Higher-Order Nonlinear Effects on Wave Structures in a Multispecies Plasma with Nonisothermal Electrons

Tarsem Singh Gill[a], Parveen Bala[b], and Harvinder Kaur[c]

[a] Department of Physics, Guru Nanak Dev University, Amritsar-143005, India
[b] Department of Math. Stat. & Physics, Punjab Agricultural University, Ludhiana-141004, India
[c] Department of Physics, Khalsa College, Amritsar-143002, India

Reprint requests to T. S. G.; E-mail: gillsema@yahoo.co.in or pravi2506@gmail.com

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In the present investigation, we have studied ion-acoustic solitary waves in a plasma consisting of warm positive and negative ions and nonisothermal electron distribution. We have used reductive perturbation theory (RPT) and derived a dispersion relation which supports only two ion-acoustic modes, viz. slow and fast. The expression for phase velocities of these modes is observed to be a function of parameters like nonisothermality, charge and mass ratio, and relative temperature of ions. A modified Korteweg-de Vries (KdV) equation with a (1+1/2) nonlinearity, also known as Schamel-mKdV model, is derived. RPT is further extended to include the contribution of higher-order terms. The results of numerical computation for such contributions are shown in the form of graphs in different parameter regimes for both, slow and fast ion-acoustic solitary waves having several interesting features. For the departure from the isothermally distributed electrons, a generalized KdV equation is derived and solved. It is observed that both rarefactive and compressive solitons exist for the isothermal case. However, nonisothermality supports only the compressive type of solitons in the given parameter regime.

Key words: Ion Acoustic Solitons; Multispecies Plasma; Nonisothermal Distribution; Renormalization Technique.

1. Introduction

The pioneer work of Zabusky and Kruskal [1] on the one-dimensional soliton solution of the Korteweg-de Vries (KdV) equation in plasma physics opened new vistas in the study of nonlinear phenomena in various branches of science. The fundamental role in modelling nonlinear processes in biology, particle physics, condensed matter physics, plasma physics, nonlinear optics, neuro physics, and more recently in Bose-Einstein condensates have been in vogue for the last several years. Of particular interest in the study of nonlinear waves, the excitation, propagation, and interaction of solitary waves is an important issue in theoretical physics. Washimi and Taniuti [2] were the first to use the reductive perturbation technique to derive a KdV equation for ion-acoustic (IA) solitons in plasma. Among the most studied structures are ion-acoustic solitons, which are beautiful manifestations of nature, arising out of the delicate balance between properties like nonlinearity and dispersion. The KdV model used to study ion-acoustic solitons (IASs) is applicable only for small amplitude solitary waves. This has been done in a number of plasma systems [3 – 5]. However, another method known as Sagdeev potential method is also used to study ion-acoustic solitary waves. This method makes use of the full nonlinearities of equations on plasma dynamics [6]. Particularly, ion-acoustic solitary waves in magnetized plasma have been restricted to study small amplitudes [7, 8]. Solitary waves and all nonlinear terms in the equation of motion have not been considered. Yu et al. [6] considered the problem of ion-acoustic solitary waves propagating in a plasma by mean of equations describing the two-dimensional wave propagation in a quasineutral magnetized plasma with cold ions. The dispersion in this plasma system is purely due to the gyro radius effect. The Sagdeev potential derived here is a function of n, V, and k_z. It is observed that for the physical viable solutions ψ is negative between $n = n$ and N if $N^2 > 1 > V^2$. In that case solitons consist of a density hump propagating at near sonic speed. Ion-acoustic solitary waves in magnetized as well as unmagnetized plasmas were reported using both the RPT as well as the Sagdeev potential method as witnessed by a large number of publications. The earlier experimental observations confirmed the existence of various solitons in laboratory [9 – 12] and is reported in an excellent review [13]. Theoretical investigations of IASs and other nonlinear wave structures such as double layers, shock waves, vortices, etc. have received considerable attention for more than over three decades as is evident from the prolific literature. Solitons and other nonlinear phenomena in plasma physics are well documented in a standard text [14].

Space plasmas contain several ions having different charge states and are usually of a multicomponent type. In situ measurements by various satellite missions provide abundant information about stationary nonlinear electrostatic/electromagnetic (ES/EM) structures occurring in a space plasma. Both, positive as well as negative ions exist in such a plasma. Further, the response of a plasma to disturbances in the presence of negative ions is significantly modified [15]. Moreover, it was well known that negative ions have a considerable effect on the characterization of ion waves [16 – 21]. Uberoi and Das [22] showed that the presence of negative ions in the lower ionosphere affect the plasma diagnostics through the study of cross over frequencies. The functional relationship of relative density and the idea of critical density of negative ions have been introduced theoretically [23, 24] and confirmed experimentally [25 – 28]. Negative ions are found in the D-region of the Earth's ionosphere [29]. As an approximation, plasma can be treated collisionless in the upper part of the D-region of the Earth's ionosphere where collision frequency is expected to be small and can be neglected [30]. As earlier mentioned, space plasmas are frequently of multicomponent type consisting of both positive as well as negative ions of different temperatures [3, 5, 31]. Two-ion species having different masses, concentrations, charge states, and temperatures [3, 5, 31] are very common in space and in the laboratory, e. g. (Ar^+, $SF_6{}^-$), (Ar^+, F^-), (H^+, $O_2{}^-$), and (H^+, H^-) plasma systems.

Recent observations of space and laboratory plasmas have shown that particle distributions play a crucial role in characterizing the physics of nonlinear waves. Both linear and nonlinear properties are influenced by the velocity distribution of the particle constituents of the plasma. Moreover, they add considerable increase in richness and variety of wave motion that can exist in plasma and further influence the conditions required for the formation of these waves. In practice, the particles may not follow a Maxwellian distribution and based on the data, particle distributions are better modelled by velocity distributions having a flat top with high energy tails. Two most commonly used non-Maxwellian type distributions are nonthermal and nonisothermal particle distributions. The former one, associated with the particle flows resulting from the force fields present in the space and astrophysical plasmas, has an abundance of superthermal particles. The second one, nonisothermal particle distribution is due to the formation of phase space holes caused by the trapping of electrons in a wave potential and is not only observed in space plasmas [32, 33] but also in laboratory [34, 35]. Moreover, the plasmas excited by an electron beam evolve towards a coherent trapped particle state rather than developing into a turbulent one as has been confirmed by experiments [36]. As a matter of fact, trapping can occur and contribute even for infinite small amplitude [37]. Several investigations, earlier [32, 33, 38] as well as the recent ones [37, 39, 40], have been reported on the study of ion-acoustic solitons with vortex-like distributions. The presence of trapped and free particles can significantly modify the characteristics in collisionless plasmas [32, 38]. Another important particle distribution, used to analyze and interpret space craft data on the Earth's magnetospheric plasma sheet, solar wind, and many space plasmas, is the kappa distribution currently being studied in many nonlinear wave structures [41, 42].

Observations made from the Freja satellite with density depletion associated with an electric field suggested that such structures are electrostatic in nature [43]. Even though several theoretical models were proposed to explain these structures, however, Cairns et al. [44] proposed the theoretical explanation of these structures by assuming a nonthermal distribution of electrons. It was found that the excess of energetic particles changes the nature of ion-acoustic (IA) solitary structures and it was possible to obtain solutions with density depletions. This opened a new area in the study of nonlinear wave structures in different plasma systems containing nonthermal particle distributions [5, 39, 45 – 59]. In a plasma with nonisothermal particle distribution both trapped and free particles are available. However, in the nonthermal model, there is an excess of energetic particles arising due to the force fields present in space plasma.

KdV or mKdV model equations describe the small amplitude IASs and include lowest-order nonlinearity and dispersion. These evolution equations are associated with a quadratic and a cubic type of nonlinearity. However, for particles described by a vortex-like distribution, the nonlinearity is of the (1+1/2) type and the resulting equation is the Schamel-mKdV equation [60]. As the wave amplitude increases, the width and velocity of a soliton deviates from the one predicted by the KdV or mKdV model. In such cases, more accurate results can be obtained by the inclusion of higher-order nonlinear and dispersive effects. For this purpose, the higher-order approximation of the reductive perturbation theory has been considered as a powerful tool [61 – 74]. More recently, Esfandyari-Kalejahi et al. [60] have studied ion-acoustic waves in a plasma consisting of adiabatic ions, nonisothermal electrons, and a weakly relativistic electron beam. They have studied both, linear and higher-order effects. Higher-order effects are shown to modify the solitary wave amplitude and may also cause a shape deformation.

In this research work, we consider higher-order nonlinear effects on the propagation of IASs in a collisionless plasma consisting of positively and negatively charged ions with nonisothermal electrons. The organisation of the paper is as follows: In Sections 2 and 3, we set up the basic equations governing the dynamics of the multispecies plasma and derive the Schamel mKdV equation. The deviation from isothermality is taken up in Section 4. In Section 5, we obtain higher-order stationary solutions under appropriate conditions using renormalization technique developed by Kodma and Taniuti [62, 63] and in Section 6, we present the discussion of numerical computation of different plasma systems. The last section is devoted to the concluding remarks.

