On Exact Solutions for Oscillatory Flows in a Generalized Burgers Fluid with Slip Condition

Tasawar Hayat[a,b], Saher Najam[c], Muhammad Sajid[b], Muhammad Ayub[a], and Said Mesloub[b]

[a] Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan
[b] Department of Mathematics, College of Sciences, KS University, P. O. Box 2455, Riyadh 11451, Saudi Arabia
[c] Theoretical Plasma Physics Division, PINSTECH, P. O. Nilore, Islamabad 4400, Pakistan

Reprint requests to T. H.; Fax: +92 51 2601171; E-mail: pensy_t@yahoo.com

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An analysis is performed for the slip effects on the exact solutions of flows in a generalized Burgers fluid. The flow modelling is based upon the magnetohydrodynamic (MHD) nature of the fluid and modified Darcy law in a porous space. Two illustrative examples of oscillatory flows are considered. The results obtained are compared with several limiting cases. It has been shown here that the derived results hold for all values of frequencies including the resonant frequency.

Key words: Slip Effects; Exact Solutions; Modified Darcy Law.

1. Introduction

The study of magnetohydrodynamic flows in a rotating frame of reference has promising applications in geophysics and astrophysics. It is well known that a number of astronomical bodies (e. g. Sun, Earth, Jupiter, Pulsars, magnetic stars) posses at least surface magnetic fields. In view of these facts much attention has been given in the past to the steady/unsteady rotating flows of viscous fluids. These flows have been also investigated by using several models of non-Newtonian fluids [1 – 10]. An excellent review to the rotating flows of viscous and non-Newtonian fluids is presented in the reference [11]. In continuation, Hayat et al. [12] studied the rotating flows of a generalized Burgers fluid filling the porous space. In the recent attempts, Asghar et al. [13] and Hayat and Abelman [14] analyzed the influence of slip condition on the rotating flows of viscous and third-grade (a subclass of differential type) fluids. It has been noted that no-slip condition is inadequate especially in polymer melts. Also the fluids exhibiting slip have applications in technology such as the polishing of artificial heart valves and internal cavities. There is no doubt that a huge amount of literature is available which deals with the slip effects on the flows in a non-rotating frame.

To our knowledge, no investigation is available yet in the literature to discuss the effects of slip condition on the rotating flows of rate type fluids. In view of this fact the purpose of the current attempt is to analyze the slip effects on the oscillatory flows of a generalized Burgers fluid (a subclass of rate type fluids). Note that the considered fluid model is more general and the results of some other subclasses of rate type fluids, namely the Maxwell, Oldroyd-B, and Burgers, can be deduced easily from the particular cases. An incompressible, homogeneous, and electrically conducting fluid occupies the porous half space. Exact analytic solutions are obtained. Graphs are prepared to display the effects of emerging flow parameters. In addition, tables are provided to make a comparison between the various fluid models.

1.1. Development of the Governing Equation

Let us consider the flow of an incompressible and electrically conducting generalized Burgers fluid occupying a semi-infinite porous space. The Cartesian coordinates are chosen in such a way that both fluid ($z > 0$) and rigid plate (at $z = 0$) possess rigid body rotation with uniform angular velocity $\mathbf{\Omega}$ about the z-axis (taken normal to the plate). The fluid is electrically conducting in the presence of a transverse applied magnetic field $\mathbf{B}_0$ in the z-direction. The induced magnetic field is neglected. Letting the velocity be $\mathbf{V} = (u(z,t)v(z,t),0)$ and the stress field $S = S(z,t)$ then the continuity equation is satisfied identically and the equation of motion in a rotating system with porous space yields

$$\rho\left(\frac{\partial u}{\partial t} - 2\Omega v\right) = -\frac{\partial \hat{p}}{\partial x} + \frac{\partial S_{xz}}{\partial z} - \sigma B_0^2 u + R_1, \quad (1)$$

$$\rho\left(\frac{\partial v}{\partial t}-2\Omega u\right)=-\frac{\partial \hat{p}}{\partial y}+\frac{\partial S_{yz}}{\partial z}-\sigma B_0^2 v+R_2, \quad (2)$$

$$0=-\frac{\partial \hat{p}}{\partial z}+\frac{\partial S_{zz}}{\partial z}, \quad (3)$$

where u and v are the velocity components parallel to x and y-axes, respectively, ρ is the fluid density, σ the electrical conductivity, R_1 and R_2 the components of the Darcy resistance $\mathbf{R}$, $\hat{p}$ $(=p-\frac{1}{2}\Omega^2(x^2+y^2))$ the modified pressure. The extra stress components S_{xz} and S_{yz} satisfy [12]

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)S_{xz}=\mu\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial z}, \quad (4)$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)S_{yz}=\mu\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial v}{\partial z}, \quad (5)$$

in which μ is the dynamic viscosity, λ_i $(i=1-4)$ are the material constants in a generalized Burgers fluid, and the Darcy resistance satisfies [12]

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\mathbf{R}=-\frac{\mu\phi}{k_1}\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\mathbf{V}, \quad (6)$$

where ϕ and k_1 are the porosity and the permeability of the porous space, respectively.

In absence of a pressure gradient one can easily write (1)–(6) as

$$\begin{aligned}&\rho\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial F}{\partial t}+2\mathrm{i}\Omega F\right)\\&+\sigma B_0^2\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)F\\&+\frac{\mu\phi}{k_1}\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)F\\&=\mu\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 F}{\partial z^2},\end{aligned} \quad (7)$$

where

$$F=u+\mathrm{i}v. \quad (8)$$

1.2. Flow Caused by General Periodic Oscillation

This section deals with the oscillatory flow caused by general periodic oscillations of a plate with slip conditions in terms of shear stress. Some results corresponding to special oscillations are also obtained. For the mathematical problem here we use (7) and the following boundary conditions:

$$\begin{aligned}&\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)F(0,t)\\&-\gamma\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial F(0,t)}{\partial z}\\&=U\sum_{k=-\infty}^{k=\infty}a_k(1+\mathrm{i}nk\lambda_1-(nk)^2\lambda_2)\mathrm{e}^{\mathrm{i}nkt},\end{aligned} \quad (9)$$

$$F(\infty,t)=0, \quad (10)$$

in which γ is a slip parameter, $n=2\pi/T_0$ (T_0 being the time period) is the oscillating frequency imposed, and $\{a_k\}$ are the Fourier series coefficients defined by

$$a_k=\frac{1}{T_0}\int_{T_0}F(t)\mathrm{e}^{-\mathrm{i}nkt}\mathrm{d}t. \quad (11)$$

