The Solution of the Regularized Long Wave Equation Using the Fourier Leap-Frog Method

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An efficient numerical method is developed for solving nonlinear wave equations by studying the propagation and stability properties of solitary waves (solitons) of the regularized long wave (RLW) equation in one space dimension using a combination of leap frog for time dependence and a pseudospectral (Fourier transform) treatment of the space dependence. Our schemes follow very accurately these solutions, which are given by simple closed formulas. Studying the interaction of two such solitons and three solitary waves interaction for the RLW equation. Our implementation employs the fast Fourier transform (FFT) algorithm.

Key words: Fourier Spectral Method; Fast Fourier Transform; Leap-Frog Method; RLW Equation; Nonlinear Waves.

1. Introduction

Discretization using finite differences in time and spectral methods in space has proved to be very useful in solving numerically nonlinear partial differential equations (PDE) describing wave propagation. The Korteweg-de Vries (KdV) equation and the generalized KdV equation use such combined schemes and analyse efficiently unidirectional solitary wave propagation in one dimension [1]. We apply a combination of spectral methods and leap frog to another well-known nonlinear PDE describing water waves, called the regularized long wave (RLW) equation. We will consider the one-dimensional RLW equation

$$u_t + u_x + \varepsilon u u_x - \mu u_{xxt} = 0. \qquad (1)$$

The regularized long wave (RLW) equation belongs to a class of nonlinear evolution equations which provide good models for predicting a variety of physical phenomena. The equation was originally introduced to describe the behaviour of undular bore development by Peregrine [2]. Since then it has been used to model a variety of phenomena such as nonlinear transverse waves in shallow water, ion-acoustic and magnetohydrodynamic waves in plasma, and phonon packets in nonlinear crystals. An analytical solution for the RLW equation was found under the restricted initial and boundary conditions. Since the analytical solution of the RLW equation is not very useful, the availability of accurate and efficient numerical methods is essential. The numerical solution of the RLW equation has been the subject of many papers. Peregrine [2] suggested the first numerical method based on the finite difference method. Various numerical techniques particularly including finite difference and finite element have been used for the solution of the RLW equation [3 – 9]. We investigate the numerical solution of the RLW equation using the Fourier leap-frog method. The method is validated by studying the motion of a single solitary wave, the development of the interaction of two positive solitary waves, and the development of three positive solitary waves interaction for the RLW equation (1).

2. Analysis

The propagation of surface waves in a shallow channel of constant depth is described by the RLW equation in the following way: u represents the dimensionless surface elevation, x the distance, and t the time. For the numerical treatment, the spatial variable x of the problem is restricted over an interval $a \le x \le b$. In this study, we consider the RLW equation (1) with the boundary conditions

$$u(a,t) = 0, \quad t > 0, \quad u(b,t) = 0, \quad t > 0. \qquad (2)$$

A numerical method is developed for the periodic initial value problem in which u is a prescribed function of x at $t = 0$ and the solution is periodic in x outside a basic interval $a \le x \le b$. For most of the problems considered, the interval may be chosen large enough so that the boundaries do not affect the wave interactions being studied.

The initial condition is

$$u(x,0) = f(x), \quad a \le x \le b, \tag{3}$$

where ε and μ are positive parameters and the subscripts x and t denote differentiation. Then (1) becomes

$$v_t = -u_x - \varepsilon u u_x, \tag{4}$$

where

$$v = u - \mu u_{xx}. \tag{5}$$

For ease of presentation the spatial period $[a,b]$ is normalized to $[0,2\pi]$, with the change of variable

$$x \to \frac{2\pi}{b-a}(x-a).$$

Let $L = b - a$.

Thus, (4) and (5) become

$$v = u - \left(\frac{2\pi}{L}\right)^2 \mu u_{xx}, \tag{6}$$

$$v_t = -\left(\frac{2\pi}{L}\right) u_x - \varepsilon \left(\frac{2\pi}{L}\right) u u_x. \tag{7}$$

$u(x,t)$ is now transformed into Fourier space with respect to x, and the derivatives (or other operators) with respect to x. This operation can be done with the fast Fourier transform (FFT). Applying the inverse Fourier transform

$$\frac{\partial^n u}{\partial x^n} = F^{-1}\{(\mathrm{i}k)^n F(u)\}, \quad n = 1,2,\cdots,$$

for $n = 1$ and $n = 2$, we obtain $u_x = F^{-1}\{\mathrm{i}kF(u)\}$ and $u_{xx} = F^{-1}\{-k^2 F(u)\}$.