2. Governing Equations

We consider a collisionless unmagnetised plasma consisting of a nonthermal electron distribution and warm positive and negative ion species having temperatures T_1 and T_2, which are divided into two distinct groups. We assume that low frequency electrostatic waves propagate in the plasma. The nonlinear behaviour of the ion-acoustic waves may be described by the following set of normalized fluid equations [3, 5]:

$$\frac{\partial n_1}{\partial t}+\frac{\partial(n_1 v_1)}{\partial x}=0, \tag{1}$$

$$\frac{\partial v_1}{\partial t}+v_1\frac{\partial v_1}{\partial x}=-\frac{1}{\delta}\frac{\partial\phi}{\partial x}-\frac{\sigma_1}{\delta Z_1}\frac{1}{n_1}\frac{\partial n_1}{\partial x}, \tag{2}$$

$$\frac{\partial n_2}{\partial t}+\frac{\partial(n_2 v_2)}{\partial x}=0, \tag{3}$$

$$\frac{\partial v_2}{\partial t}+v_2\frac{\partial v_2}{\partial x}=\frac{\varepsilon_z}{\delta\eta}\frac{\partial\phi}{\partial x}-\frac{\sigma_2}{\delta\eta Z_1}\frac{1}{n_2}\frac{\partial n_2}{\partial x}, \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_\mathrm{e}-\frac{n_1}{1-\alpha\varepsilon_z}+\frac{\alpha\varepsilon_z}{1-\alpha\varepsilon_z}n_2, \tag{5}$$

where

$$\begin{aligned}&\delta=\frac{\eta+\alpha\varepsilon_z^2}{\eta(1-\alpha\varepsilon_z)},\quad \alpha=\frac{n_{20}}{n_{10}},\quad \varepsilon_z=\frac{Z_2}{Z_1},\\ &\eta=\frac{m_2}{m_1},\quad \sigma_1=\frac{T_1}{T_\mathrm{e}},\quad \sigma_2=\frac{T_2}{T_\mathrm{e}}.\end{aligned} \tag{6}$$

In the above equations n_1, v_1 and n_2, v_2 are the densities and fluid velocities of positive and negative ion species, respectively. n_{10}, n_{20} are the equilibrium densities of two ion components, respectively. Further, ϕ is the electrostatic potential, η is the mass ratio of the negative ion species to the positive ion species, α is the equilibrium density ratio of the negative ion to positive ion species, and ε_z is the charge multiplicity ratio of the negative ion to positive ion species. In (1) – (6), velocities (v_1, v_2), potential (ϕ), time (t), and space coordinate (x) have been normalized with respect to the ion-acoustic speed in the mixture, $C_\mathrm{s}=\sqrt{T_\mathrm{e}\delta Z_1/m_1}$, thermal potential T_e/e, inverse of ion plasma frequency in the mixture $\omega_\mathrm{pi}^{-1}=\sqrt{m_1/4\pi n_\mathrm{eo}\delta Z_1}$, Debye length $\lambda_\mathrm{D}=\sqrt{T_\mathrm{e}/4\pi n_\mathrm{eo}e^2}$, respectively. Ion densities n_1, n_2 and electron density n_e are normalized with their corresponding equilibrium values.

3. Formulation of the Problem

To study the effect of nonisothermal electrons on the characterization of nonlinear ion-acoustic waves in nonrelativistic plasma, we employ the vortex-like electron distribution of Schamel [32, 75, 76], which solves the electron Vlasov equation. Thus, we have

$$\begin{aligned}&f_\mathrm{ef}=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}(v^2-2\phi)},\quad |v|>\sqrt{2\phi},\\ &f_\mathrm{et}=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2\beta}(v^2-2\phi)},\quad |v|\le\sqrt{2\phi}.\end{aligned} \tag{7}$$

Here f_ef (f_et) represents the free (trapped) electron distribution. The velocity v is normalized to the electron thermal velocity C_s and $\beta=T_\mathrm{ef}/T_\mathrm{et}$ is the ratio of free electron temperature to trapped electron temperature. The parameter β determines the number of trapped electrons. It has been assumed that the velocity of nonlinear ion-acoustic waves is small in comparison with the electron thermal velocity. Also, it becomes obvious from this distribution that $\beta = 1$ ($\beta = 0$) represents a Maxwellian (flat-topped) distribution, whereas $\beta < 0$ represents a vortex-like excavated trapped electron distribution.

The electron distribution functions (7) can be readily integrated over the velocity space to get the electron number density [75 – 78].

$$n_e = I(\phi) + \frac{e^{\beta\phi}}{\sqrt{|\beta|}}\mathrm{erf}(\sqrt{\beta\phi}), \quad \beta \geq 0,$$
$$n_e = I(\phi) + \frac{2}{\sqrt{\pi|\beta|}} W\sqrt{\beta\phi}, \quad \beta < 0, \tag{8}$$

where

$$I(\phi) = [1-\mathrm{erf}(\sqrt{\phi})]e^{\phi},$$
$$\mathrm{erf}(\sqrt{\beta\phi}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\beta\phi}} e^{-y^2}\,dy, \tag{9}$$
$$W(\sqrt{-\beta\phi}) = e^{\beta\phi}\int_0^{\sqrt{-\beta\phi}} e^{y^2}\,dy.$$

In the small amplitude limit, we can expand n_e and on keeping terms up to ϕ^2, it is found that n_e is the same for both $\beta \geq 0$ and $\beta < 0$. We find

$$n_e = 1 + \phi + \frac{1}{2}\phi^2 - \frac{4}{3}b\phi^{3/2}, \tag{10}$$

where $\beta = 1 - \beta/\sqrt{\pi}$ and $\beta = T_{ef}/T_{et}$.

In order to derive a nonlinear dynamical equation for the ion-acoustic waves from equations (1) – (5) and (10), we must find an appropriate co-ordinate from where the wave can be described smoothly. For this purpose, we need to know the thickness and the nonlinear velocity v_0 of the wave which can be taken from the equilibrium theory using a vortex-like electron distribution [32, 38]. Thus, we find $\Delta \propto \varepsilon^{1/4}$ and $(v_0 - 1) \propto \varepsilon^{1/2}$, where ε is a small parameter measuring the weakness of nonlinearity. This immediately leads to the following stretched co-ordinates:

$$\xi = \varepsilon^{1/4}(x - \lambda t), \quad \tau = \varepsilon^{3/2}t, \tag{11}$$

where λ is the phase velocity of ion-acoustic waves and ε measures the size of the perturbation amplitude. Furthermore, the dependent variables are expanded as power series in ε about their equilibrium values as:

$$n_{1,2} = 1 + \sum_{r=1}^{\infty}\varepsilon^{(r+1)/2}n_{1,2}^{(n)}, \quad u_{1,2} = \sum_{r=1}^{\infty}\varepsilon^{(r+1)/2}v_{1,2}^{(n)},$$
$$\phi = \sum_{r=1}^{\infty}\varepsilon^{(r+1)/2}\phi_r, \tag{12}$$

in which the index numbers 1,2 refer to positive and negative ions, respectively. Substituting the stretched coordinates (11) and the perturbation relations (12) to the basic set of equations (1) – (5) and following the usual reductive perturbation theory [2], the first-order equations give

$$n_1^{(1)} = \frac{Z_1}{\delta Z_1\lambda^2 - \sigma_1}\phi_1, \quad n_2^{(1)} = \frac{-Z_2}{\delta Z_2\lambda^2 - \sigma_2}\phi_1,$$
$$v_1^{(1)} = \frac{Z_1\lambda_g}{\delta Z_1\lambda^2 - \sigma_2}\phi_1, \quad v_2^{(1)} = \frac{-Z_2\lambda_g}{\delta Z_1\lambda^2 - \sigma_1}\phi_1. \tag{13}$$

The Poisson's equation gives the following dispersion relation:

$$\lambda^2 = \left(\frac{1}{2} + \frac{\sigma^2 + \eta\sigma_1}{2\delta\eta Z_1}\right) \pm \left[\left(\frac{1}{2} + \frac{\sigma^2 + \eta\sigma_1}{2\delta\eta Z_1}\right)^2 - \frac{1}{\eta\delta^2 Z_1}\left(\frac{\sigma_1\sigma_2}{Z_1} + \frac{\sigma^2 + \sigma_1\alpha\varepsilon_z^2}{1 - \alpha\varepsilon_z}\right)\right]^{\frac{1}{2}}, \tag{14}$$

where positive sign is for the fast ion-acoustic mode and negative sign for the slow ion-acoustic mode.