Setting

$$z^*=\frac{zU}{\nu},\quad F^*=\frac{F}{U},\quad t^*=\frac{tU^2}{\nu},\quad n^*=\frac{n\nu}{U^2},$$
$$\Omega^*=\frac{\Omega\nu}{U^2},\quad \lambda_1^*=\lambda_1\frac{U^2}{\nu},\quad \lambda_3^*=\lambda_3\frac{U^2}{\nu},$$
$$\lambda_2^*=\lambda_2\frac{U^4}{\nu^2},\quad \lambda_4^*=\lambda_4\frac{U^4}{\nu^2},\quad \gamma^*=\frac{\gamma u}{\nu},$$
$$M^2=\frac{\sigma B_0^2}{\rho U},\quad \frac{1}{k}=\frac{\phi\nu}{Uk_1},$$

and omitting the asterisks, we obtain

$$\begin{aligned}&\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial F}{\partial t}+2\mathrm{i}\Omega F\right)\\&+M^2\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)F\\&+\frac{1}{K}\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)F\\&=\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 F}{\partial z^2},\end{aligned} \quad (12)$$

$$\begin{aligned}&\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)F(0,t)\\&-\gamma\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial F(0,t)}{\partial z}\\&=\sum_{k=-\infty}^{k=\infty}a_k(1+\mathrm{i}nk\lambda_1-(nk)^2\lambda_2)\mathrm{e}^{\mathrm{i}nkt},\end{aligned} \quad (13)$$

$$F(\infty,t)=0. \quad (14)$$

The above problem can be solved by the Fourier transform pair defined by

$$\bar{F}(z,\omega)=\int_{-\infty}^{\infty}F(z,t)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t, \quad (15)$$

$$F(z,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{F}(z,\omega)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega. \quad (16)$$

Noting

$$\int_{-\infty}^{\infty}\frac{\mathrm{D}^n}{\mathrm{D}t^n}[F(z,t)]\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t=(\mathrm{i}\omega)^nF(z,\omega) \quad (17)$$

and using (15) and (16), one can write

$$\begin{aligned}F(z,t)=&\sum_{k=-\infty}^{k=\infty}a_k(1+\mathrm{i}nk\lambda_1-(nk)^2\lambda_2)\\&\cdot\big[(1+\mathrm{i}nk\lambda_1-(nk)^2\lambda_2)+\gamma(\xi+\mathrm{i}\eta)\\&\quad\cdot(1+\mathrm{i}nk\lambda_3-(nk)^2\lambda_4)\big]^{-1}\mathrm{e}^{-\xi_kz+\mathrm{i}(nkt-\eta_kz)},\end{aligned} \quad (18)$$

$$\xi_k=\frac{1}{\sqrt{2}}\sqrt{a_{1k}+\sqrt{a_{1k}^2+a_{2k}^2}}, \quad (19)$$

$$\eta_k=\sqrt{\frac{a_{2k}^2}{2\left(a_{1k}+\sqrt{a_{1k}^2+a_{2k}^2}\right)}}, \quad (20)$$

$$\begin{aligned}a_{1k}=&\Big\{kn(kn+2\Omega)[(\lambda_3-\lambda_1)+(\lambda_1\lambda_4-\lambda_2\lambda_3)(kn)^2]\\&+M^2[1-(\lambda_2+\lambda_4)(kn)^2+(\lambda_1\lambda_3-\lambda_2\lambda_4(kn)^2)(kn)^2]\\&+\frac{1}{K}[(1-\lambda_4(kn)^2)^2+(\lambda_3kn)^2]\Big\}\\&\quad\cdot\Big\{(1-\lambda_4(kn)^2)^2+(\lambda_3kn)^2\Big\}^{-1},\end{aligned} \quad (21)$$

$$\begin{aligned}a_{2k}=&\Big\{(kn+2\Omega)[1-(kn)^2(\lambda_2+\lambda_4)\\&\quad+(kn)^2(\lambda_1\lambda_3-\lambda_2\lambda_4(kn)^2)]\\&+M^2(kn)[(\lambda_1-\lambda_3)+(\lambda_2\lambda_3-\lambda_1\lambda_4)(kn)^2]\Big\}\\&\quad\cdot\Big\{(1-\lambda_4(kn)^2)^2+(\lambda_3kn)^2\Big\}^{-1},\end{aligned} \quad (22)$$

in which ξ_k and η_k are real and positive. It is interesting to point out that (18) describes the flow induced by a general periodic oscillations of a plate. The special flows due to oscillations $\mathrm{e}^{\mathrm{i}nt}$, $\cos nt$, $\sin nt$, $\begin{bmatrix}1, & |t|<\frac{T_1}{2}\\ 0, & T_1<|t|<\frac{T_0}{2}\end{bmatrix}$, and $\sum\limits_{k=\infty}^{\infty}\delta(t-kT_0)$ are given by

$$F_1(z,t)=\mathrm{e}^{-\xi_1z+\mathrm{i}(nt-\eta_1z)}, \quad (23)$$

$$F_2(z,t)=\frac{1}{2}(\mathrm{e}^{-\xi_1z+\mathrm{i}(nt-\eta_1z)}+\mathrm{e}^{-\xi_{-1}z-\mathrm{i}(nt+\eta_{-1}z)}), \quad (24)$$

$$F_3(z,t)=\frac{1}{2i}(\mathrm{e}^{-\xi_1z+\mathrm{i}(nt-\eta_1z)}-\mathrm{e}^{-\xi_{-1}z-\mathrm{i}(nt+\eta_{-1}z)}), \quad (25)$$

$$F_4(z,t)=\sum_{k=-\infty}^{\infty}\frac{\sin knT_1}{k\pi}\mathrm{e}^{-\xi_kz+\mathrm{i}(knt-\eta_kz)},\quad k\neq0, \quad (26)$$

$$F_5(z,t)=\frac{1}{T_0}\sum_{k=-\infty}^{\infty}\mathrm{e}^{-\xi_kz+\mathrm{i}(knt-\eta_kz)}. \quad (27)$$

1.3. Flow Due to Elliptic Harmonic Oscillations

In this section the flow is generated by elliptic harmonic oscillations of a plate. There is no disturbance in the flow far away from the plate. The resulting flow problem is governed by (7), (10), and the following slip boundary condition:

$$\begin{aligned}&\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)F(0,t)\\&-\gamma\left(1+\lambda_3\frac{\partial}{\partial t}+\lambda_4\frac{\partial^2}{\partial t^2}\right)\frac{\partial F(0,t)}{\partial z}\\&=(a\mathrm{e}^{\mathrm{i}nt}+b\mathrm{e}^{\mathrm{i}nt})(1-\lambda_2n^2)+\mathrm{i}\lambda_1n(a\mathrm{e}^{\mathrm{i}nt}-b\mathrm{e}^{\mathrm{i}nt}).\end{aligned} \quad (28)$$

We search a solution of the form

$$F(z,t)=F_1(z)a\mathrm{e}^{\mathrm{i}nt}+F_2(z)b\mathrm{e}^{\mathrm{i}nt}, \quad (29)$$

where $a=a_1+\mathrm{i}a_2$ and $b=b_1+\mathrm{i}b_2$.

