Interesting Band Properties of One-Dimensional Photonic Crystals Containing Epsilon-Negative Layers

Fei Xu and Yulin Chen

College of Mathematics & Physics, Nanjing University of Information Science and Technology, Nanjing 210044, China

Reprint requests to F. X.; E-mail: xufei77@gmail.com

Z. Naturforsch. **65a,** 329 – 334 (2010); received September 24, 2008 / revised June 2, 2009

The transmission properties of one-dimensional photonic crystals constituted by a periodic repetition of positive-index layers and epsilon-negative layers are studied theoretically. This structure shows some interesting properties including a wide gap in the low frequency range for small period number and a comb-like transmission band in the gap. The properties of the comb-like transmission band are sensitive to the period number of the structure. In contrast to the zero-$\bar{n}$ gap and the zero-ϕ_{eff} gap, the transmission properties are dependent on the structure parameters. A general method to decide the position of gap and transmission band in this kind of structure is also presented.

Key words: One-Dimensional Photonic Crystals; Epsilon-Negative; Transmission.
PACS numbers: 42.70.Qs, 78.20Ci, 41.20.Jb

Photonic crystals (PCs) have attracted considerable attention in recent years owing to their unique ability to control and manipulate light [1]. Recently, double negative refraction (DNG) materials, i. e. left-handed materials (LHM) with simultaneously negative permittivity and negative permeability, have been reported for their peculiar properties, such as the reversal of Doppler shift and the famous negative refraction [2 – 12]. It is demonstrated that stacking alternating layers of positive-index and double negative-index media leads to a type of photonic band gap (PBG) corresponding to a zero-averaged refractive index. A number of unique properties of the zero-$\bar{n}$ gap on the beam shaping effect have been studied [13 – 15]. The zero-$\bar{n}$ gap differs fundamentally from the usual PBG induced by the Bragg scattering, e. g. it is independent of scaling and insensitive to the disorder, and the edge of such a zero-$\bar{n}$ gap is insensitive to incident angle and polarization. In addition to the DNG materials, another material called the single-negative (SNG) material has also been studied. The SNG materials consist of the mu-negative (MNG) materials with negative μ but positive ε, and the epsilon-negative (ENG) materials with negative ε but positive μ. It has been found that a one-dimensional photonic crystal (1DPC) constituted by a periodic repetition of MNG and ENG layers can possess another type of photonic gap with effective phase ϕ_{eff} of zero called the SNG gap or the zero-ϕ_{eff} gap [16 – 18]. Similar to the zero-$\bar{n}$ gap, the SNG gap is invariant with a change of scale length and is insensitive to thickness fluctuation [17]. However, in contrast to a zero-$\bar{n}$ gap, the SNG gap can be made very wide by varying the ratio of the thicknesses of two media.

In this paper, we suggest another structure of 1DPC that is constituted by a periodic repetition of air layers and ENG layers. We study its band and find some new interesting properties with it. They are different from all cases of the usual PBG, the zero-$\bar{n}$ gap, and the SNG gap. In practice, the ENG layers can be fabricated by using wire elements and the fabrication of the ENG-air structure may be less intricate than those of the DNG structure.

Consider the 1DPC with the finite periodic structure of $(AB)^{\text{N}}$, where A represents ENG materials and B represents air, and N is the number of periods. The thickness of A and B are firstly supposed to be $d_a = d_b = d/2$. In the following calculation, we choose a basic frequency $\omega_0 = \pi/d$. In our calculation, all frequencies are in unit of ω_0, thus the value of d doesn't influence all the following results. Corresponding to light with the basic frequency, the period length of the 1DPC is just half of the wavelength. For the A layers, the relative permittivity and permeability in the ENG materials are given by [17]

$$\varepsilon_a = 1 - \omega_{\text{ep}}^2/\omega^2, \quad \mu_a = 1, \qquad (1)$$