Then (6) and (7) become

$$v = u - \left(\frac{2\pi}{L}\right)^2 \mu F^{-1}\{-k^2 F(u)\}, \tag{8}$$

$$\begin{aligned} v_t = &-\left(\frac{2\pi}{L}\right) F^{-1}\{\mathrm{i}kF(u)\} \\ &- \varepsilon \left(\frac{2\pi}{L}\right) u F^{-1}\{\mathrm{i}kF(u)\}. \end{aligned} \tag{9}$$

In practice, we need to discretize (6) and (7). For any integer $N > 0$, we consider

$$x_j = j\Delta x = \frac{2\pi j}{N}, \quad j = 0,1,\ldots,N-1.$$

Let $u(x,t)$ be the solution of (7). Then, we transform it into the discrete Fourier space as

$$\begin{aligned} &\hat{u}(k,t) = F(u) = \frac{1}{N} \sum_{J=0}^{N-1} u(x_j,t) \mathrm{e}^{-\mathrm{i}kx_j}, \\ &-\frac{N}{2} \le k \le \frac{N}{2} - 1. \end{aligned} \tag{10}$$

From this, using the inversion formula, we get

$$\begin{aligned} &u(x_j,t) = F^{-1}(\hat{u}) = \sum_{k=-N/2}^{N/2-1} \hat{u}(k,t) \mathrm{e}^{-\mathrm{i}kx_j}, \\ &0 \le j \le N-1. \end{aligned} \tag{11}$$

Replacing F and F^{-1} by their discrete counterparts, and discretizing (8) and (9) gives

$$v(x_j,t) = u(x_j,t) - \left(\frac{2\pi}{L}\right)^2 \mu F^{-1}\{-k^2 F(u)\}, \tag{12}$$

$$\begin{aligned} \frac{\partial v(x_j,t)}{\partial t} = &-\left(\frac{2\pi}{L}\right) F^{-1}\{\mathrm{i}kF(u)\} \\ &- \varepsilon \left(\frac{2\pi}{L}\right) u(x_j,t) F^{-1}\{\mathrm{i}kF(u)\}. \end{aligned} \tag{13}$$

Letting be $\mathbf{U} = [u(x_0,t), u(x,t), \cdots, u(x_{N-1},t)]^T$.

Then (13) can be written in the vector form

$$\mathbf{V}_t = \mathbf{F}(\mathbf{U}), \tag{14}$$

where $\mathbf{F}$ defines the right hand side of (13).

3. Fourier Leap-Frog Method for RLW Equation

A time integration known as a leap-frog method (a two step scheme) is given by

$$\mathbf{V}_t = \frac{\mathbf{V}(x,t+\Delta t) - \mathbf{V}(x,t-\Delta t)}{2\Delta t}.$$

Using the leap-frog scheme to advance in time, we obtain

$$\mathbf{V}(t+\Delta t) = \mathbf{V}(t-\Delta t) + 2\Delta t F(\mathbf{U}(t)).$$

This is called the Fourier leap-frog (FLF) scheme for the RLW equation (14). FLF needs two levels of initial data, we begin with $u(x,0)$ to get $v(x,0)$ from (12):

$$v^n = F^{-1}\left[(F(u^n))\left(1+\left(\frac{2\pi}{L}\right)^2\mu k^2\right)\right], \quad (15)$$

$$v(x,0) = F^{-1}\left[(F(u(x,0)))\left(1+\left(\frac{2\pi}{L}\right)^2\mu k^2\right)\right]. \quad (16)$$