Inserting (29) into (7), (10), and (28) and then solving the resulting systems of F_1 and F_2, we get the following expression for $n<2\Omega$:

$$\begin{aligned}F(z,t)=&\,a[(1-\lambda_2n^2)+\mathrm{i}\lambda_1n][(1-\lambda_4n^2)-\mathrm{i}\lambda_3n]^{1/2}\\&\cdot\Big\{\gamma(\alpha_1+\mathrm{i}\beta_1)[(1-\lambda_4n^2)B_1-\lambda_3nA_1]^{1/2}\\&\quad\cdot[(1-\lambda_4n^2)+\mathrm{i}\lambda_3n]^{1/2}+[(1-\lambda_2n^2)+\mathrm{i}\lambda_1n]\\&\quad\cdot[(1-\lambda_4n^2)-\mathrm{i}\lambda_3n]^{1/2}\Big\}^{-1}\mathrm{e}^{-\psi_1\alpha_1+\mathrm{i}(nt-\psi_1\beta_1)}\\&+b[(1-\lambda_2n^2)-\mathrm{i}\lambda_1n][(1-\lambda_4n^2)+\mathrm{i}\lambda_3n]^{1/2}\\&\quad\cdot\Big\{[(1-\lambda_4n^2)B_2+\lambda_3nA_2]^{1/2}[(1-\lambda_4n^2)-\mathrm{i}\lambda_3n]^{1/2}\\&\qquad+[(1-\lambda_2n^2)-\mathrm{i}\lambda_1n][(1-\lambda_4n^2)+\mathrm{i}\lambda_3n]^{1/2}\Big\}^{-1}\\&\cdot\mathrm{e}^{-\psi_2\alpha_2-\mathrm{i}(nt+\psi_2\beta_2)}.\end{aligned} \quad (30)$$

For $n > 2\Omega$ we have

$$\begin{aligned}
F(z,t) &= a[(1-\lambda_2 n^2)+\mathrm{i}\lambda_1 n][(1-\lambda_4 n^2)-\mathrm{i}\lambda_3 n]^{1/2} \\
&\cdot \Big\{\gamma(\alpha_1+\mathrm{i}\beta_1)[(1-\lambda_4 n^2)B_1-\lambda_3 n A_1]^{1/2} \\
&\quad \cdot [(1-\lambda_4 n^2)+\mathrm{i}\lambda_3 n]^{1/2}+[(1-\lambda_2 n^2)+\mathrm{i}\lambda_1 n] \\
&\quad \cdot [(1-\lambda_4 n^2)-\mathrm{i}\lambda_3 n]^{1/2}\Big\}^{-1} \mathrm{e}^{-\psi_1\alpha_1+\mathrm{i}(nt-\psi_1\beta_1)} \\
&+ b[(1-\lambda_2 n^2)-\mathrm{i}\lambda_1 n][(1-\lambda_4 n^2)+\mathrm{i}\lambda_3 n]^{1/2} \\
&\cdot \Big\{\gamma(\alpha_3+\mathrm{i}\beta_3)[(1-\lambda_4 n^2)B_3-\lambda_3 n A_3]^{1/2} \\
&\quad \cdot [(1-\lambda_4 n^2)-\mathrm{i}\lambda_3 n]^{1/2}+[(1-\lambda_2 n^2)-\mathrm{i}\lambda_1 n] \\
&\quad \cdot [(1-\lambda_4 n^2)+\mathrm{i}\lambda_3 n]^{1/2}\Big\}^{-1} \mathrm{e}^{-\psi_3\alpha_3-\mathrm{i}(nt-\psi_3\beta_3)}.
\end{aligned} \tag{31}$$

The solution for $n = 2\Omega$ is

$$\begin{aligned}
F(z,t) &= a[(1-\lambda_2 n^2)+\mathrm{i}\lambda_1 n][(1-\lambda_4 n^2)-\mathrm{i}\lambda_3 n]^{1/2} \\
&\cdot \Big\{\gamma(\alpha_1+\mathrm{i}\beta_1)[(1-\lambda_4 n^2)B_1-\lambda_3 n A_1]^{1/2} \\
&\quad \cdot [(1-\lambda_4 n^2)+\mathrm{i}\lambda_3 n]^{1/2}+[(1-\lambda_2 n^2)+\mathrm{i}\lambda_1 n] \\
&\quad \cdot [(1-\lambda_4 n^2)-\mathrm{i}\lambda_3 n]^{1/2}\Big\}^{-1} \mathrm{e}^{-\psi_1\alpha_1-\mathrm{i}(nt-\psi_1\beta_1)} \\
&+ b[(1-\lambda_2 n^2)-\mathrm{i}\lambda_1 n][(1-\lambda_4 n^2)+\mathrm{i}\lambda_3 n]^{1/2} \\
&\cdot \Big\{\gamma(\alpha_0+\mathrm{i}\beta_0)[(1-\lambda_4 n^2)B_0+\lambda_3 n A_0]^{1/2} \\
&\quad \cdot [(1-\lambda_4 n^2)-\mathrm{i}\lambda_3 n]^{1/2}+[(1-\lambda_2 n^2)-\mathrm{i}\lambda_1 n] \\
&\quad \cdot [(1-\lambda_4 n^2)+\mathrm{i}\lambda_3 n]^{1/2}\Big\}^{-1} \mathrm{e}^{-\psi_0\alpha_0-\mathrm{i}(nt+\psi_0\beta_0)}.
\end{aligned} \tag{32}$$