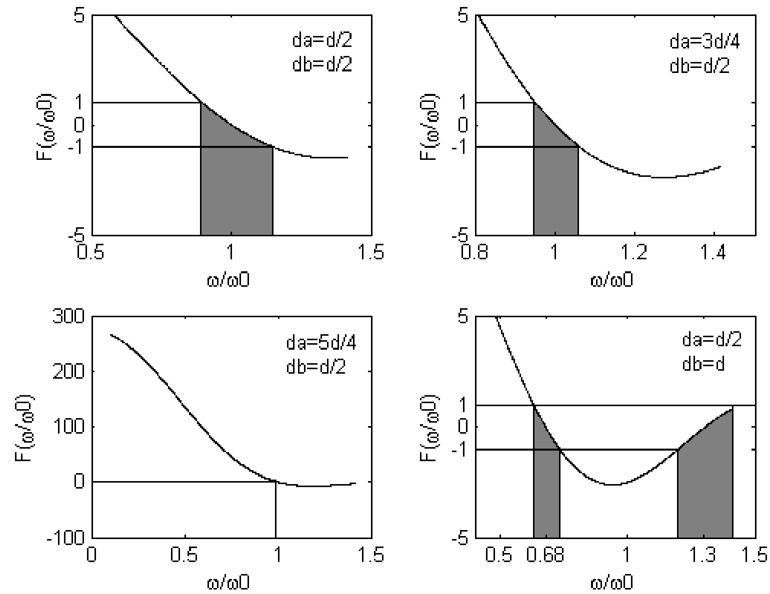

Fig. 1. Values of $F(\omega/\omega_0)$ in the range of ENG frequencies for different thicknesses of pair layers and the corresponding range of transmission bands.

where ω_{ep} is the electronic plasma frequency. Without loss of generality, we suppose $\omega_{\text{ep}} = 2\omega_0$. The ENG frequency is determined by $\omega < \omega_{\text{ep}}$. When $\omega > \omega_{\text{ep}}$, the layers A are turned into positive-index materials. For layers B, we suppose $\varepsilon_b = \mu_b = 1$.

Firstly, we consider an infinite periodic structure ($N \to \infty$), according to Bloch's theorem, the dispersion at any incident angle follows the relation [19]

$$\cos\beta_z(d_a+d_b) = \cos[k_z^{(A)}d_a]\cos[k_b^{(B)}d_b] - \frac{1}{2}\left(\frac{q_B}{q_A}+\frac{q_A}{q_B}\right)\sin[k_z^{(A)}d_a]\sin[k_b^{(B)}d_b], \tag{2}$$

where β_z is the z-component of the Bloch wave vector, and $k_z^j = \omega/c\sqrt{\varepsilon_j}\sqrt{\mu_j}\sqrt{1-(\sin^2\theta/\varepsilon_j\mu_j)}$ is the z-component of the wave vector $\boldsymbol{k}^j$ in the ith layer (c is the light velocity in the vacuum and θ the incident angle). It is for TE wave, $q_i = \sqrt{\varepsilon_j}/\sqrt{\mu_j}\sqrt{1-(\sin^2\theta/\varepsilon_j\mu_j)}$; for TM wave, $q_j = \sqrt{\mu_j}/\sqrt{\varepsilon_j}\sqrt{1-(\sin^2\theta/\varepsilon_j\mu_j)}$. The condition of (2) having no real solution for β_z is $|\cos\beta_z(d_a+d_b)| > 1$, which corresponds to the band gap of 1DPC and is well known as the Bragg condition. In the range of ENG frequencies, $k_z^{(A)}$ is an imaginary number due to the negative value of ε_a, $\cos[k_z^{(A)}d_a] = \cosh[|k_z^{(A)}d_a|]$, and $\sin[k_z^{(A)}d_a] = \sinh[|k_z^{(A)}d_a|]$. For normal incidence ($\theta = 0$) and TE wave valids $q_A = \sqrt{1-2\frac{\omega_0^2}{\omega^2}}$ and $q_B = 1$.