Apparently, the phase velocities are the function of several parameters including density parameter α, mass ratio η, and ion temperatures σ_1, σ_2. Thus, the system supports two types of ion-acoustic modes which propagate with different phase velocities given by (14). The mode with smaller phase velocity is the slow ion-acoustic mode whereas the mode with larger phase velocity is known as the fast ion-acoustic mode. Consequently, the system supports two types of ion-acoustic solitons, viz. slow ion-acoustic solitons and fast ion-acoustic solitons. Earlier investigations [3, 4] reported two cases in which the system does not support the slow ion-acoustic mode, viz.:

(a) When both species are cold i. e., $\sigma_1 = \sigma_2 = 0$, then from (14), the phase velocity of the slow ion-acoustic mode becomes zero indicating that the slow ion-acoustic mode does not exist, only the fast acoustic mode exists.

(b) However, when $\sigma_1 = \sigma_2$ and $\eta = 1$, the slow ion-acoustic mode exists.

It may be mentioned here that the expressions for the phase velocities in case of slow and fast mode are in complete agreement with the results obtained by Mishra and Chhabra [3]. It is also clear from (14) that λ is independent of β.

If we consider the next order in ε, we obtain a system of equations in the second-order perturbed quantities. Solving this system using (13) and (14), we obtain the Schamel-type mKdV equation

$$\frac{\partial \phi_1}{\partial \tau} + 2bA\sqrt{\phi_1}\frac{\partial \phi_1}{\partial \xi} + A\frac{\partial^3 \phi_1}{\partial \xi^3} = 0. \tag{15}$$

Here

$$A = \frac{1-\alpha\varepsilon_z}{2\delta\lambda}\left[\frac{Z_1^2}{(\delta Z_1\lambda^2-\sigma_1)^2} + \frac{\alpha\eta Z_2^2}{(\eta\delta Z_1\lambda^2-\sigma_2)^2}\right]^{-1}. \tag{16}$$

4. Deviation from Isothermality

Traditionally, with Boltzmann distributed electrons, solitary ion-acoustic waves have both positive and negative potential. There exists a critical ratio of negative to positive ion concentration below which compressive and above which rarefactive solitons are admissible. This aspect is discussed in detail, where literature on this issue is cited both from theoretical and experimental point of view. To consider this aspect how situation changes, we have derived a generalized KdV equation to investigate the effect of small deviation from isothermality of electrons on ion-acoustic solitary waves. In this case, the previous analysis based on the modified KdV equation (15) is no longer valid and we go back to the original governing equations (1) – (5) and (10). Using stretching coordinates as $\xi = \varepsilon^{1/2}(x-\lambda t)$ and $\tau = \varepsilon^{3/2}t$ and adopting the standard procedure of Tagare and Chakrabarti [82], we arrive at the following generalized KdV equation:

$$\frac{\partial \phi_1}{\partial \tau} + (y_1\phi_1 + y_2\sqrt{\phi_1})\frac{\partial \phi_1}{\partial \xi} + y_3\frac{\partial^3 \phi_1}{\partial \xi^3} = 0, \tag{17}$$

where $y_1 = AQ, y_2 = 2b_1A, y_3 = A$, and $b_1 = \varepsilon^{1/2}b$ [82]. A is given by (16) and Q is given by

$$Q = \frac{Z_1^2}{(1-\alpha\varepsilon_z)}\left[\frac{(3\delta Z_1\lambda^2)}{(\delta Z_1\lambda^2-\sigma_1)^3} - \alpha\varepsilon_z^3\frac{(3\eta\delta Z_1\lambda^2)}{(\eta\delta Z_1\lambda^2-\sigma_1)^3}\right] - 1. \tag{18}$$

We seek the stationary soliton-like solutions of the generalized KdV equation (17) subject to appropriate boundary conditions, namely $\phi_1(\chi) \to 0$, $\mathrm{d}\phi_1/\mathrm{d}\chi \to 0$, and $\mathrm{d}^2\phi_1/\mathrm{d}^2\chi^2 \to 0$ as $\chi = |\xi - v_0\tau| \to \pm\infty$. Solitary wave like solution of this generalized KdV equation as [82] is given as follows:

$$\phi_{g1} = \left[\frac{4y_2}{15v_0} + \left(\frac{16y_2}{225v_0^2} + \frac{y_1}{3v_0}\right)^{\frac{1}{2}}\cosh(\chi/\Delta)\right], \tag{19}$$

where $\Delta = \sqrt{4y_3/v_0}$.

5. Higher-Order Nonlinearity and Stationary Solutions

To the next order in ε, we get the equation

$$\frac{\partial \phi_2}{\partial \tau} + 2bA\frac{\partial \phi_2\sqrt{\phi_1}}{\partial \xi} + A\frac{\partial^3 \phi_2}{\partial \xi^3} = S(\phi_1), \tag{20}$$

where the source term $S(\phi_1)$ is given by

$$S(\phi_1) = (4b^2D - 2AC)\phi_1\frac{\partial \phi_1}{\partial \xi} + D\frac{\partial^5 \phi_1}{\partial \xi^5} + \frac{3bD}{\sqrt{\phi_1}}\frac{\partial \phi_1}{\partial \xi}\frac{\partial^2 \phi_1}{\partial \xi^2} - \frac{b}{2}\frac{D}{\phi_1^{3/2}}\left(\frac{\partial \phi_1}{\partial \xi}\right)^3 + 4bD\sqrt{\phi_1}\frac{\partial^3 \phi_1}{\partial \xi^3}, \tag{21}$$

with $D = A^2B$ and the coefficients B and C occurring in (22) are given by

$$B = \frac{\delta Z_1^2 A}{(1-\alpha\varepsilon_z)}\left[\frac{1}{(\delta Z_1\lambda^2-\sigma_1)^2} - \alpha\varepsilon_z^2\frac{\eta}{(\eta\delta Z_1\lambda^2-\sigma_2)^2}\right] - \frac{4\lambda^2\delta^2 Z_1^3 A}{(1-\alpha\varepsilon_z)}\left[\frac{1}{(\delta Z_1\lambda^2-\sigma_1)^3} - \alpha\varepsilon_z^2\frac{\eta^2}{(\delta Z_1\lambda^2-\sigma_1)^3}\right], \tag{22}$$

$$C = \frac{Z_1^2}{2(1-\alpha\varepsilon_z)}\left[\frac{(3\delta Z_1\lambda^2-\sigma_1)}{(\delta Z_1\lambda^2-\sigma_1)^3} - \alpha\varepsilon_z^3\frac{(3\eta\delta Z_1\lambda^2-\sigma_2)}{(\eta\delta Z_1\lambda^2-\sigma_2)^3}\right] - \frac{1}{2}. \tag{23}$$

The basic set of equations (1) – (5) is reduced to a nonlinear mKdV equation (15) in terms of ϕ_1 and a linear homogeneous differential equation (20) in terms of ϕ_2, for which the source term of (20) is described by a known function ϕ_1. In the following, we shall see how the two evolution equations may be solved analytically.

We employ the renormalization method developed by Kodma and Taniuti [62] in order to solve (15) and (20). Using this method, (15) and (20) get modified as

$$\frac{\partial \tilde{\phi}_1}{\partial \tau} + 2bA\sqrt{\tilde{\phi}_1}\frac{\partial \tilde{\phi}_1}{\partial \xi} + A\frac{\partial^3 \tilde{\phi}_1}{\partial \xi^3} + \delta v\frac{\partial \tilde{\phi}_1}{\partial \xi} = 0 \tag{24}$$

and

$$\frac{\partial\tilde{\phi}_2}{\partial\tau}+2bA\frac{\partial\tilde{\phi}_2\sqrt{\tilde{\phi}_1}}{\partial\xi}+A\frac{\partial^3\tilde{\phi}_2}{\partial\xi^3}\delta\nu\frac{\partial\tilde{\phi}_2}{\partial\xi}=S(\tilde{\phi}_1)\delta\nu\frac{\partial\tilde{\phi}_1}{\partial\xi}, \tag{25}$$

where $\tilde{\phi}_1$ and $\tilde{\phi}_2$ refers to the renormalization variables.