Then we evaluate the second level of the initial solution $v(x,\Delta t)$ by using a higher-order one-step method, for example, a fourth-order Runge-Kutta method (RK4):

$$\begin{aligned}
K_1 &= F(U(x,0),0),\\
K_2 &= F\left(U(x,0)+\frac{1}{2}\Delta t K_1,\frac{1}{2}\Delta t\right),\\
K_3 &= F\left(U(x,0)+\frac{1}{2}\Delta t K_2,\frac{1}{2}\Delta t\right), \quad (17)\\
K_4 &= F\left(U(x,0)+\Delta t K_3,\Delta t\right),\\
V(x,\Delta t) &= V(x,0)+\frac{\Delta t}{6}[K_1+2K_2+2K_3+K_4].
\end{aligned}$$

We substitute in

$$u^{n+1} = F^{-1}\left(\frac{F(v^{n+1})}{1+\left(\frac{2\pi}{L}\right)^2\mu k^2}\right) \quad (18)$$

to get $u(x,\Delta t)$. Thus, the time discretization for (13) is given by

$$\begin{aligned}
v(x,t+\Delta t) &= v(x,t-\Delta t)\\
&\quad -2\Delta t\left[\left(\frac{2\pi}{L}\right)F^{-1}\{\mathrm{i}kF(u)\}\right. \quad (19)\\
&\quad \left.+\,\varepsilon\left(\frac{2\pi}{L}\right)u(x_j,t)F^{-1}\{\mathrm{i}kF(u)\}\right].
\end{aligned}$$

We substitute $v(x,0)$ and $u(x,\Delta t)$ in (19) to evaluate $v(x,2\Delta t)$ and then insert in (18) to evaluate $u(x,2\Delta t)$, so we have $v(x,\Delta t)$ and $u(x,2\Delta t)$, substitute in (19) to evaluate $v(x,3\Delta t)$, $u(x,3\Delta t)$, from (18) and so on, until we evaluate $u(x,t)$ at time t.

4. Numerical Examples and Results

In order to show how good the numerical solutions are in comparison with the exact ones, we will use the L_2 and L_∞ error norms defined by [7]

$$\begin{aligned}
L_2 &= \|u^{\text{exact}}-u^{\text{num}}|_2 = \left[\Delta x\sum_{i=1}^{N}|u_i^{\text{exact}}-u_i^{\text{num}}|^2\right]^{\frac{1}{2}}, \quad (20)\\
L_\infty &= \|u^{\text{exact}}-u^{\text{num}}|_\infty = \max_i |u_i^{\text{exact}}-u_i^{\text{num}}|.
\end{aligned}$$

The RLW equation (1) satisfies only three conservation laws given as

$$\begin{aligned}
I_1 &= \int_{-\infty}^{\infty} u\,\mathrm{d}x = \Delta x\sum_{j=1}^{n} u(x_j,t),\\
I_2 &= \int_{-\infty}^{\infty}[u^2+\mu(u_x)^2]\mathrm{d}x\\
&= \Delta x\sum_{j=1}^{n}[(u(x_j,t),)^2+\mu(u_x(x_j,t))^2], \quad (21)\\
I_3 &= \int_{-\infty}^{\infty}[u^3+3u^2]\mathrm{d}x\\
&= \Delta x\sum_{j=1}^{n}((u(x_j,t),)^3+3(u_x(x_j,t))^2),
\end{aligned}$$

which correspond to mass, momentum, and energy, respectively [10]. In the simulations the invariants I_1, I_2, and I_3 are monitored to check the conservation of the numerical scheme. For the computation of u_x in (21), we used the Fourier transform. To implement the performance of the method, three test problems will be considered: the motion of a single solitary wave, the development of two positive solitary waves interaction, and the development of three positive solitary wave interaction. For the purpose of comparing with the earlier work, all computations are done for the parameters $\mu = 1$, $\varepsilon = 1$.