If we define $x = \omega/\omega_0 = \omega/(2\pi c/d)$, the right side of (2) can be regarded as a function of x with the form

$$F(x) = \cosh\left[\left(\sqrt{2/x^2-1}\right)(x\frac{2\pi}{d})d_a\right]\cos[(x\frac{2\pi}{d})d_b] - \frac{1}{2}\left(\frac{1}{\sqrt{1-\frac{2}{x^2}}}+\sqrt{1-\frac{2}{x^2}}\right) \cdot\sinh\left[\left(\sqrt{\frac{2}{x^2}-1}\right)(x\frac{2\pi}{d})d_a\right]\sin[(x\frac{2\pi}{d})d_b]. \tag{3}$$

If x satisfies the condition that $F(x)$ is a real number and $|F(x)| \le 1$, the corresponding range of ω values becomes a transmission band, otherwise it becomes the ENG-air gap. Giving definite values of d_a and d_b, we can decide the ENG-air band structure by means of (3). By numerical calculation, Figure 1 plots the values of $F(\omega/\omega_0)$ in the range of the ENG frequencies for different thicknesses of pair layers. From Figure 1, we easily decide the positions of transmission bands and ENG-air gaps. The transmission bands are shown by gray areas. Although we are studying a finite structure, the above result still provides us a useful reference.

For a finite periodic structure of $(AB)^{\text{N}}$ and TE waves, let a plane wave be injected from vacuum into the 1DPC at an incident angle θ, then the transmission ratio $t(\omega)$ for both TE and TM waves and the field distribution inside the structure can be obtained by the transfer matrix method [17]. In the case of normal in-

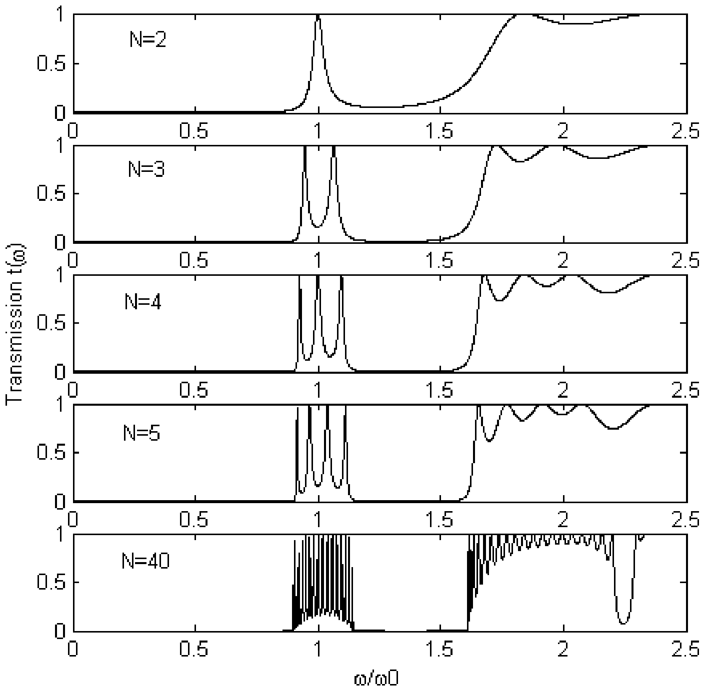

Fig. 2. Transmission ratio $t(\omega)$ for different period number N and structure parameters of $d_a = d_b = d/2$.

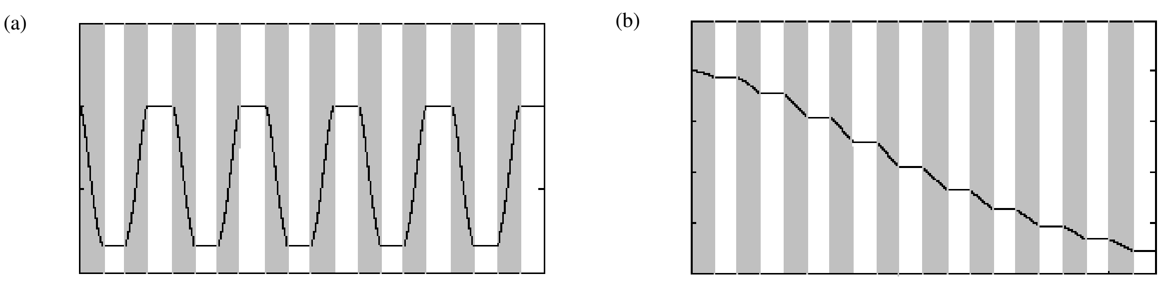

Fig. 3. Field distribution in the ENG-air structure for $N = 10$ and $d_a = d_b = d/2$ with two frequencies of $\omega = \omega_0$ (a) and $\omega = 0.1\omega_0$ (b). The gray area stands for A layer and the white area stands for B layer.