Let us have the stationary solution by defining a new variable ψ as

$$\psi=\xi-(\nu+\delta\nu)\tau, \tag{26}$$

where the parameter ν is related to the Mach number $M=V/C_s$ [66, 79, 80] by $\nu+\delta\nu=M-1=\Delta M$, where V is the soliton velocity. Under this transformation, (24) and (25) become

$$A\frac{\partial^3\tilde{\phi}_1}{\partial\psi^3}+2bA\sqrt{\tilde{\phi}_1}\frac{\partial\tilde{\phi}_1}{\partial\psi}-\nu\frac{\partial\tilde{\phi}_1}{\partial\psi}=0 \tag{27}$$

and

$$A\frac{\partial^3\tilde{\phi}_2}{\partial\psi^3}+2bA\frac{\partial\tilde{\phi}_2\sqrt{\tilde{\phi}_1}}{\partial\psi}-\nu\frac{\partial\tilde{\phi}_2}{\partial\psi}=S(\tilde{\phi}_1)+\delta\nu\frac{\partial\tilde{\phi}_1}{\partial\psi}. \tag{28}$$

It may be mentioned that (27) and (28) are ordinary differential equations (ODEs) which are easily integrated under the boundary conditions

$$\tilde{\phi}_1=\tilde{\phi}_2=\frac{\partial\tilde{\phi}_1}{\partial\xi}=\frac{\partial\tilde{\phi}_2}{\partial\psi}=\frac{\partial^2\tilde{\phi}_1}{\partial\psi^2}=\frac{\partial^2\tilde{\phi}_2}{\partial\psi^2}=0, \quad \text{as } |\psi|\to\infty. \tag{29}$$

Using above boundary conditions for $\tilde{\phi}_1$ and $\tilde{\phi}_2$, (27) and (28) can be integrated with respect to the variable ψ and their derivatives up to the second-order:

$$\frac{\mathrm{d}^2\tilde{\phi}_1}{\mathrm{d}\psi^2}+\left(\frac{4b}{3}\tilde{\phi}_1{}^{1/2}-\frac{\nu}{A}\right)\tilde{\phi}_1=0, \tag{30}$$

$$\frac{\mathrm{d}^2\tilde{\phi}_2}{\mathrm{d}\psi^2}+\left(2b\tilde{\phi}_1{}^{\frac{1}{2}}-\frac{\nu}{A}\right)\tilde{\phi}_2=\int_{-\infty}^{\psi}\left(S(\tilde{\phi}_1)+\delta\nu\frac{\mathrm{d}\tilde{\phi}_1}{\mathrm{d}\psi}\right)\mathrm{d}\psi. \tag{31}$$

The one-soliton solution of (30) is given by

$$\tilde{\phi}_1=\phi_0\,\mathrm{sech}^4(\psi W^{-1}), \tag{32}$$

where the amplitude (ϕ_0) and width (W) of the solitary wave are given by

$$\phi_0=\left(\frac{15\nu}{16bA}\right)^2 \quad \text{and} \quad W=\sqrt{\frac{16A}{\nu}}. \tag{33}$$

It may be mentioned that A is assumed to be positive to ensure the reality of the solutions. (32) shows the existence of the compressive solitons only.

The source term of (25) becomes

$$\begin{aligned}&\int_{-\infty}^{\psi}\left(S(\tilde{\phi}_1)+\delta\nu\frac{\mathrm{d}\tilde{\phi}_1}{\mathrm{d}\psi}\right)\mathrm{d}\psi\\&=\frac{\phi_0}{A}\Big[(\delta_\nu-B\nu^2)\,\mathrm{sech}^4(\psi W^{-1})\\&\qquad+(A/2-CA)\phi_0\,\mathrm{sech}^8(\psi W^{-1})\Big].\end{aligned} \tag{34}$$

In order to cancel the secular terms in $S(\tilde{\phi}_1)$, we have to put

$$\delta\nu=B\nu^2. \tag{35}$$

To solve (34), we define a new independent variable

$$\mu=\tanh(\psi W^{-1}). \tag{36}$$

Using this transformation, (31) thereby becomes

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left[(1-\mu^2)\frac{\mathrm{d}\tilde{\phi}_2}{\mathrm{d}\mu}\right]+\left[30-\frac{16}{1-\mu^2}\right]\tilde{\phi}_2\\&=F(1-\mu^2)^3=T(\mu),\end{aligned} \tag{37}$$

where

$$F=\frac{(15)^4}{b^4}\frac{\nu^3}{(16A)^3}(1/2-C). \tag{38}$$

The two independent solutions of (39) are given by associated Legendre functions of the first and second kind. These are given by

$$\begin{aligned}P_5^4&=945\mu(1-\mu^2)^2,\\Q_5^4&=\frac{945}{2}\mu(1-\mu^2)\ln\frac{1+\mu}{1-\mu}-334(1-\mu^2)^2\\&\quad+975\mu^2(1-\mu^2)+630\mu^4+264\frac{\mu^6}{1-\mu^2}\\&\quad+48\frac{\mu^3}{(1-\mu^2)^2}.\end{aligned} \tag{39}$$

By using the method of variation of parameters, the particular solution of (37) can be written as

$$\tilde{\phi}_{2p}(\mu)=q_1(\mu)P_5^4+q_2(\mu)Q_5^4, \tag{40}$$

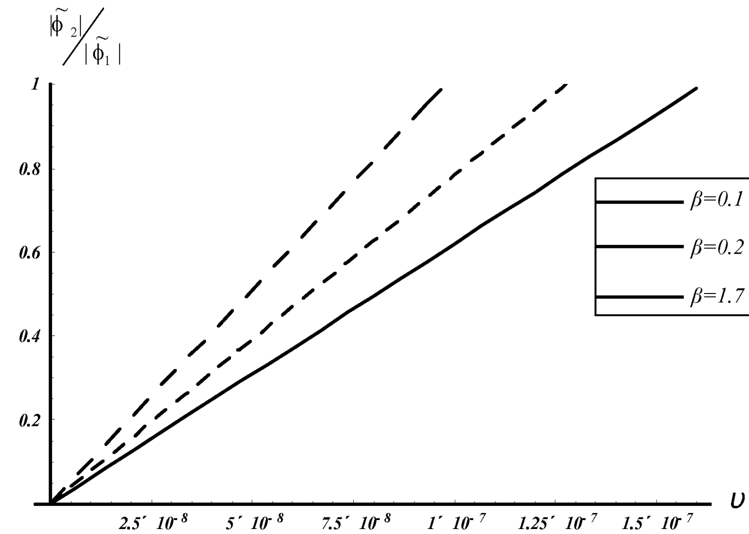

Fig. 1(a). For slow mode, plot of $|\tilde{\phi}_2|/|\tilde{\phi}_1|$ as a function of ν for three different values of β with $\alpha = 0.3$, $\eta = 1$, $\sigma_1 = 0.1$, $\sigma_2 = 0.01$.

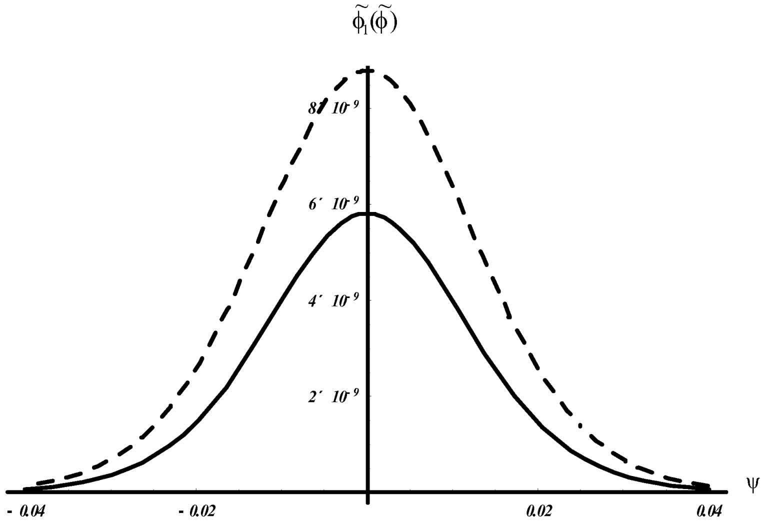

Fig. 1(b). For slow mode, plot showing the comparison of KdV soliton solution $\tilde{\phi}_1$ [given by (32) and shown by solid curve] and higher-order soliton solution $\tilde{\phi}$ [given by (43) and shown by dotted curve] as a function of ψ with $\nu = 5 \times 10^{-8}$ and $\beta = 1.7 (> 1)$. The other parameters are same as in Figure 1(a).

where

$$q_1(\mu) = -\frac{F}{945 \times 384} \int Q_5^4(\mu)(1-\mu^2)^3 \mathrm{d}\mu, \quad q_2(\mu) = \frac{F}{945 \times 384} \int P_5^4(\mu)(1-\mu^2)^3 \mathrm{d}\mu. \tag{41}$$

The complementary solution of (37) is given by

$$\tilde{\phi}_{2c}(\mu) = c_1(\mu) P_5^4 + c_2(\mu) Q_5^4. \tag{42}$$

Here, the first term is the secular one which can be eliminated by renormalizing the amplitude. Also, $c_2 = 0$, as a result of vanishing boundary conditions for $\tilde{\phi}_2(\psi)$ as $|\psi| \to \infty$. In terms of the variable ψ, the stationary solution for the potential of the ion-acoustic wave is given by

$$\begin{aligned} \tilde{\phi}(\psi) &= \tilde{\phi}_1(\psi) + \tilde{\phi}_2(\psi) \\ &= \phi_0 \operatorname{sech}^4(\psi W^{-1}) + \frac{F}{6} \operatorname{sech}^4(\psi W^{-1}) \\ &\quad - \frac{F}{12} \operatorname{sech}^6(\psi W^{-1}). \end{aligned} \tag{43}$$