In the above solutions

$$\alpha_j = \frac{1}{\sqrt{2}}[S_j+(S_j^2+1)^{1/2}]^{1/2},$$
$$\beta_j = \frac{1}{\sqrt{2}}[S_j+(S_j^2+1)^{1/2}]^{1/2}, \quad j=0,1,2,3,$$

$$\psi_1 = \left[\frac{(1-\lambda_4 n^2)B_1-\lambda_3 n A_1}{(1-\lambda_4 n^2)^2+(\lambda_3 n)^2}\right]^{1/2} z,$$
$$\psi_2 = \left[\frac{(1-\lambda_4 n^2)B_2+\lambda_3 n A_2}{(1-\lambda_4 n^2)^2+(\lambda_3 n)^2}\right]^{1/2} z,$$
$$\psi_3 = \left[\frac{(1-\lambda_4 n^2)B_3-\lambda_3 n A_3}{(1-\lambda_4 n^2)^2+(\lambda_3 n)^2}\right]^{1/2} z,$$
$$\psi_0 = \left[\frac{(1-\lambda_4 n^2)B_0-\lambda_3 n A_0}{(1-\lambda_4 n^2)^2+(\lambda_3 n)^2}\right]^{1/2} z,$$

$$S_1 = \frac{(1-\lambda_4 n^2)A_1+\lambda_3 n B_1}{(1-\lambda_4 n^2)B_1-\lambda_3 n A_1},$$
$$S_2 = \frac{(1-\lambda_4 n^2)A_2-\lambda_3 n B_2}{(1-\lambda_4 n^2)B_2+\lambda_3 n A_2},$$
$$S_3 = \frac{(1-\lambda_4 n^2)A_3+\lambda_3 n B_3}{(1-\lambda_4 n^2)B_3+\lambda_3 n A_3},$$
$$S_0 = \frac{(1-\lambda_4 n^2)A_0+\lambda_3 n B_0}{(1-\lambda_4 n^2)B_0-\lambda_3 n A_0},$$

$$A_1 = M^2(1-\lambda_2 n^2)-\lambda_1 n(n+2\Omega)+\frac{1}{K}(1-\lambda_4 n^2),$$
$$B_1 = M^2\lambda_1 n+(n+2\Omega)(1-\lambda_2 n^2)+\frac{\lambda_3 n}{K},$$
$$A_2 = M^2(1-\lambda_2 n^2)+\lambda_1 n(2\Omega-n)+\frac{1}{K}(1-\lambda_4 n^2),$$
$$B_2 = -M^2\lambda_1 n+(2\Omega-n)(1-\lambda_2 n^2)+\frac{\lambda_3 n}{K},$$
$$A_3 = M^2(1-\lambda_2 n^2)-\lambda_1 n(n-2\Omega)+\frac{1}{K}(1-\lambda_4 n^2),$$
$$B_3 = M^2\lambda_1 n+(n-2\Omega)(1-\lambda_2 n^2)+\frac{\lambda_3 n}{K},$$
$$A_0 = M^2(1-\lambda_2 n^2)+\frac{1}{K}(1-\lambda_4 n^2),$$
$$B_0 = M^2\lambda_1 n+\frac{\lambda_3 n}{K}.$$

2. Results and Discussion

In this section our main emphasis is to discuss the influence of a slip parameter on the real and imaginary parts of the velocity. The effects have been investigated for six different fluids, namely the generalized Burgers, Burgers, Oldroyd-B, Maxwell, second-grade, and viscous fluids. The physical problems of the oscillatory flow due to general periodic oscillations and elliptic harmonic oscillations (for $n < 2\Omega$, $n > 2\Omega$, and $n = 2\Omega$) are analyzed. To see the influence of the slip parameter on the velocity components Figures 1 – 12 have been displayed. In all these figures Panel (a) corresponds to the real component of the velocity and Panel (b) corresponds to the imaginary part of the velocity. In all these graphs the values of the parameters M and K are fixed. Moreover, the oscillating frequency is fixed.

In Figures 1 – 6, the graphs for the oscillation of type $\cos nt$ are considered. The velocity u decreases by an

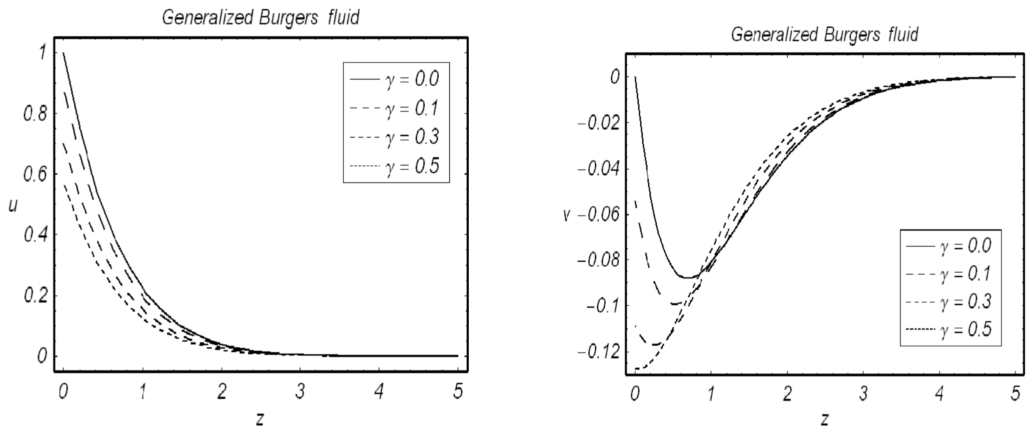

Fig. 1. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0.75$, $\lambda_3 = 0.5$, $\lambda_4 = 0.25$, $M = K = 1$, $t = 0.1$, and $n = \Omega = 0.5$ are fixed.

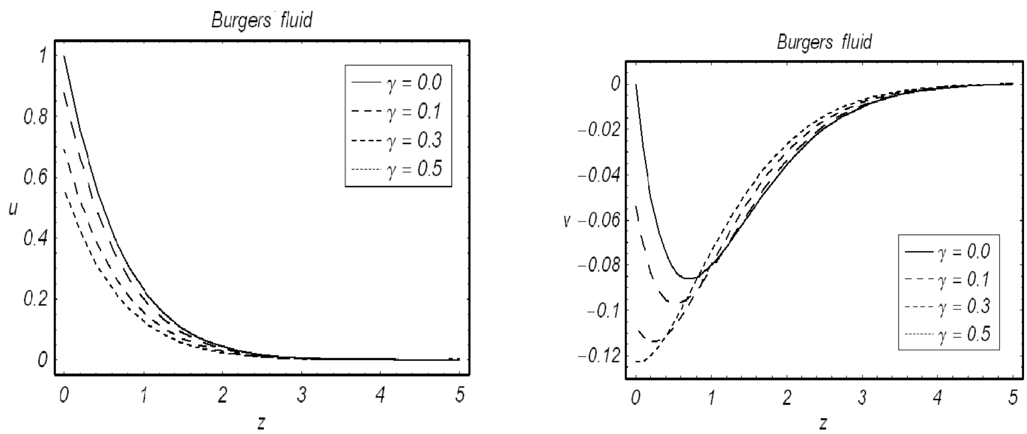

Fig. 2. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0.75$, $\lambda_3 = 0.5$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = \Omega = 0.5$ are fixed.