cident, TE and TM waves have the same results. All the plane waves are incident on the 1DPC at normal direction in the following study.

Figure 2 gives the transmission ratio $t(\omega)$ for different period number N. From it, we find the band properties quite different from the usual Bragg band. Except a comb-like transmission band around $\omega = \omega_0$, there is a wide band gap in the range of low frequencies. The wide gap bases on small period numbers, which lead to more compact geometries necessary for applications in the microwave range. From (1), when $\omega < \sqrt{2}\omega_0$, $\varepsilon_a < 0$, thus most of the wide gap results from the ENG-air structure (we call it as the ENG-air gap in the later). As we know, the SNG gap originates from the interaction of evanescent waves, while the zero-$\bar{n}$ gap and the usual Bragg gap both originate from the interaction of the propagating modes. However, the ENG-air gap originates from the interaction of evanescent waves and propagation wave, thus it takes on different properties from all above band gaps, which will be discussed later. It is interesting that the number of the comb-like transmission peaks is just $N - 1$. What's more, the value of transmission ratio at $\omega = \omega_0$ is just at peak with even period number, while it corresponds to a trough with odd period number. Although the figure only shows finite period number, the conclusion can extend to any period number. Therefore, the value of $t(\omega)$ in the comb-like transmission band is sensitive to the period number of the ENG-air structure, which becomes its unique property.

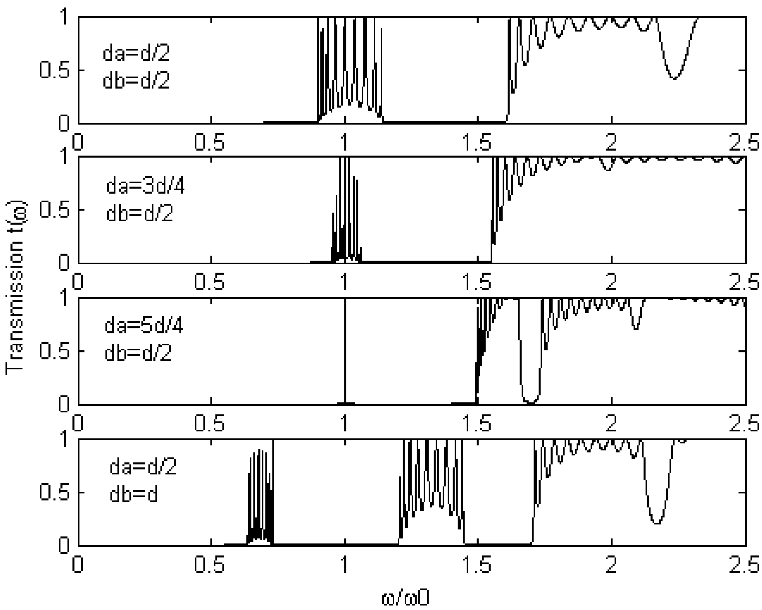

Fig. 4. The transmission ratio $t(\omega)$ for $N = 40$ and different structure parameters.