4.1. The Motion of a Single Solitary Wave

We first consider (1) with the boundary $u \to 0$ as $x \to \pm\infty$. We take the RLW equation of the form (1) with the periodic boundary condition

$$u(a,t) = u(b,t) = 0, \quad a \le x \le b. \quad (22)$$

The theoretical solution [7] is

$$U(x,t) = 3c\,\mathrm{sech}^2(p(x-x_0-(1+\varepsilon c)t)). \quad (23)$$

This solution corresponds to the motion of a single solitary wave with amplitude $3c$, the wave velocity $v = 1+\varepsilon c$, and width p, initially centered at x_0, where $p = \frac{1}{2}\left(\frac{\varepsilon c}{\mu(1+\varepsilon c)}\right)^{1/2}$, ε and μ are positive parameters. The initial condition is

$$u(x,0) = 3c\,\mathrm{sech}^2(p(x-x_0)). \quad (24)$$

This solution will also be used over an interval $-L/2 \le x \le L/2$, $\Delta x = 1$, and several tests have been made for the wave solution verifying for various values of $N =$

128 to 1024 and time step $\Delta t = 0.0001$ to 0.02. The invariants from the theoretical values are

$$I_1 = \frac{6c}{p}, \quad I_2 = \frac{12c^2}{p} + \frac{48pc^2\mu}{5}, \\ I_3 = \frac{36c^2}{p}\left(1+\frac{4c}{5}\right). \tag{25}$$

Example 1

We compute the numerical solutions $u(x,20)$ using the Fourier leap-frog scheme at $c = 0.1$ (magnitude is 0.3 at $t = 0$), $N = 1024$, and $\Delta t = 0.001$. We plotted the exact solution and the numerical solution is given in Figure 1 at time $t = 20$. The magnitude of

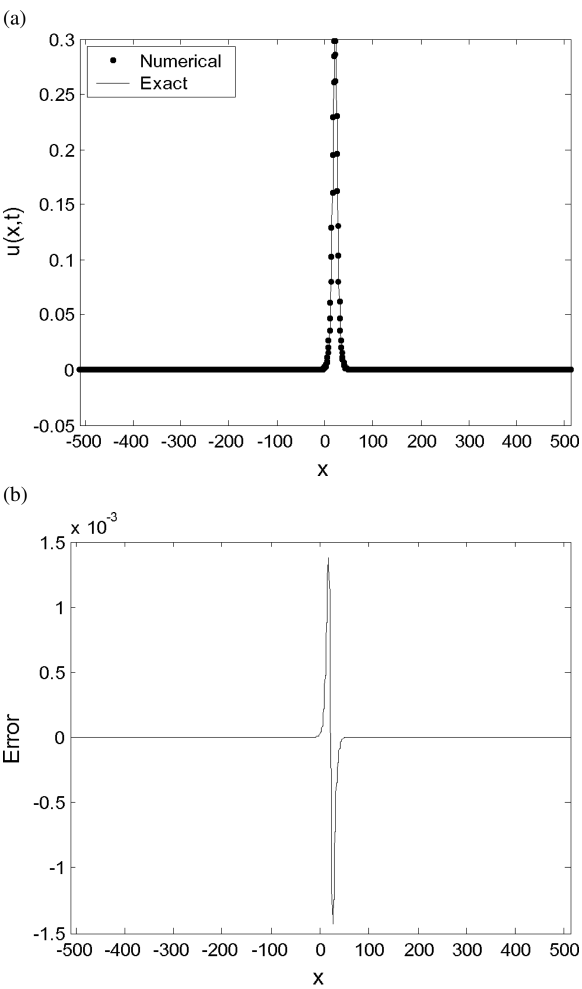

Fig. 1. (a) Fourier spectral solution and (b) error distributions in FLF scheme for the RLW equation with $c = 0.1$ and $N = 1024$ at $t = 20$.

