written as [21]

$$q^{(1)} = 2b_1 \sqrt{\frac{2\alpha_1(z)}{\alpha_2(z)}} \operatorname{sech}\theta_1 \mathrm{e}^{\mathrm{i}\xi_1}, \quad (16)$$

where δ_1, ϕ_1 are arbitrary real constants. The above bright soliton solution is identical to the one obtained using the Darboux transformation [17]. When $\alpha_3(z) = 0$ ($\alpha_4(z)$ and $\alpha_5(z)$ turn out be zero by virtue of (2a)) and $\alpha_1(z)$ and $\alpha_2(z)$ are constants, then the femtosoliton reduces to the standard NLS soliton with constant amplitude given by

$$\begin{aligned} q^{(1)} = &2b_1 \operatorname{sech}[2b_1(t - 4a_1 z + 2\delta_1)] \\ &\cdot \exp[\mathrm{i}(2a_1 t - 4(a_1^2 - b_1^2)z - 2\phi_1)]. \end{aligned} \quad (17)$$

Looking at (16), one understands that the amplitude of the femtosoliton $2b_1\sqrt{\frac{2\alpha_1(z)}{\alpha_2(z)}}$ depends on the coefficient of GVD ($\alpha_1(z)$) and SPM ($\alpha_2(z)$) while the velocity is determined by $(-12a_1^2 + 4b_1^2)\alpha_3(z) - 4a_1\alpha_1(z)$.

From the above, it is also evident that using the gauge transformation approach, one can derive the N-soliton solution from a trivial input solution and the vacuum eigenfunction $\Psi^0(t,z,\lambda)$ makes this approach quite superior to other analytical methods. For example, the two-soliton solution can be written as

$$q^{(2)} = \sqrt{\frac{2\alpha_1(z)}{\alpha_2(z)}} \frac{A_1 + A_2 + A_3 + A_4}{B_1 + B_2}, \quad (18)$$

where

$$A_1 = \{-2b_2[(a_2-a_1)^2-(b_1^2-b_2^2)] - 4\mathrm{i}b_1b_2(a_2-a_1)\}\mathrm{e}^{(\theta_1+\mathrm{i}\xi_2)},$$
$$A_2 = -2b_2[(a_2-a_1)^2+(b_1^2+b_2^2)]\mathrm{e}^{(-\theta_1+\mathrm{i}\xi_2)},$$
$$A_3 = \{-2b_1[(a_2-a_1)^2+(b_1^2-b_2^2)] + 4\mathrm{i}b_1b_2(a_2-a_1)\}\mathrm{e}^{(\mathrm{i}\xi_1+\theta_2)},$$
$$A_4 = -4\mathrm{i}b_1b_2[(a_2-a_1)-\mathrm{i}(b_1-b_2)]\mathrm{e}^{(\mathrm{i}\xi_1-\theta_2)},$$
$$B_1 = -4b_1b_2[\sinh(\theta_1)\sinh(\theta_2)+\cos(\xi_1-\xi_2)],$$
$$B_2 = 2\cosh(\theta_1)\cosh(\theta_2)[(a_2-a_1)^2+(b_1^2+b_2^2)],$$

and

$$\theta_k = 2b_k\left(t-12a_k^2\int\alpha_3(z)\mathrm{d}z+4b_k^2\int\alpha_3(z)\mathrm{d}z -4a_k\int\alpha_1(z)\mathrm{d}z\right)+2\delta_k,$$
$$\xi_k = 2a_kt+24a_kb_k^2\int\alpha_3(z)\mathrm{d}z-8a_k^3\int\alpha_3(z)\mathrm{d}z -4(a_k^2-b_k^2)\int\alpha_1(z)\mathrm{d}z-2\phi_k,$$
$$k=1,2.$$

Thus, one observes that an interplay between dispersion and nonlinearity can generate various femtosoliton profiles. In fact, we have identified the following favourable conditions to facilitate femtosoliton propagation in optical fibers.

Case (i): Periodically chirped bright soliton array

Allowing the fiber parameters $\alpha_1(z)$, $\alpha_2(z)$, and $\alpha_3(z)$ to vary periodically with the propagation distance z, we find that there is a periodic chirping of intensity of the bright soliton pulses along the fiber (Fig. 1). It should be mentioned that this periodic chirping of the intensity is brought about by periodic variations of the third-order dispersion $\alpha_3(z)$ after counter balancing GVD ($\alpha_1(z)$) and SPM ($\alpha_2(z)$) and chirping would disappear when $\alpha_3(z)$ becomes a constant.

Case (ii): Compression of bright soliton trains

When the effect of GVD, SPM, and TOD are all equal, one of the soliton trains gets compressed enormously during the interaction as shown in Figure 2 while the intensity of the other pulse decreases after the interaction. It is worth highlighting at this juncture that this enormous compression of one of the soliton pulses arises due to the transfer of energy from one bound state to the other. Such enormous compression of soliton pulses normally arises only in the Manakov model [22], a two component NLS equation, and one would not witness this in the standard NLS equation. This again underscores the role of third-order dispersion $\alpha_3(z)$.