In order to further demonstrate the unique feature, Figure 3 plots the electric field distribution of light with $\omega = \omega_0$ and $\omega = 0.1\omega_0$ inside the ENG-air structure for $d_a = d_b = d/2$ and $N = 10$, respectively. Clearly, for the case of $\omega = \omega_0$, the field value changes periodically with the layer number increasing and the maximum values and the minimum values alternately occur within the air layers, which further demonstrates the result of Figure 2. For this property we can give a qualitative explain. Due to the sudden change of impedance from the air to the ENG layer, there is a large reflection on the interface between two layers. All the reflection light interact each other. The number of the reflection light beams is equal to the period number of the structure. Because the phase difference between two adjacent reflection beams is just π according to the parameters of Figure 2, the reflection light beams with even number will cancel by interaction with each other, which leads to the maximum values of t. The reflection beams with odd number can not fully cancel, which leads to the minimum values of t. That is the reson why the field inside the structure and the value of t alternately change with the period number of the ENG-air structure. In addition, the field value in A layers alternately goes up and down, while in B layers keeps invariant. Clearly, the field evolvement in A layers is dependent on the field value of the two nearest B layers, which is also different from that of the SNG structure. In the SNG structure the fields corresponding to the band edges are localized at each interface of two media (see Fig. 3 of [17]). For the case of $\omega = 0.1\omega_0$ in Figure 3, i. e. in the ENG-air gap, the field in all A layers becomes evanescent and decreases quickly with the period number increasing. According to the above results, the ENG-air structure can be well used as multiple channeled filtering, because the position and number of the comb-like transmission peaks can be controlled easily and exactly.

As we have known, the zero-$\bar{n}$ gap is independent of scaling and insensitive to the disorder. One may ask whether or not the ENG-air band structure is dependent of thickness fluctuation of layers A and B. Figure 4 plots $t(\omega)$ for different d_a and d_b with $N = 40$. For the cases of $d_b = d/2$, with the value of d_a increasing, the width of the comb-like transmission band becomes more and more narrow, though its center position keeps invariant. Especially, when $d_a = 5/4d$, the band becomes one line at $\omega = \omega_0$. This property makes it serve as a single frequency filtering with high Q value. Comparing Figures 2 and 4 with Figure 1, we find there is an excellent agreement among them, which further demonstrates our calculations. Moreover, according to the property of scaling invariant of photonic crystals [20], if d change to $d' = sd$ (s is a scale parameter), the transmission spectra of $t(\omega)$ and the basic frequency become $t(\omega')$ ($\omega' = \omega/s$) and $\omega_0' = \pi/d' = \omega_0/s$, respectively. If we use ω_0' as frequency unit, due to $\omega/\omega_0 = (\omega')/\omega_0'$, the change of d doesn't influence