Table 1. Invariants and error norms for the single soliton using FLF scheme with $c = 0.1$, $N = 1024$, and $\Delta t = 0.001$.

t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0	3.976063	0.809699	2.576489	0.0000	0.0000
5	3.976063	0.809699	2.576489	0.5752	0.1622
10	3.976063	0.809699	2.576489	1.2829	0.3651
15	3.976063	0.809699	2.576489	1.8605	0.5341
20	3.976063	0.809699	2.576489	2.3083	0.6656
25	3.976063	0.809699	2.576489	2.8898	0.8363

Table 2. Invariants and error norms for the single soliton using finite difference cubic method [3,7] with $c = 0.1$, $\Delta x = 0.1$, $\Delta t = 0.1$, and over the region $-40 \le x \le 60$.

t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0	3.979 92	0.810 459	2.579 01	0.00	0.00
4	4.420 17	0.899 873	2.863 39	39.82	13.74
8	4.418 22	0.899 236	2.861 06	79.46	27.66
12	4.416 23	0.898 601	2.858 63	118.8	41.35
16	4.414 23	0.897 967	2.856 13	157.7	54.60
20	4.412 19	0.897 342	2.853 61	196.1	67.35

the solitary wave measured at $t = 20$ is 0.2982877. It is clear from Figure 1a that the proposed method performs the motion of the propagation of a solitary wave satisfactorily, which moved to the right with preserved amplitude and shape. The error distribution at time $t = 20$ is drawn in Figure 1b, from which it can be seen that the maximum errors happened just around the peak position of the solitary wave. Table 1 displays the values of the invariants and the error norms obtained with $\Delta t = 0.001$. As it is seen from the table, the numerical values of invariants obtained from (21) are in agreement with their analytical values obtained from (25). The quantities in the invariants remain almost constant during the computer run. At times $t = 0$ and $t = 25$, I_1, I_2, and I_3 are exact up to the last recorded digit. The error norms at each time obtained by the present method are smaller than those given in [4,11]. The error norms at $t = 20$ and $N = 1024$ are $L_2 = 2.3083 \times 10^{-3}$ and $L_\infty = 0.6656 \times 10^{-3}$ whereas they are $L_2 = 191.1 \times 10^{-3}$ and $L_\infty = 67.35 \times 10^{-3}$ for the cubic finite difference method (see Table 2). Table 3 displays the values of the error norms obtained at different values of N with $\Delta t = 0.001$. As it is seen from the tables, the error norms decrease when N increases.

Example 2

Similar experiments were undertaken when a smaller amplitude of 0.09 of the solitary wave is used. Our results are documented at time $t = 20$, $c = 0.03$, $N = 1024$, and $\Delta t = 0.001$ as shown in Figure 2.

N	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$	amplitude
128	3.948856	0.804358	2.558859	18.5031	5.3549	0.2996516
256	3.964403	0.807410	2.568933	9.2411	2.6692	0.2997597
512	3.972176	0.808936	2.573970	4.6179	1.3324	0.2999208
1024	3.976063	0.809699	2.576489	2.3083	0.6656	0.2999480

Table 3. Invariants, error norms, and amplitudes for the single soliton using FLF scheme with $c = 0.1$, $\Delta t = 0.001$, and different values of N at $t = 20$.