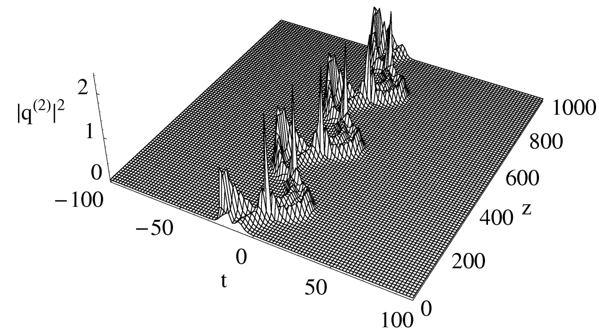

Fig. 1. Periodically chirped solitons for the parametric choice $a_1 = 0.2$, $a_2 = 0.25$, $b_1 = 0.15$, $b_2 = 0.25$, $\alpha_1(z) = \alpha_2(z) = \alpha_3(z) = 0.2\sin(0.02z)$, and $\Gamma(z) = 0$.

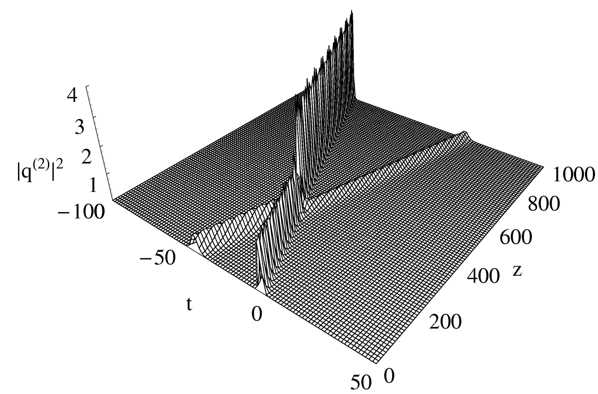

Fig. 2. Compression of soliton pulses with $a_1 = 0.1$, $a_2 = 0.09$, $b_1 = 0.2$, $b_2 = 0.5$, $\alpha_1(z) = \alpha_2(z) = \alpha_3(z) = 0.12$, and $\Gamma(z) = 0$.

Case (iii): Dispersion managed femtosolitons

Allowing the fiber parameters $\alpha_1(z)$, $\alpha_2(z)$, and $\alpha_3(z)$ to decrease exponentially with propagation distance z, one observes that the intensity of the two soliton pulses initially decreases and then later settles down at a constant value and propagates undistorted for about 1000 dispersion lengths as shown in Figure 3. Thus, it is clear that the combined effects of exponentially decreasing both dispersion and nonlinearity coefficient restricts the interaction between neighbouring solitons to a great extent. It can be observed that even-

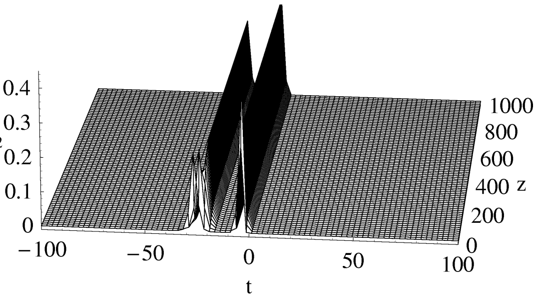

Fig. 3. Dispersion managed femtosolitons with $a_1 = 0.3$, $a_2 = 0.5$, $b_1 = 0.5$, $b_2 = 0.25$, $\alpha_1(z) = 0.12\exp[-0.1z]$, $\alpha_2(z) = 0.52\exp[-0.1z]$, $\alpha_3(z) = 0.02\exp[-0.025z]$, and $\Gamma(z) \approx 0\,(-6.950\times 10^{-18})$.

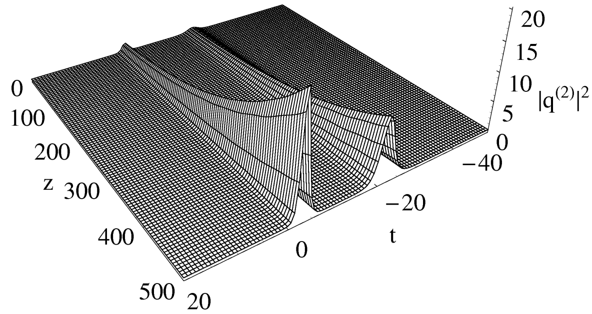

Fig. 5. Continuous enhancement of amplitude and width of 'optical similaritons' with $a_1 = 0.3$, $a_2 = 0.5$, $b_1 = 0.5$, $b_2 = 0.25$, $\alpha_1(z) = \alpha_3(z) = \cos(0.1z)$, $\alpha_2(z) = \cos(0.1z)\exp(-0.005z)$, and $\Gamma(z) = 0.0025$.