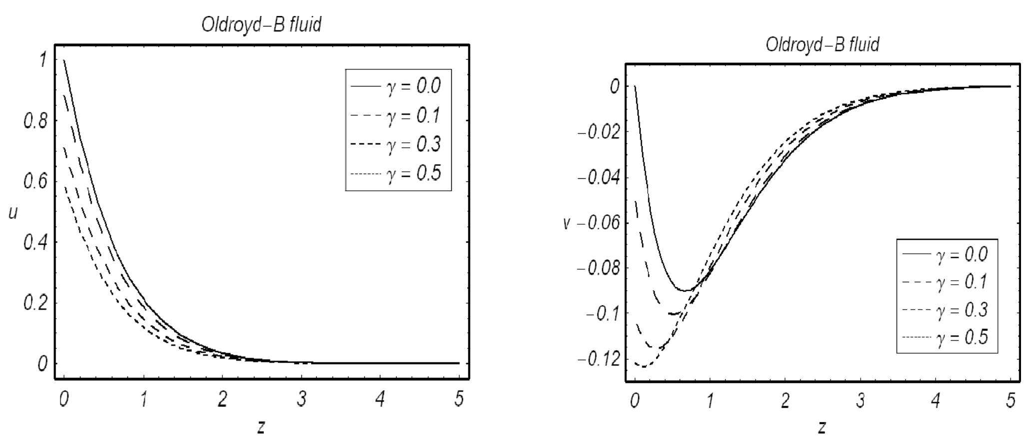

Fig. 3. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 = 0.5$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = \Omega = 0.5$ are fixed.

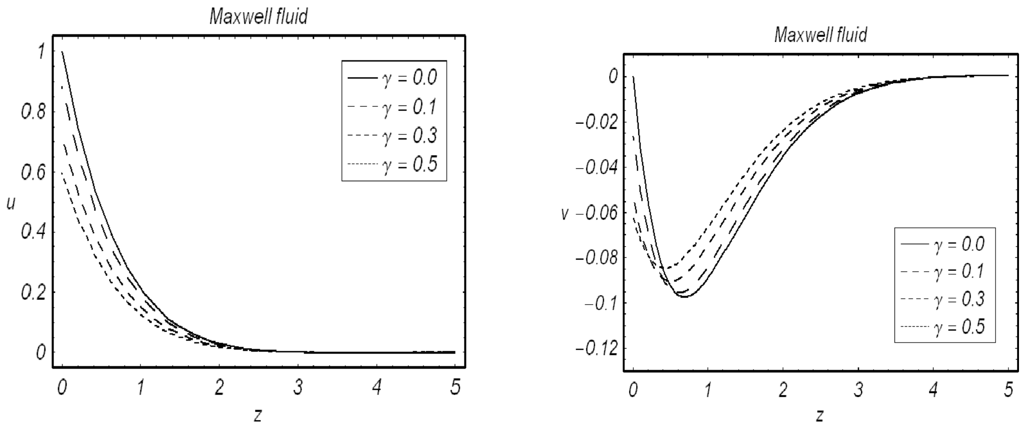

Fig. 4. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = \Omega = 0.5$ are fixed.

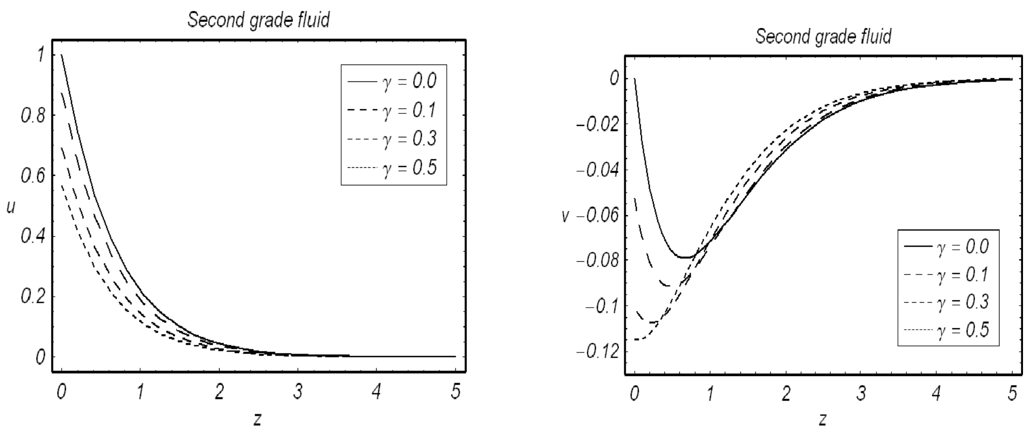

Fig. 5. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0.5$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = \Omega = 0.5$ are fixed.

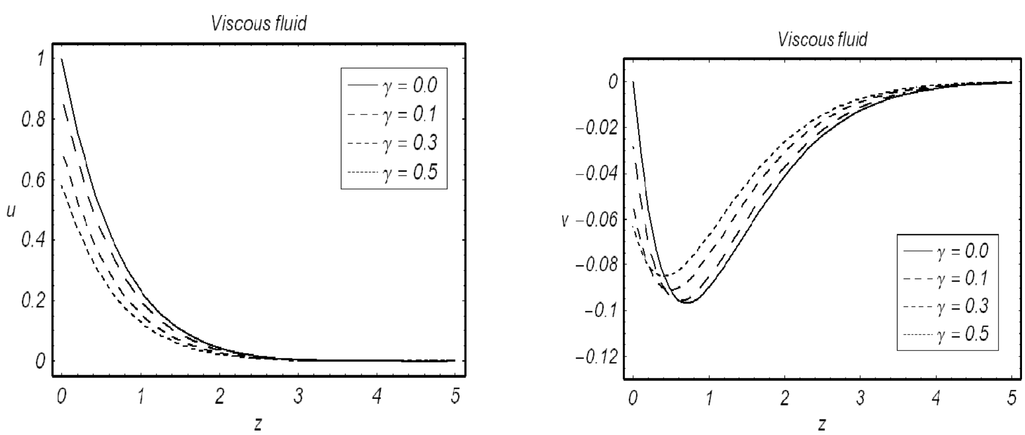

Fig. 6. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = \Omega = 0.5$ are fixed.