6. Discussion of Numerical Results

We have considered a plasma model consisting of positive and negative ions with hot electrons obeying a trapped/vortex-like distribution. Using the reductive perturbation theory, we have studied nonlinear ion-acoustic solitons in this plasma system taking

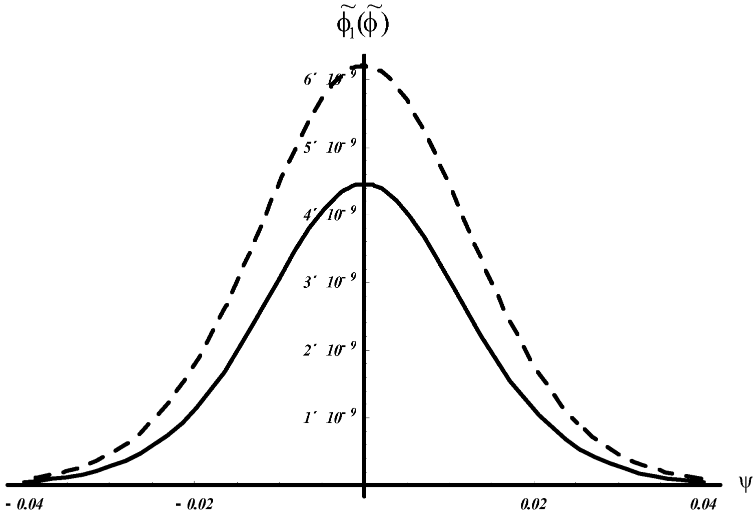

Fig. 1(c). For slow mode, the same plot as in Figure 1(b) with $\beta = 0.2 (< 1)$.

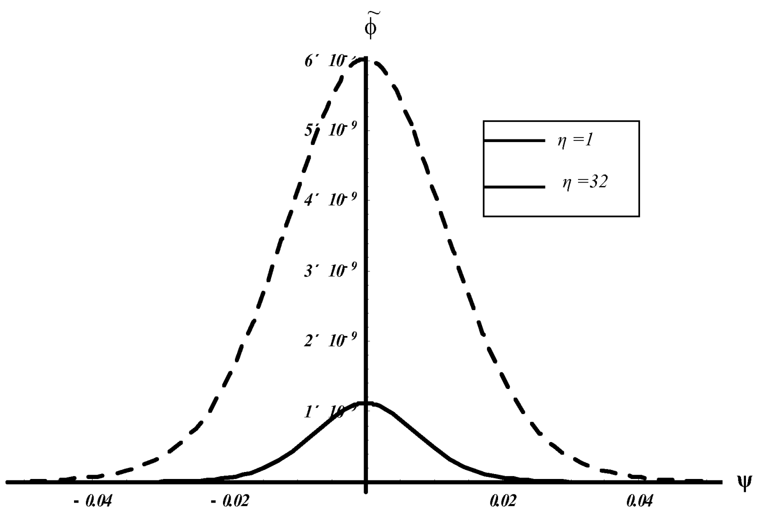

Fig. 1(d). For slow mode, variation of higher-order soliton solution $\tilde{\phi}$ as a function of ψ for two different values of η with $v = 2 \times 10^{-8}$ and $\beta = 1.7$. The other parameters are same as in Figure 1(a).

into account the contribution of higher-order nonlinear and dispersive terms. The basic set of fluid equations governing the dynamics, leads to the derivation of the Schamel-type equation (20) in the lowest-order of perturbation theory. It may be mentioned that (20) permits only compressive type of solitons. These results are consistent with those of Mamun and Shukla [81]. The solitary type solution of (15) having sech^4 profile, contains a peak amplitude which is a function of several parameters as relative density of negative ions α, relative ion-temperatures σ_1, σ_2, soliton velocity v, and temperature ratio of free to trapped electrons $\beta\,(= T_{\text{ef}}/T_{\text{et}})$, i. e. the trapped electron parameter. As already mentioned (14) is quadratic in λ^2 and correspondingly we have two modes, viz. slow and fast ion-acoustic modes, propagating with different phase velocities.

We have nonlinear ion-acoustic waves associated with these modes and here, we consider the effect of higher-order nonlinearity and dispersive effects on these slow and fast ion-acoustic modes. In the following, we shall discuss these modes and associated ion-acoustic solitons. To examine further the effect of these parameters on the nature of solitary waves associated with these modes, we numerically analyse the solution for an appropriate set of parameters relevant to ionosphere [3, 5] and laboratory plasma [28].

It is worth noting here that underlying the principle rule of reductive perturbation theory [2], for inclusion of higher effects, the following condition has to be sat-

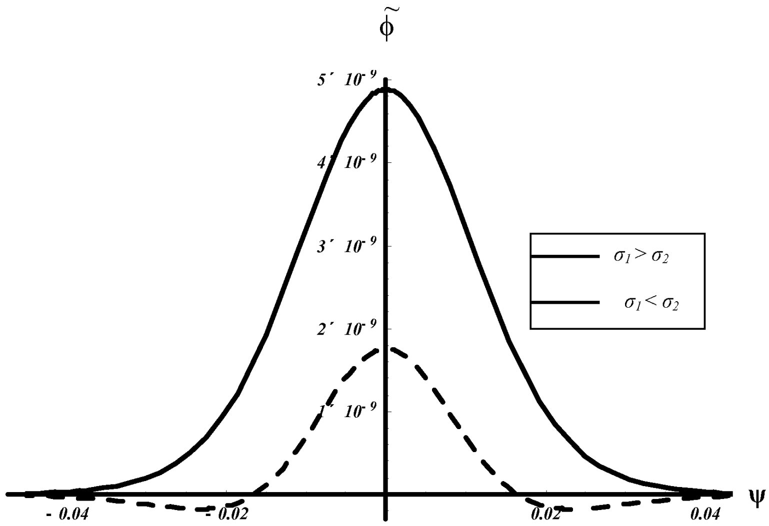

Fig. 1(e). For slow mode, variation of higher-order soliton solution $\tilde{\phi}$ as a function of ψ for two sets of σ_1, σ_2 with $\eta = 1$ and $v = 2\times10^{-8}$. Here the solid curve is for $\sigma_1 = 0.1$, $\sigma_2 = 0.01$ ($\sigma_1 > \sigma_2$) and the dotted curve for $\sigma_1 = 0.01$, $\sigma_2 = 0.1$ ($\sigma_1 < \sigma_2$).

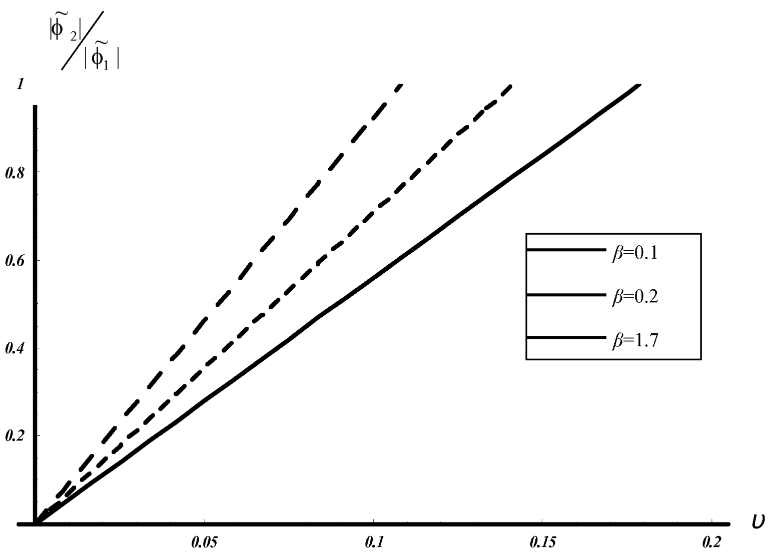

Fig. 2(a). For fast mode, plot of $|\tilde{\phi}_2|/|\tilde{\phi}_1|$ as a function of v for three different values of β with the other parameters same as in Figure 1(a).

isfied:

$$\frac{|\tilde{\phi}_2|}{|\tilde{\phi}_1|} \le 1. \tag{44}$$

This requirement stems from the proper ordering in ε. This means we have to choose an appropriate solitary excitation velocity. It is important to discuss the two modes separately and compare the results accordingly.