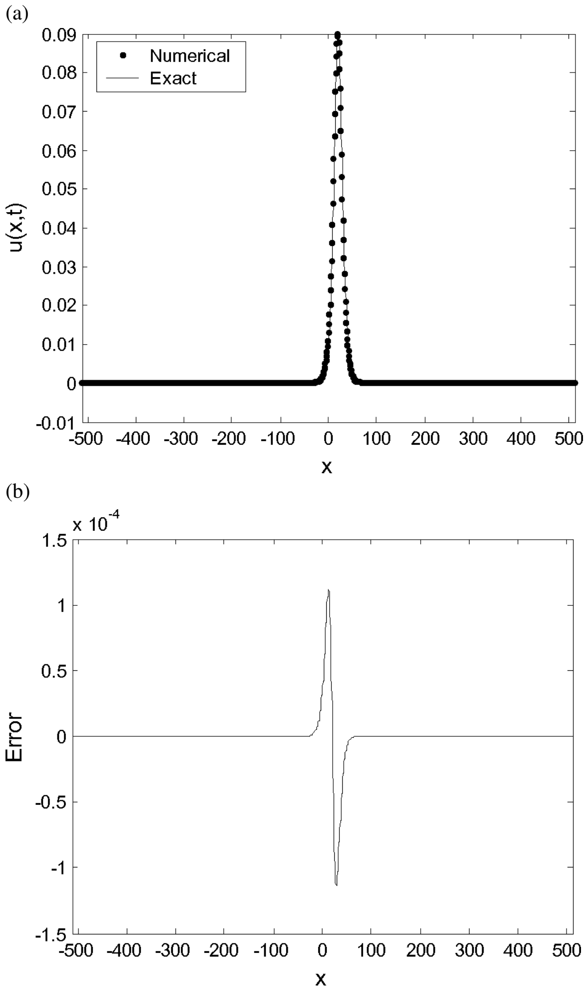

Fig. 2. (a) Fourier spectral solution and (b) error distributions in FLF scheme for the RLW equation with $c = 0.03$ and $N = 1024$ at $t = 20$.

Tables 4 and 5 display the numerical values of invariants and the error norms obtained at different values of N with $\Delta t = 0.001$. As it is seen from the tables, the error norms decrease when N increases. The magnitude of the solitary wave was 0.0899925 at $t = 20$.

4.2. The Interaction of Two Positive Solitary Waves

Secondly, the interaction of two positive solitary waves is studied by using the initial condition given by the linear sum of two separate solitary waves of various amplitudes,

$$u(x,0) = u_1 + u_2,$$
$$u_j = 3A_j \operatorname{sech}^2(p_j(x - x_j)),$$
$$A_j = \frac{4p_j^2}{1 - 4p_j^2}, \quad j = 1,2. \tag{26}$$

Table 4. Invariants and error norms for the single soliton using FLF scheme with $c = 0.03$, $N = 1024$, and $\Delta t = 0.001$.

t	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
0	2.107348	0.127179	0.388426	0.0000	0.0000
5	2.107348	0.127179	0.388426	0.1313	0.0282
10	2.107348	0.127179	0.388426	0.2905	0.0625
15	2.107348	0.127179	0.388426	0.4217	0.0909
20	2.107348	0.127179	0.388426	0.5251	0.1132
25	2.107348	0.127179	0.388426	0.6564	1.1416

The boundary conditions $u(0,t) = u(128,t) = 0$ were used with parameters $p_1 = 0.4$, $x_1 = 15$, $p_2 = 0.3$, and $x_2 = 35$. These parameters provide solitary waves of magnitudes about 5.33338 and 1.68598 at $t = 0$ and their peaks are positioned at $x = 15$ and $x = 35.1$. The calculation is carried out with a time step of $\Delta t = 0.001$ and $N = 512$ over the region $0 \le x \le 128$. The initial function was placed with the larger wave to the left of the smaller one as seen in the Figure 3a. Both waves move to the right with velocities dependent upon their magnitudes. The shapes of the two solitary waves are graphed during the interaction at time $t = 15$ and after the interaction at time $t = 30$, which is seen to have separated the larger wave from the smaller one as shown in Figure 3. According to Figure 3, the larger wave catches up the smaller wave at about $t = 10$, the overlapping process continues until $t = 20$, then two solitary waves emerge from the interaction and resume their former shapes and amplitudes. At $t = 30$, the magnitude of the smaller wave is 1.68388 on reaching position $x = 78$, and of the larger wave 5.33144 having the position $x = 101$, so that the difference in amplitudes is 0.00362 for the smaller wave and 0.00194 for the larger wave. Table 6 displays the values of the invariants obtained by the present method. It is observed that the obtained values of the invariants remain almost constant during the computer run. The change in I_2 is

N	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$	amplitude
128	2.092859	0.126319	0.385768	4.2274	0.9111	0.0899858
256	2.101168	0.126810	0.387287	2.1029	0.4540	0.0899897
512	2.105288	0.127056	0.388047	1.0506	0.2266	0.0899916
1024	2.107348	0.127179	0.388426	0.5251	0.1132	0.0899925

Table 5. Invariants, error norms, and amplitudes for the single soliton using FLF scheme with $c = 0.03$, $\Delta t = 0.001$, and different values of N at $t = 20$.