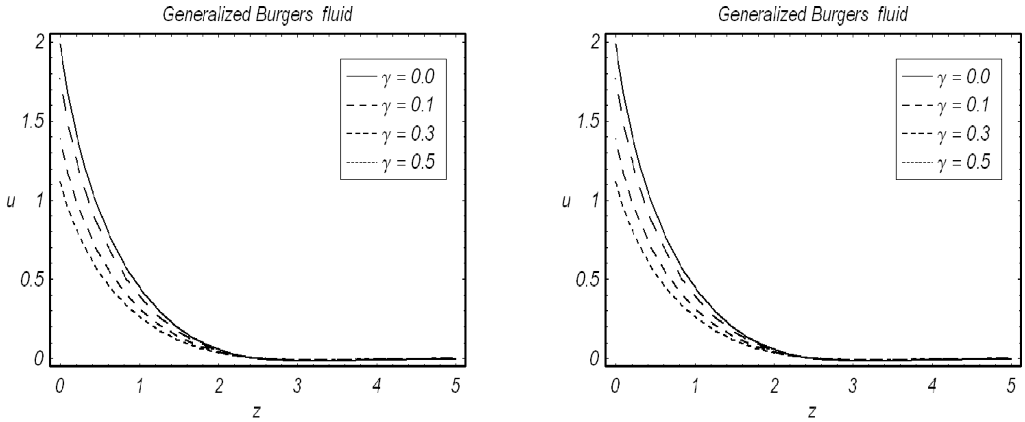

Fig. 7. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0.75$, $\lambda_3 = 0.5$, $\lambda_4 = 0.25$, $M = K = 1$, $t = 0.1$, and $n = 1$, $\Omega = 0.5$ are fixed.

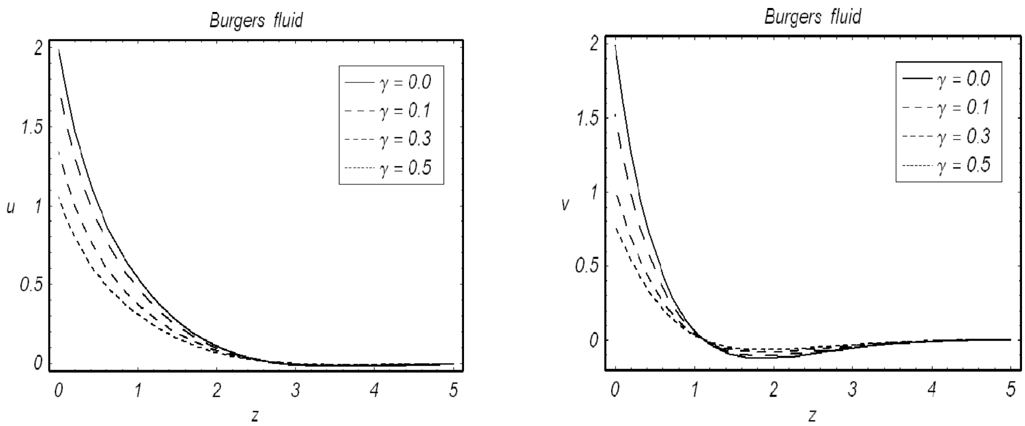

Fig. 8. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0.75$, $\lambda_3 = 0.5$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = 1$, $\Omega = 0.5$ are fixed.

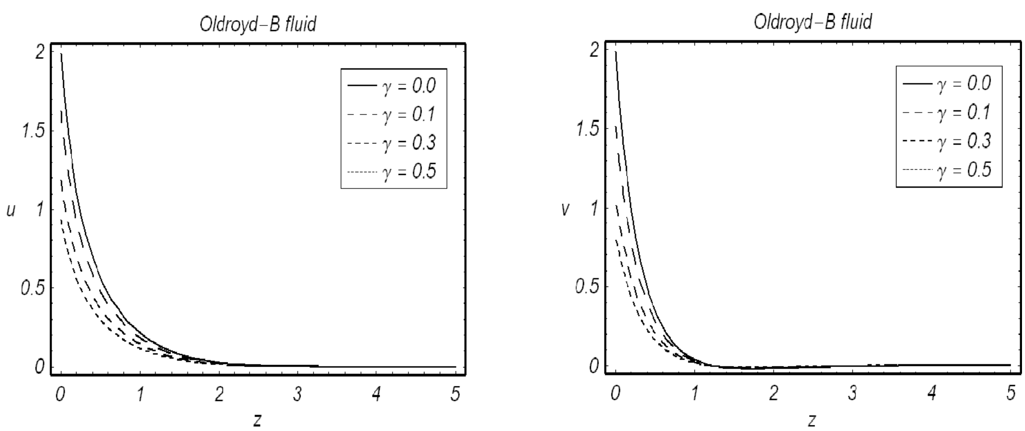

Fig. 9. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 = 0.5$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = 1$, $\Omega = 0.5$ are fixed.

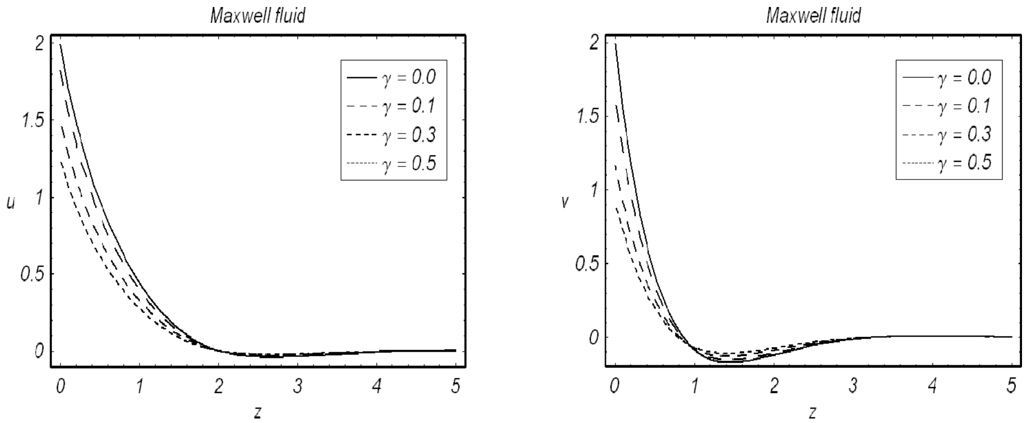

Fig. 10. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = 1$, $\Omega = 0.5$ are fixed.