For the slow mode, the variation of $|\tilde{\phi}_2|/|\tilde{\phi}_1|$ at $\psi = 0$ vs. v is shown in Figure 1(a) for three different values of β ($\beta > 1$ and $\beta < 1$). The other parameters are chosen as follows:

$$\alpha = 0.3, \quad \eta = 1, \quad \varepsilon_z = Z_1 = 1, \quad \sigma_1 = 0.1, \quad \text{and } \sigma_2 = 0.01.$$

These parameters are relevant to (H^+, H^-) plasma that occur in the D-region of the ionosphere. As earlier mentioned, we have assumed the collision frequency as small, and so it has been neglected. As evident from the graphs shown in Figure 1(a), the increase in β allows the inequality (44) be satisfied for lower values of v. This is true for the case of $\beta < 1$, where the higher temperature of the trapped electrons is considered. However, opposite is true for the case with $\beta > 1$

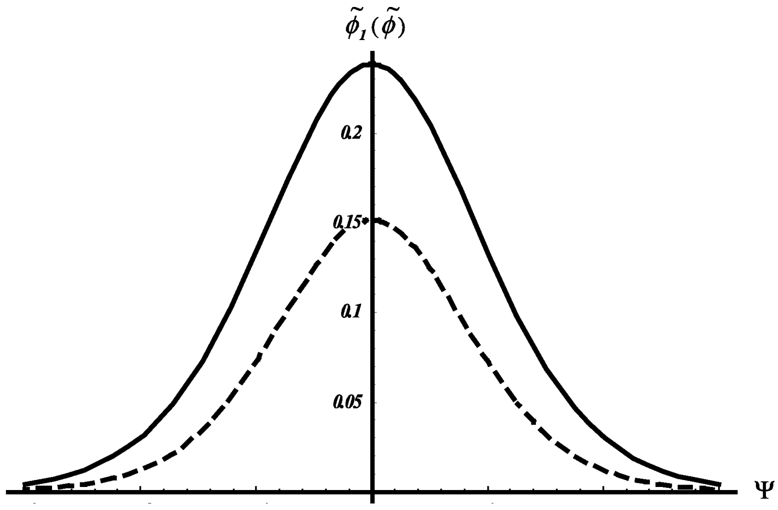

Fig. 2(b). For fast mode, plot showing the comparison of KdV soliton solution $\tilde{\phi}_1$ [shown by solid curve] and higher-order soliton solution $\tilde{\phi}$ [shown by dotted curve] as a function of ψ with $\alpha = 0.2$, $\nu = 0.1$ and $\beta = 1.7(> 1)$.

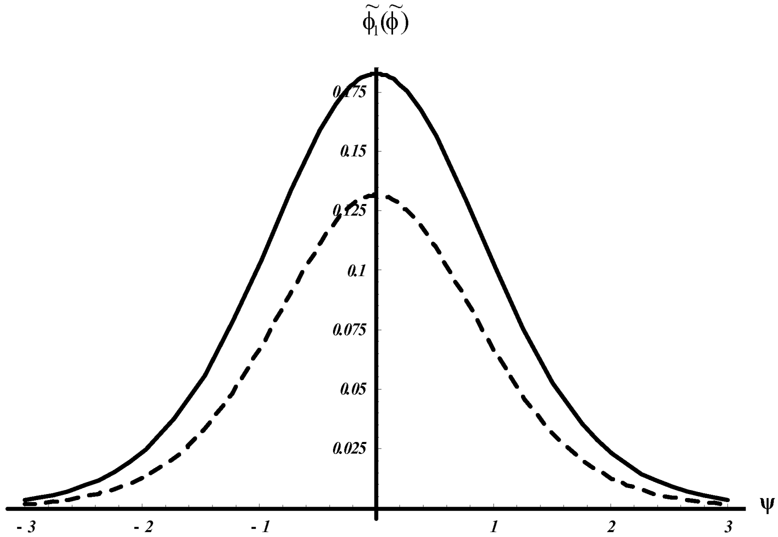

Fig. 2(c). For fast mode, the same plot as in Figure 1(b) with $\beta = 0.2(< 1)$.

[not shown]. A similar behaviour is observed in case of the fast mode as obvious from the variation of $|\tilde{\phi}_2|/|\tilde{\phi}_1|$ at $\psi = 0$ vs. ν shown in Figure 2(a).

To highlight the importance of the inclusion of higher-order effects, it is important to compare $\tilde{\phi}_1$, the soliton solution (32) of the slow ion-acoustic mode, with $\tilde{\phi}(= \tilde{\phi}_1 + \tilde{\phi}_2)$, the combined second-order solution given by (43). We have treated both cases of trapped electron parameter, viz. $\beta > 1$ and $\beta < 1$, with the other parameters taken for numerical computation as follows:

$$\alpha = 0.3, \quad \eta = 1, \quad \sigma_1 = 0.1, \quad \sigma_2 = 0.01, \quad \text{and} \quad \nu = 5 \times 10^{-8}.$$

The results are shown in the graphs plotted in Figures 1(b) and 1(c). As observed in both cases ($\beta \lesseqgtr 1$), the effect of second-order nonlinearity leads to an increase in the amplitude of the slow ion-acoustic solitary waves. This is consistent with the finding of earlier investigations [60, 74], where the wave amplitude is predicted to increase and consequently width decreases with the inclusion of higher-order nonlinearity. When similar results are evaluated for the fast mode [Figs. 2(b) and 2(c)], we obtain graphs contrary to those of the slow mode, i. e. the peak amplitude of the compressive solitons decreases on the introduction of higher-order nonlinearity for $\beta > 1$ as well as for $\beta < 1$.

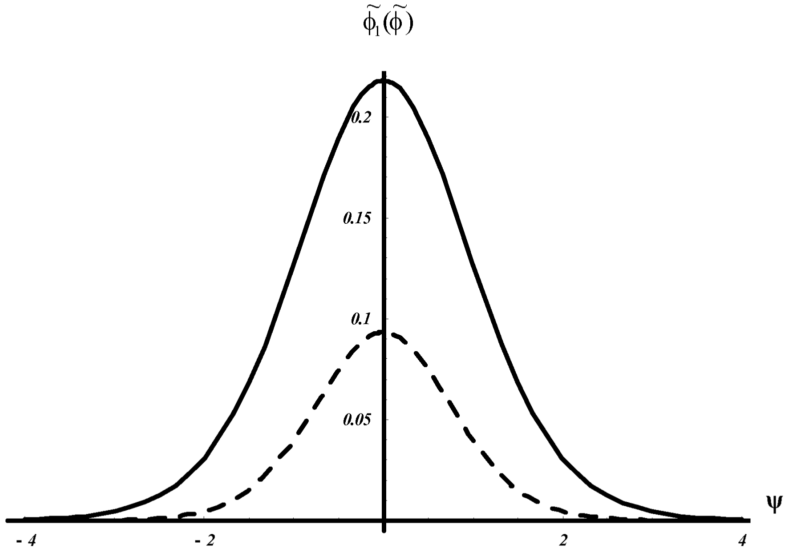

Fig. 2(d). For fast mode, variation of higher-order soliton solution $\tilde{\phi}$ as a function of ψ for two different values of η with $\nu = 0.1$ and the other parameters same as in Figure 1(d).

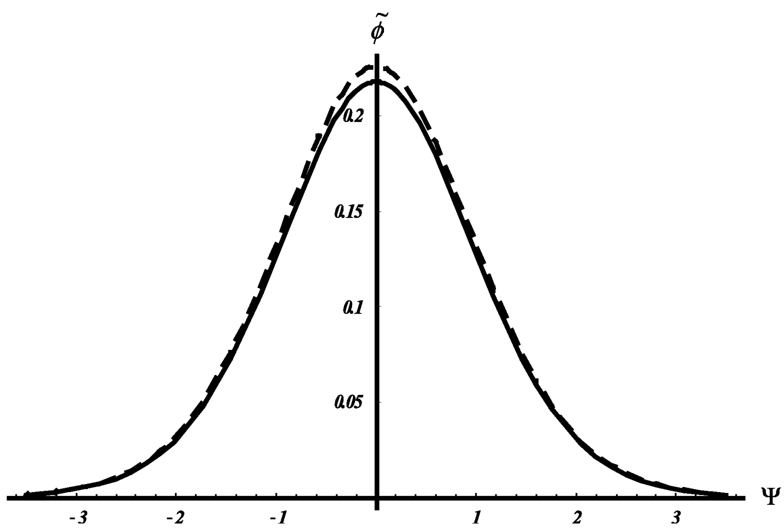

Fig. 2(e). For fast mode, variation of higher-order soliton solution $\tilde{\phi}$ as a function of ψ for two sets of σ_1, σ_2 with the other parameters as $\alpha = 0.4$, $\beta = 1.7$, $\eta = 1$, and $\nu = 0.1$. Here, the solid curve is for $\sigma_1 = 0.1$, $\sigma_2 = 0.01$ ($\sigma_1 > \sigma_2$) and the dotted curve for $\sigma_1 = 0.01$, $\sigma_2 = 0.1$ ($\sigma_1 < \sigma_2$).