Table 6. Invariants of interaction of two positive solitary waves of the RLW equation using FLF scheme with $\Delta t = 0.001$ and $N = 512$ over the region $0 \le x \le 128$.

t	I_1	I_2	I_3
0	37.916519	120.523234	744.081209
2	37.916519	120.523193	744.080870
4	37.916519	120.523193	744.080869
5	37.916519	120.523193	744.080872
6	37.916519	120.523191	744.080856
8	37.916519	120.523193	744.080870
10	37.916519	120.523181	744.080804
12	37.916519	120.523193	744.080870
14	37.916519	120.523193	744.080870
15	37.916519	120.523153	744.080655
16	37.916519	120.523193	744.080870
18	37.916519	120.523193	744.080870
20	37.916519	120.523193	744.080872
22	37.916519	120.523193	744.080870
24	37.916519	120.523193	744.080868
25	37.916519	120.523193	744.080868
26	37.916519	120.523193	744.080869
28	37.916519	120.523193	744.080869
30	37.916519	120.523193	744.080869

0.000081, in I_3 0.000554, and I_1 is exact up to the last recorded digit.

4.3. The Interaction of Three Positive Solitary Waves

We consider (1) with the boundary condition

$$u(0,t) = u(320,t) = 0$$

and the initial conditions given by a linear sum of three well separated solitary waves of various amplitudes:

$$\begin{aligned} u(x,0) &= u_1(x,0) + u_2(x,0) + u_3(x,0), \\ u(x,0) &= 3c_1 \operatorname{sech}^2(p_1(x-x_1)) \\ &\quad + 3c_2 \operatorname{sech}^2(p_2(x-x_2)) \\ &\quad + 3c_3 \sec h^2(p_3(x-x_3)), \end{aligned} \tag{27}$$

where $c_1, c_2, c_3, x_1, x_2, x_3$, are arbitrary constants. With the parameters

$$p_1 = \frac{1}{2}\left(\frac{\varepsilon c_1}{\mu(1+\varepsilon c_1)}\right)^{\frac{1}{2}}, \quad p_2 = \frac{1}{2}\left(\frac{\varepsilon c_2}{\mu(1+\varepsilon c_2)}\right)^{\frac{1}{2}},$$
$$p_3 = \frac{1}{2}\left(\frac{\varepsilon c_3}{\mu(1+\varepsilon c_3)}\right)^{\frac{1}{2}},$$

Table 7. Invariants of interaction of three positive solitary waves of the RLW equation using FLF scheme with $\Delta t = 0.001$ and $N = 512$ over the region $0 \le x \le 320$.

t	I_1	I_2	I_3
0	56.75675	181.67263	1144.01884
10	56.75675	181.67256	1144.01829
20	56.75675	181.67255	1144.01820
30	56.75675	181.67254	1144.01813
40	56.75675	181.67255	1144.01819
50	56.75675	181.67255	1144.01821
60	56.75675	181.67255	1144.01822
70	56.75675	181.67256	1144.01823
80	56.75675	181.67256	1144.01823
90	56.75675	181.67255	1144.01816

$\mu = 1, \varepsilon = 1, c_1 = 2, c_2 = 1, c_3 = 0.5, x_1 = 30, x_2 = 60, x_3 = 90$, $0 \le x \le 320$, $N = 512$, and $\Delta t = 0.001$, we use the FLF scheme to march the solution in time and the Fourier spectral method to take care of the spatial domain. Figure 4 shows the three solitary waves interaction at different times. Table 7 displays the values of the invariants obtained by the present method. It is observed that the obtained values of the invariants remain almost constant during the computer run. The change in I_2 is 0.00009, in I_3 0.00071, and I_1 is exact up to the last recorded digit.

5. Conclusions

We applied the leap-frog scheme combined with the Fourier spectral collocation, called the Fourier leap-frog method, to find the numerical solution of the RLW equation. The method is tested on the problems of the motion of a single solitary wave, the development of interaction of two positive solitary waves, and the development of three positive solitary waves interaction for the RLW equation. In the experiments of soliton simulation, the usual features of keeping shape and conserved quantities remained almost the same, and high accuracy was achieved with the L_2 and L_∞ error norms. It is apparently seen that the Fourier spectral collocation method is a powerful and efficient technique in finding numerical solutions for wide classes of nonlinear partial differential equations.