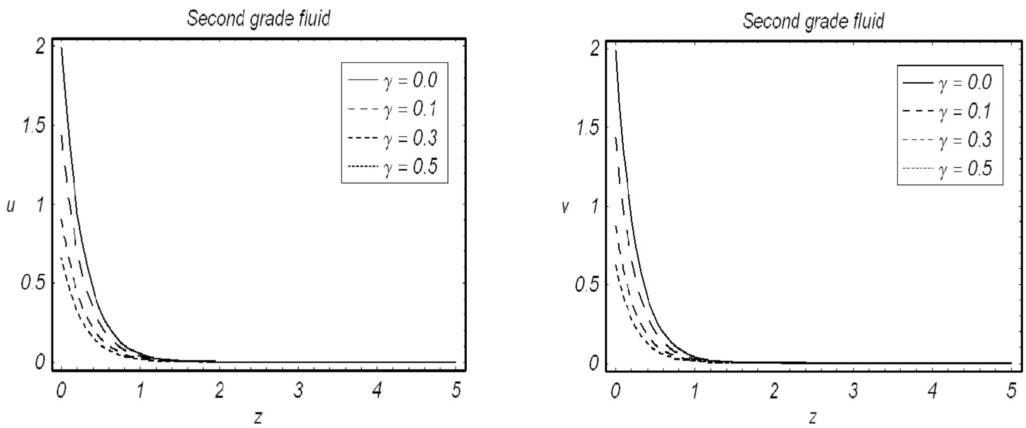

Fig. 11. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0.5$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = 1$, $\Omega = 0.5$ are fixed.

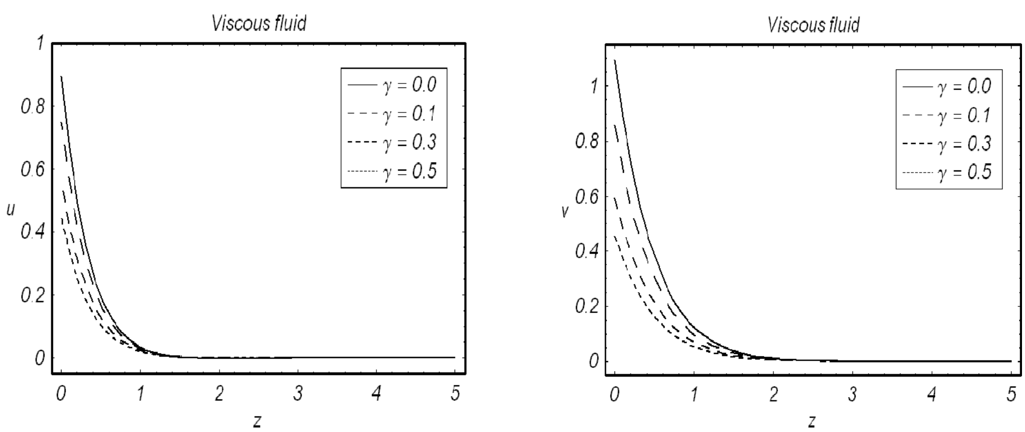

Fig. 12. Variation of slip parameter γ on the velocity profiles u (left) and v (right) for periodic oscillation when $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $M = K = 1$, $t = 0.1$, and $n = 1$, $\Omega = 0.5$ are fixed.

Table 1. Comparison of velocity for six different fluids when $M = K = 1$, $n = \Omega = 0.5$, $t = 0.1$ are fixed in the flow due to periodic oscillation $\cos nt$.

Type of fluid	Rheological parameter	z	γ	u	v
generalized Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0.25$	0.5	0	0.484796	−0.084034
			0.1	0.274169	−0.109551
			0.3	0.182681	−0.09334
			0.5	0.135357	−0.077664
Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0$	0.5	0	0.491778	−0.081612
			0.1	0.273139	−0.105776
			0.3	0.181125	−0.089091
			0.5	0.134045	−0.073679
Oldroyd-B	$\lambda_2 = \lambda_4 = 0, \lambda_1 = 1, \lambda_3 = 0.5$	0.5	0	0.471317	−0.0870815
			0.1	0.272516	−0.108587
			0.3	0.184718	−0.0934285
			0.5	0.138244	−0.0785571
Maxwell	$\lambda_2 = \lambda_3 = \lambda_4 = 0, \lambda_1 = 1$	0.5	0	0.478182	−0.0927006
			0.1	0.283909	−0.0842761
			0.3	0.19967	−0.0687988
			0.5	0.153253	−0.0570548
Second grade	$\lambda_1 = \lambda_2 = \lambda_4 = 0, \lambda_3 = 0.5$	0.5	0	0.474396	−0.0762363
			0.1	0.260979	−0.0973591
			0.3	0.175887	−0.0819407
			0.5	0.131869	−0.0683074
Viscous	$\lambda_i = 0, i = 1,2,3,4$	0.5	0	0.491376	−0.0915004
			0.1	0.279384	−0.084572
			0.3	0.193367	−0.068185
			0.5	0.147507	−0.0560756

Table 2. Comparison of velocity for six different fluids when $M = K = 1$, $n = \Omega = 0.5$, $t = 0.1$ are fixed in the flow due to periodic oscillation $n < 2\Omega$.