Since space environment plasmas are a rich source of multispecies plasmas, it is useful to highlight the effect of the mass ratio of negative to positive ions, both for slow and fast ion-acoustic solitons. To this end, we have considered the effect of $\eta = (m_2/m_1)$, and numerically evaluated $\tilde{\phi}_1$ as a function of ψ for both cases ($\beta > 1$ or $\beta < 1$) with the other parameters taken as follows:

$$\alpha = 0.3, \quad \beta = 1.8, \quad \sigma_1 = 0.1, \quad \sigma_2 = 0.01,$$
$$\nu = 2 \times 10^{-8}, \text{ and } \eta = 0.476, 1, 32.$$

It is observed that in the case of slow mode, for given β (< 1 or > 1), the amplitude first decreases with the increase in mass of negative ions [Fig. 1(d)]. However, the amplitude increases for high values of η. This may be due to the contribution of the mass of positive ions. The behaviour is different in case of fast mode, where the peak amplitude of compressive solitons decreases with increase in the mass of negative ions. This behaviour is predicted in Figure 2(d). We can say that for the formation of large amplitude solitons, heavier negative ions are favourable for the slow ion-acoustic mode while heavier positive ions are favourable in case of fast ion-acoustic mode.

Remarkably interesting features are observed on the introduction of higher-order terms when $\sigma_1 > \sigma_2$ or

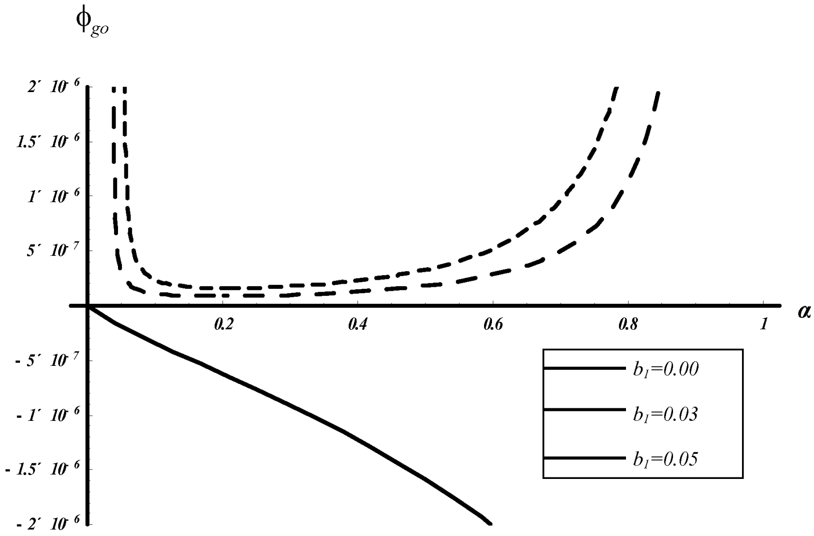

Fig. 3(a). For slow mode, plot showing the variation of peak amplitude (ϕ_{g0}) of generalized KdV equation (17) as a function of negative to positive ion density ratio α for three different values of b_1. The other parameters are $\eta = 1$, $Z_1 = Z_2 = \varepsilon_z = 1$, $\sigma_1 = 0.1$, $\sigma_2 = 0.01$, and $v_0 = 1.6 \cdot 10^{-6}$.

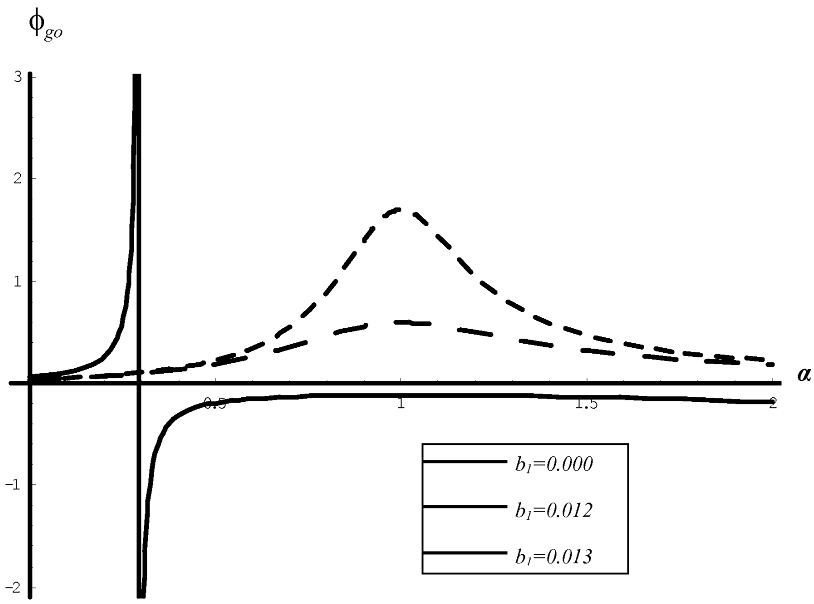

Fig. 3(b). For fast mode, plot showing the variation of peak amplitude (ϕ_{g0}) of generalized KdV equation (17) as a function of negative to positive ion density ratio α for three different values of b_1. The other parameters are $\eta = 1$, $Z_1 = Z_2 = \varepsilon_z = 1$, $\sigma_1 = 0.1$, $\sigma_2 = 0.01$, and $v_0 = 0.01$.

$\sigma_1 < \sigma_2$ and $\beta > 1$. This becomes obvious when we plot $\tilde{\phi}$ as a function of ψ for the parameters

$$\alpha = 0.3, \quad \beta = 1.7, \quad \eta = 1, \quad v = 3 \times 10^{-8}, \quad \text{and} \ \sigma_1 > \sigma_2 \ \text{or} \ \sigma_1 < \sigma_2.$$

A large peak value of $\tilde{\phi}$ is obtained when $\sigma_1 > \sigma_2$ as shown in Figure 1(e). However, when $\sigma_1 < \sigma_2$, we obtain a W-type shape. Such behaviour is unphysical as the contribution of higher-order terms exceed the $\tilde{\phi}_1$, and it leads to a violation of the inequality (44). Similar behaviour is observed for the case of $\beta < 1$ [not shown]. On the contrary, we observe W-type shape for $\sigma_1 > \sigma_2$ and the peak amplitude increases for $\sigma_1 < \sigma_2$ for fast ion-acoustic solitons as shown in Figure 2(e). The role of $\beta > 1$ or $\beta < 1$ is not significant except for the increase in peak amplitude.

To highlight the role of the isothermality/nonisothermality, we show how the amplitude $\phi_{g0} = \phi_1(\chi = 0)$ changes with $\alpha = n_2{}^{(0)}/n_1{}^{(0)}$. In Figure 3(a), we have shown ϕ_{g0} as a function of α, both for isothermal ($b_1 = 0$) and finite but small values of the nonisothermal parameter. While the rarefactive solitons are obtained in the isothermally distributed electrons, situation changes drastically when $b_1 \neq 0$ as shown in Figure 3(a), where compressive solitons are obtained for slow IASs. It becomes more interesting when we consider the case of fast ion-acoustic solitary waves. Figure 3(b) exhibit the total potential profile ϕ_{g0} as a function of α as observed for $b_1 = 0$ (isothermal) and $b_1 \neq 0$ (nonisothermal). Obviously, both compressive and rarefactive solitons are obtained for isothermally distributed electrons. On the other hand, only compressive solitons are admissible for the nonisothermal case.

In conclusion, scaling laws for isothermal and nonisothermal electron distributions are different. A small contribution resulting from resonant particle interaction with a trapped particle distribution leads to nullify the effect producing rarefactive solitons in isothermally distributed electrons. This feature becomes more lucid from analyzing the contribution to ϕ_{g0} of the generalized KdV equation. It may be mentioned that the present investigation is relatant to study astrophysical and cosmic plasma environments.

7. Conclusion

In the present investigation, the authors have studied the effect of higher-order nonlinear effects in a multispecies plasma consisting of positive and negative ions with a nonisothermal distribution of the electrons. Two ion-acoustic-modes, viz. slow and fast ion-acoustic-modes, are observed corresponding to different phase velocities. We have further investigated solitons corresponding to these modes. The effect of the nonisothermal parameter β, mass ratio η, and ion temperatures have been thoroughly studied. Associated profiles of sech^4 types are significantly modified by the inclusion of higher-order dispersive and nonlinear effects. The deviation of the isothermal behaviour is studied by deriving the generalized mKdV equation by Tagare and Chakrabarti [82]. It is observed that while compressive as well as rarefactive solitons are admissible for isothermally distributed electrons, only compressive solitons result for the nonisothermal case. This investigation is relevant to study astrophysical and cosmic plasma environment.