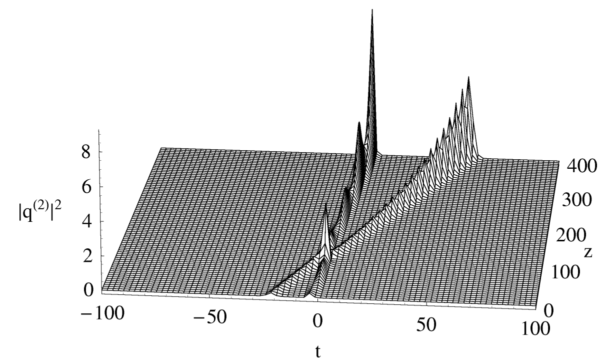

Fig. 4. Divergent ultrashort solitons for the choice $a_1 = 0.3$, $a_2 = 0.5$, $b_1 = 0.5$, $b_2 = 0.25$, $\alpha_1(z) = 0.02$, $\alpha_2(z) = 0.1\exp[-0.01z]$, $\alpha_3(z) = 0.052$, and $\Gamma(z) = 0.005$.

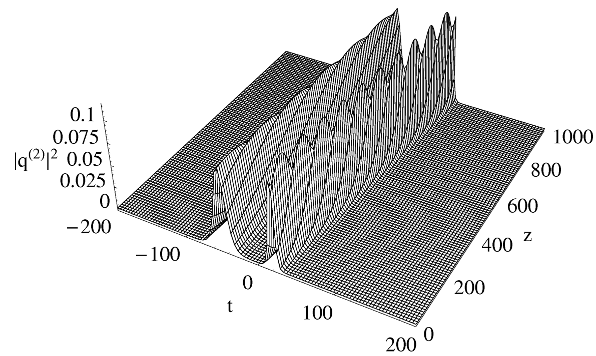

Fig. 6. Two non-interacting solitons for the following parametric choice $a_1 = 0.1$, $a_2 = 0.06$, $b_1 = 0.12$, $b_2 = 0.05$, $\alpha_1(z) = 0.1$, $\alpha_2(z) = 0.1$, $\alpha_3(z) = 0.02$, and $\Gamma(z) = 0$.

though the gain $\Gamma(z)$ is negative in this case, it is negligibly small and hence there is neither amplification nor absorption of the pulses as they travel along the fiber.

Case (iv): Divergent ultrashort solitons

Allowing the fiber parameter $\alpha_2(z)$ to decrease exponentially with z and keeping $\alpha_1(z) < \alpha_3(z)$, we observe that the two solitons not only get compressed, but also traverse along two different trajectories and the positive gain indicates the transfer of energy from the fiber to the pulses during propagation (Figure 4). From this, one understands that this complete divergence (or isolation of two soliton pulses) is again brought about by the condition $\alpha_3(z) > \alpha_1(z)$. Thus, it is obvious that the compression of the pulses is brought about by both SPM ($\alpha_2(z)$) and the positive gain $\Gamma(z)$.

Case (v): Optical similaritons

Allowing $\alpha_1(z)$ and $\alpha_3(z)$ to vary periodically with z while driving the SPM by a periodic damping function of z, one not only witnesses continuous enhancement of intensities of soliton pulses, but also continuum enhancement of width as well. One calls such solutions as 'optical similaritons' [23] whose amplitude and width simply scale with propagation distance. It is also obvious that the positive gain $\Gamma(z)$ which is a function of propagation distance z arising from the inhomogeneities (defects) in the fiber continuously feeds the soliton trains enhancing their intensities as they propagate along the fiber (Fig. 5).

Case (vi): Two non-interacting soliton trains

Perfect balance between GVD and SPM ensures that, no matter however TOD varies with distance z, the two soliton trains hardly interact with each other and the separation distance remains a constant for almost 1000 dispersion lengths as shown in Figure 6. This clearly shows that the GVD can be made more predominant under special circumstances.

3. Conclusion

In this paper, we have investigated the higher-order inhomogeneous nonlinear Schrödinger (NLS) equation describing femtosecond soliton propagation through optical fibers and generated bright solitons using gauge transformation approach from their linear eigenvalue problem for a given parametric restriction. We have shown how the interplay between dispersion and nonlinearity can generate various favourable femtosoliton profiles with very high intensities while suffering minimum dispersion during propagation through fibers. Our investigations show that the effects of GVD, SPM, and TOD will have to be equal to enhance the intensities of one of the femtosecond pulses while for optimum dispersion management, GVD, SPM, and TOD should decrease exponentially with propagation distance. We also observe that perfect balance between GVD and SPM makes sure that the two optical pulses do not interact at all which we believe could be exploited for technological applications. The identification of optical similaritons arising by virtue of higher-order effects in the femtosecond regime is an interesting development which may have wider technological ramifications. The stability of these solutions against finite perturbations remains to be investigated.

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