Type of fluid	Rheological parameter	z	γ	u	v
generalized Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0.25$	0.5	0	0.526323	0.361837
			0.1	0.263575	0.179496
			0.3	0.176824	0.121156
			0.5	0.133111	0.091515
Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0$	0.5	0	0.524945	0.366551
			0.1	0.256581	0.182252
			0.3	0.170629	0.1229
			0.5	0.127866	0.092765
Oldroyd-B	$\lambda_2 = \lambda_4 = 0, \lambda_1 = 1, \lambda_3 = 0.5$	0.5	0	0.529003	0.366047
			0.1	0.271231	0.172942
			0.3	0.183745	0.115013
			0.5	0.139055	0.086259
Maxwell	$\lambda_2 = \lambda_3 = \lambda_4 = 0.5, \lambda_1 = 1$	0.5	0	0.486643	0.303932
			0.1	0.251512	0.18495
			0.3	0.168849	0.131674
			0.5	0.126955	0.102047
Second grade	$\lambda_1 = \lambda_2 = \lambda_4 = 0, \lambda_3 = 0.5$	0.5	0	0.634764	0.517002
			0.1	0.312256	0.204198
			0.3	0.207012	0.122379
			0.5	0.15465	0.086568
Viscous	$\lambda_i = 0, i = 1,2,3,4$	0.5	0	0.438672	0.366101
			0.1	0.182537	0.137633
			0.3	0.11576	0.0847221
			0.5	0.0847925	0.0611824

Table 3. Comparison of velocity for six different fluids when $M = K = 1$, $n = 1$, $\Omega = 0.0001$, $t = 0.1$ are fixed in the flow due to periodic oscillation $n > 2\Omega$.

Type of fluid	Rheological parameter	z	γ	u	v
generalized Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0.25$	0.5	0	0.855242	0.855155
			0.1	0.501827	0.501759
			0.3	0.34721	0.347162
			0.5	0.264028	0.263992
Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0$	0.5	0	0.920905	0.920824
			0.1	0.512444	0.512385
			0.3	0.341106	0.34107
			0.5	0.253392	0.253368
Oldroyd-B	$\lambda_2 = \lambda_4 = 0, \lambda_1 = 1, \lambda_3 = 0.5$	0.5	0	0.615355	0.615304
			0.1	0.319268	0.319234
			0.3	0.214798	0.214774
			0.5	0.161729	0.16171
Maxwell	$\lambda_2 = \lambda_3 = \lambda_4 = 0.5, \lambda_1 = 1$	0.5	0	0.87127	0.871146
			0.1	0.563396	0.563284
			0.3	0.40917	0.409081
			0.5	0.31972	0.319647
Second grade	$\lambda_1 = \lambda_2 = \lambda_4 = 0, \lambda_3 = 0.5$	0.5	0	0.166977	0.166983
			0.1	0.043759	0.0437462
			0.3	0.024638	0.0246285
			0.5	0.017116	0.0171093
Viscous	$\lambda_i = 0, i = 1,2,3,4$	0.5	0	0.447247	0.447228
			0.1	0.265393	0.0429465
			0.3	0.145063	0.0030364
			0.5	0.097674	−0.001763

Table 4. Comparison of velocity for six different fluids when $M = K = 1$, $n = 1$, $\Omega = 0.5$, $t = 0.1$ are fixed in the flow due to periodic oscillation $n = 2\Omega$.

Type of fluid	Rheological parameter	z	γ	u	v
generalized Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0.25$	0.5	0	0.916195	0.494761
			0.1	0.532411	0.229771
			0.3	0.367935	0.153833
			0.5	0.28079	0.1166719
Burgers	$\lambda_1 = 1, \lambda_2 = 0.75, \lambda_3 = 0.5, \lambda_4 = 0$	0.5	0	0.994518	0.589213
			0.1	0.550119	0.271677
			0.3	0.368743	0.184398
			0.5	0.276742	0.141164
Oldroyd-B	$\lambda_2 = \lambda_4 = 0, \lambda_1 = 1, \lambda_3 = 0.5$	0.5	0	0.561449	0.346499
			0.1	0.298798	0.163279
			0.3	0.204377	0.108784
			0.5	0.155442	0.081732
Maxwell	$\lambda_2 = \lambda_3 = \lambda_4 = 0, \lambda_1 = 1$	0.5	0	0.965515	0.426629
			0.1	0.609618	0.171083
			0.3	0.435452	0.121332
			0.5	0.337893	0.089199
Second grade	$\lambda_1 = \lambda_2 = \lambda_4 = 0, \lambda_3 = 0.5$	0.5	0	0.314498	0.287117
			0.1	0.107517	0.084833
			0.3	0.063579	0.047959
			0.5	0.045039	0.033286
Viscous	$\lambda_i = 0, i = 1,2,3,4$	0.5	0	0.194662	0.375476
			0.1	0.103252	0.160432
			0.3	0.069337	0.101432
			0.5	0.052104	0.074243

increase in the slip parameter. The magnitude of influence of the slip parameter in a Burgers fluid is slightly greater than that of a generalized Burgers fluid. There is a slight variation observed in the case of Oldroyd-B and Maxwell fluids when compared with the Burgers fluid. The magnitude of variation in velocity is relatively strong for second-grade and viscous fluids when compared with the other fluid models under consideration. The similar kind of effects for the case of elliptic oscillations is observed when $a_1 = a_2 = b_1 = b_2 = 1$. Here we are including the Figures 7 – 12 to see the influence of the slip parameter for elliptic harmonic oscillation when $n = 2\Omega$. However, the results are similar for $n < 2\Omega$ and $n > 2\Omega$ in a qualitative sense. In view of that we are excluding the figures for $n < 2\Omega$ and $n > 2\Omega$. In all these cases the variation in the velocity is of order 10^{-1}, therefore, one can not easily observe the variation through graphs. Due to this fact the velocity has been tabulated for $z = 0.5$. It is noted from Table 1 that the magnitudes of u and v are large for the small slip parameter. Furthermore, the magnitude of u is large in comparison to the magnitude of v in both no-slip and slip conditions. The comparison of Tables 1 – 4 reveals that the velocity in an elliptic harmonic oscillation is much larger than the velocity in the case of $\cos nt$. Table 2 shows that the velocity u in an Oldroyd-B fluid is less than in the Maxwell fluid in both slip and no-slip condition cases. Table 2 depicts that u in a generalized Burgers fluid is greater than in a Burgers fluid. This table also witnesses that u and v in a second-grade fluid show larger values in comparison to the viscous fluid. In presence of slip condition u in the Maxwell and Oldroyd-B fluids is much larger than v for $n < 2\Omega$, $n > 2\Omega$, and $n = 2\Omega$ (Tables 2 – 4).

3. Concluding Remarks

In this article a mathematical analysis has been carried out for two flow problems in a generalized Burgers fluid occupying a porous half space. The computations has been performed in the presence of slip condition. To the best of our knowledge such condition in the case of rate type fluid models has been introduced for the first time in literature. Closed form solutions have been derived for the two oscillatory flows. It is noticed that the derived results are valid for all values of the frequencies including the resonant frequency. The effect of the slip parameter on the velocity is sketched and analyzed. A comparison between the velocities of several fluid models is also included.

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