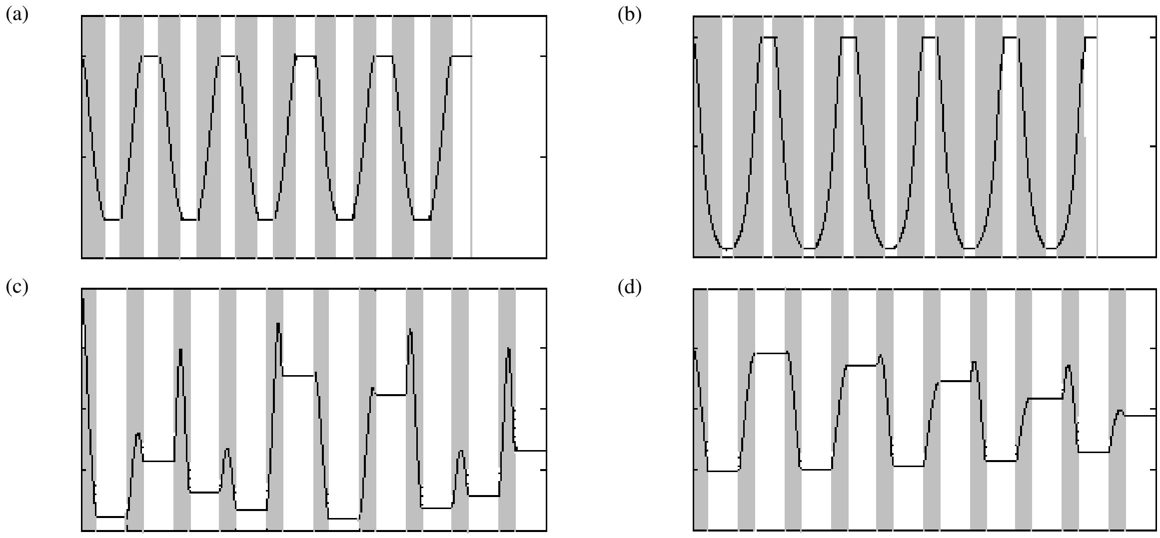

Fig. 5. Field distribution in the ENG-air structure with different structure parameters corresponding to Figure 4 and light frequencies. (a) $d_a = 3/4d$, $d_b = d/2$, $\omega = \omega_0$; (b) $d_a = 5/4d$, $d_b = d/2$, $\omega = \omega_0$; (c) $d_a = d/2$, $d_b = d$, $\omega = 0.68\omega_0$; (d) $d_a = d/2$, $d_b = d$, $\omega = y1.3\omega_0$. The gray area stands for A layer and the white area stands for B layer.

the calculation results. Therefore we can adjust the basic frequency according to our need. For the case of $d_a = d/2$ and $d_b = d$, there are two transmission bands, which are at $\omega = 0.68\omega_0$ and $\omega = 1.3\omega_0$, respectively. The first band is narrower than the second. Thus, we can conclude that the value of d_b decides the number and position of transmission bands while the value of d_a decides the width of transmission bands. Based on Figure 4, Figure 5 plots the field distribution in the ENG-air structure with different structure parameters. All the frequencies are selected at the center of the transmission bands. It is obviously that the results of Figures 4a, 4b, and Figure 1a are almost alike. In Figures 4c and 4d, the field in layers A takes on multiple behaviour.

In conclusion, the ENG-air band structure is sensitive to the thickness fluctuation of pair layers and the period number, which is different from the zero-$\bar{n}$ gap and the SNG gap. However, the dependence of the ENG-air band structure on thicknesses of pair layers can help us devise our needed optical device. The exact position of the ENG-air gap and transmission band can be obtained by a numerical calculation basing on Bloch's theorem.

[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[3] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[4] D. R. Smith and N. Kroll, Phys. Rev. Lett. **85**, 2933 (2000).
[5] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[6] R. A. Shelby, D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, Appl. Phys. Lett. **78**, 489 (2001).
[7] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[8] Z. M. Zhang and C. J. Fu, Appl. Phys. Lett. **80**, 1097 (2002).
[9] J. Li, L. Zhou, C. T. Chan, and P. Sheng, Phys. Rev. Lett. **90**, 083901 (2003).
[10] I. V. Shadrivov, A. A. Sukhorukov, and Y. S. Kivshar, Appl. Phys. Lett. **82**, 3820 (2003).
[11] A. A. Houck, J. B. Brock, and I. L. Chuang, Phys. Rev. Lett. **90**, 137401 (2003).
[12] Y. Fang, Q. Zhou, Appl. Phys. B **83**, 587 (2006).
[13] J. Li, L. Zhou, C. T. Chan, and P. Sheng, Phys. Rev. Lett. **90**, 083901 (2003).
[14] I. V. Shadrivov, A. A. Sukhorukov, and Y. S. Kivshar, Appl. Phys. Lett. **82**, 3820 (2003).
[15] H. Jiang, H. Chen, H. Li, and Y. Zhang, Appl. Phys. Lett. **83**, 5386 (2003).
[16] H. Jiang, H. Chen, H. Li, Y. Zhang, J. Zi, and S. Zhu, Phys. Rev. E **69**, 066607 (2004).
[17] L. Wang, H. Chen, and S. Zhu, Phys. Rev. B **70**, 245102 (2004).

[18] S. M. Wang, C. J. Tang, T. Pan, and L. Gao, Phys. Lett. A **348**, 424 (2006).
[19] M. Centini, C. Sibilia, M. Scalora, G. D'Aguanno, M. Bertolotti, M. J. Bloemer, C. M. Bowden, and I. Nefedov, Phys. Rev. E **60**, 4891 (1999).
[20] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade. Photonic Crystals Molding the flow of light [M]. In the United Kindom: University Press, Princeton 2008, p. 20.