[1] N. J. Zabusky and M. D. Kruskal, Phys. Rev. Lett. **15**, 240 (1965).
[2] H. Washimi and T. Taniuti, Phys. Rev. Lett. **17**, 996 (1966).
[3] M. K. Mishra and R. S. Chhabra, Phys. Plasmas **3**, 4446 (1996).
[4] M. K. Mishra, A. K. Arora, and R. S. Chhabra, Phys. Rev. E **66**, 046402 (2002).
[5] T. S. Gill, P. Bala, H. Kaur, N. S. Saini, S. Bansal, and J. Kaur, Eur. Phys. J. D **31**, 91 (2004).
[6] M. Y. Yu, P. K. Shukla, and S. Bujarbarua, Phys. Fluids **23**, 2146 (1980).
[7] V. E. Zakharov and E. A. Kuznetsov, Zh. Eksp. Teor. Fiz. **66**, 594 (1974).
[8] P. K. Shukla and M. Y. Yu, J. Math. Phys. **19**, 2506 (1978).
[9] H. Ikezi, R. J. Taylor, and T. R. Baker, Phys. Rev. Lett. **25**, 11 (1970); H. Ikezi, Phys. Fluids **16**, 1668 (1973).
[10] M. Q. Tran, Phys. Scripta **20**, 3197 (1979).
[11] Y. Nakamura, IEEE Trans. Plasma Sci. **PS-10**, 180 (1982).
[12] K. E. Lonngren, Plasma Phys. **25**, 943 (1983).
[13] K. E. Lonngren, Opt. Quantum Electron. **30**, 615 (1998).
[14] R. C. Davidson, Methods in Nonlinear Plasma Theory. Academic Press, New York 1972.
[15] G. C. Das and S. G. Tagare, Plasma Phys. **17**, 1025 (1975).
[16] Y. Nakamura, in: Nonlinear and Environmental Electromagnetics, (Ed. H. Kikuchi), Elsevier Science Publishers B. V. Amsterdam 1985, p. 139.
[17] G. O. Ludwig, J. L. Ferreira, and Y. Nakamura, Phys. Rev. Lett. **52**, 275 (1984).
[18] Y. Nakamura and I. Tsukabayashi, Phys. Rev. Lett. **52**, 2356 (1984).
[19] Y. Nakamura, T. Odagiri, and I. Tsukabayashi, Plasma Phys. Controlled Fusion **39**, 105 (1997).
[20] F. B. Rizzato, R. S. Schneider, and D. Dillenburg, Plasma Phys. Controlled Fusion **29**, 1127 (1987).
[21] F. Verheest, J. Plasma Phys. **39**, 71 (1988).
[22] C. Uberoi and G. C. Das, Plasma Phys. **16**, 669 (1974).
[23] G. C. Das, Plasma Phys. **19**, 363 (1977).
[24] G. C. Das, Plasma Phys. **21**, 257 (1979).
[25] G. C. Das, Kh. I. Singh, and B. Karmakar, Plasma Phys. **17**, 1025 (1989).
[26] Y. Nakamura, H. Bailung, and K. E. Lonngren, Phys. Plasmas **6**, 3467 (1999).
[27] B. Handique and H. Bailung, G. C. Das, and J. Chutia, Phys. Plasmas **6**, 1636 (1999).
[28] H. Bailung, J. Chutia, and Y. Nakamura, Chaos, Solitons, and Fractals **7**, 21 (1996).
[29] W. Swider, in: Ionospheric Modelling, (Ed. J. N. Korenkov), Birkhäuser, Basel 1988, p. 403.
[30] M. K. Mishra, R. S. Chhabra, and S. R. Sharma, Phys. Rev. E **51**, 4790 (1995).

[31] M. K. Mishra, R. S. Chhabra, and S. R. Sharma, J. Plasma Phys. **52**, 409 (1994).
[32] H. Schamel, J. Plasma Phys. **9**, 373 (1973).
[33] H. Schamel, Phys. Rep. **140**,161 (1986).
[34] K. Saeki, P. Michelsen, H. L. Pecseli, and J. J. Rasmussen, Phys. Rev. Lett. **42**, 501 (1979).
[35] P. Guio, S. Borve, L. K. Daldorff, J. P. Lynov, P. Michelsen, H. L. Pecseli, J. J. Rasmussen, K. Saeki, and J. Trulsen, Nonlinear Processes Geophys. **10**, 75 (2003).
[36] J. P. Lynov, P. Michelson, H. L. Pecseli, and J. Rasmussen, Phys. Lett. **80**, 23 (1980).
[37] T. S. Gill, H. Kaur, and N. S. Saini, Phys. Plasmas **10**, 3927 (2003).
[38] H. Schamel, Plasma Phys. **14**, 905 (1972).
[39] A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas **3**, 2610 (1996).
[40] A. A. Mamun, Phys. Plasmas **5**, 322 (1998).
[41] N. S. Saini and I. Kourakis, Phys. Plasmas **15**, 123701 (2008).
[42] K. Aoutou, M. Tribeche, and T. H. Zerguini, Phys. Plasmas **15**, 013702 (2008).
[43] P. O. Dovner, A. I. Eriksson, R. Boström, and B. Holback, Geophys. Res. Lett. **21**, 1827 (1994).
[44] R. A. Cairns, A. A. Mamun, R. Bostrom, R. O. Dendy, C. M. C. Nairn, and P. K. Shukla, Geophys. Res. Lett. **22**, 2709 (1995).
[45] R. A. Cairns, A. A. Mamun, R. Bingham, and P. K. Shukla, Phys. Scripta **T63**, 80 (1996).
[46] A. A. Mamun, Eur. Phys. J. D **11**, 143 (2000).
[47] A. Bandyopadhyay and K. P. Das, Phys. Scripta **61**, 92 (2000).
[48] A. Bandyopadhyay and K. P. Das, Phys. Scripta **63**, 145 (2001).
[49] S. N. Paul, S. Chattopadhyaya, S. K. Bhattacharya, and B. Bera, Pramana J. Phys. **60**, 1217 (2003).
[50] B. Sahu and R. Roychoudhury, Phys. Plasmas **12**, 052106 (2005).
[51] T. S. Gill, H. Kaur, and N. S. Saini, Chaos, Solitons, and Fractals **30**, 1020 (2005).
[52] B. Sahu and R. Roychoudhury, Phys. Plasmas **13**, 072302 (2006).
[53] M. Lin and W. Duan, Chaos, Solitons, and Fractals **33**, 1189 (2007).
[54] T. S. Gill, H. Kaur, S. Bansal, N. S. Saini, and P. Bala, Eur. Phys. J. D. **41**, 151 (2007).
[55] F. Verheest and S. R. Pillay, Nonlinear Processes Geophys. **15**, 551 (2008).
[56] F. Verheest and S. R. Pillay, Phys. Plasmas **15**, 013703 (2008).
[57] R. Sabry, W. M. Moslem, and P. K. Shukla, Phys. Plasmas **16**, 032302 (2009).
[58] H. R. Pakzad, Phys. Lett. A **373**, 847 (2009).
[59] F. Verheest, Phys. Plasmas **16**, 013704 (2009).
[60] A. Esfandyari-Kalejahi, I. Kourakis, and P. K. Shukla, Phys. Plasmas **15**, 022303 (2008).
[61] S. Watanabe and B. Jiang, Phys. Fluids B **5**, 409 (1993).
[62] Y. Kodma and T. Taniuti, J. Phys. Soc. Jpn. **45**, 298 (1978); **45**, 1765 (1978).
[63] Y. Kodma, J. Phys. Soc. Jpn. **45**, 311 (1978).
[64] Y. H. Ichikawa, T. Mitsuhashi, and K. Konno, J. Phys. Soc. Jpn. **41**, 1382 (1976).
[65] N. Sugimoto and T. Kakutani, J. Phys. Soc. Jpn. **43**, 1469 (1977).
[66] C. S. Lai, Can. J. Phys. **57**, 490 (1979).
[67] H. Demiay, J. Phys. Soc. Jpn. **68**, 1833 (1999).
[68] H. Demiay, J. Phys. Soc. Jpn. **71**, 1921 (2002).
[69] W. M. Moslem, J. Plasma Phys. **61**, 177 (1999).
[70] W. M. Moselm, J. Plasma Phys. **63**, 139 (2000).
[71] S. K. El-Labany, W. M. Moslem, E. K. El-Shewy, and A. E. Mowafy, Chaos, Solitons, and Fractals **23**, 581 (2005).
[72] W. F. El-Taibany and W. M. Moslem, Phys. Plasmas **12**, 032307 (2005).
[73] K. K. Mondal, FIZIKA A (Zagreb) **16**, 11-24 (2007).
[74] T. S. Gill, P. Bala, and H. Kaur, Phys. Plasmas **15**, 122309 (2008).
[75] H. Schamel, Plasma Phys. **13**, 491 (1971).
[76] J. P. Holloway and J. J. Dorning, Phys. Rev. A **44**, 3856 (1991).
[77] H. Schamel, Phys. Plasma **7**, 4831 (2000).
[78] H. Schamel, Phys. Rev. Lett. **79**, 2811 (1997).
[79] S. K. El-Labany, Astrophys. Space Sci. **191**, 185 (1992).
[80] S. K. El-Labany and S. M. Shaaban, J. Plasma Phys. **53**, 245 (1995).
[81] A. A. Mamun and P. K. Shukla, J. Geophys. Res. **107**, SIA 15 (2002).
[82] S. G. Tagare and A. Chakrabarti, Phys. Fluids **17**, 1331 (1974).