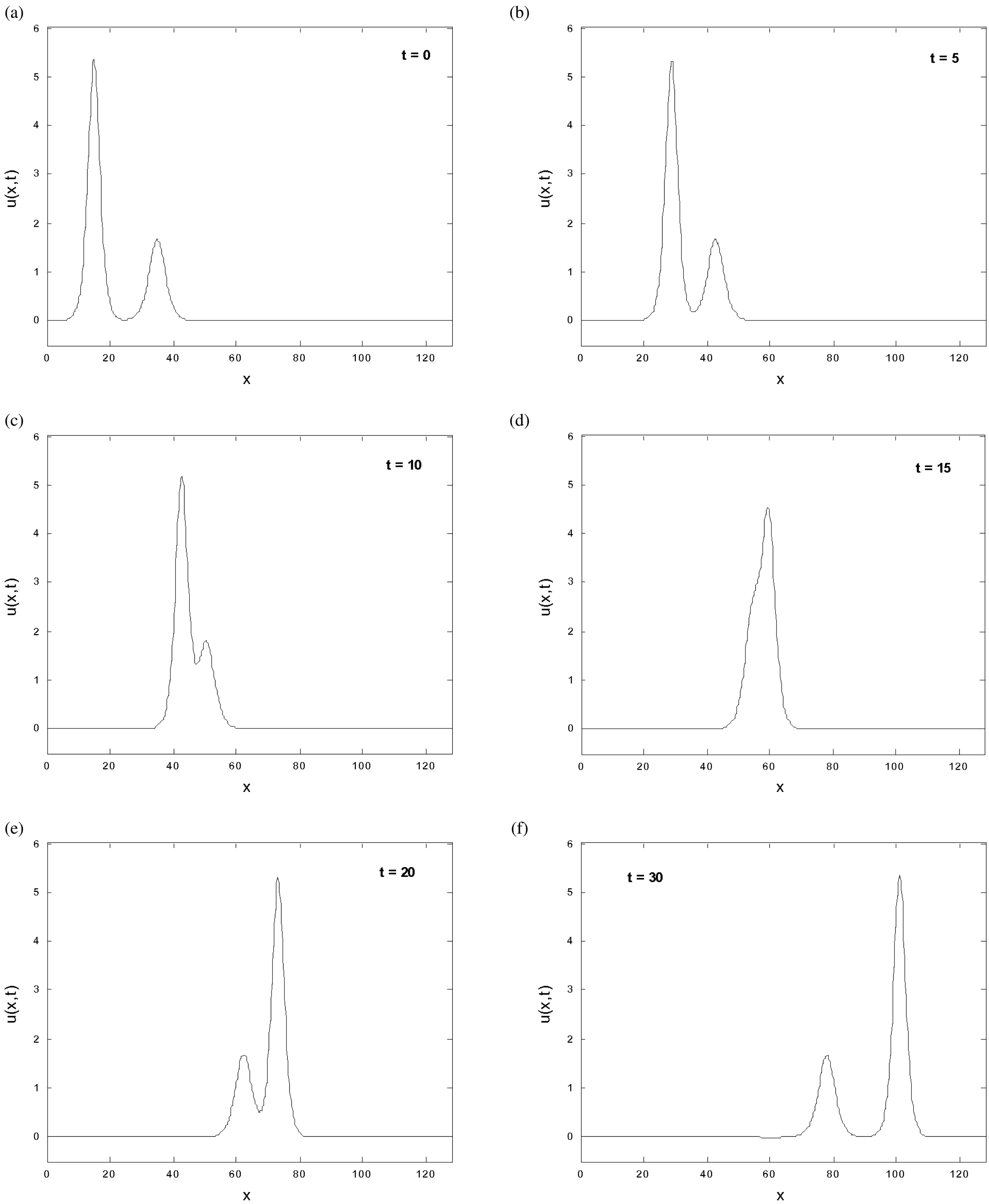

Fig. 3. Fourier spectral solution of interaction of two positive solitary waves of the RLW equation using FLF scheme with $N = 512$.

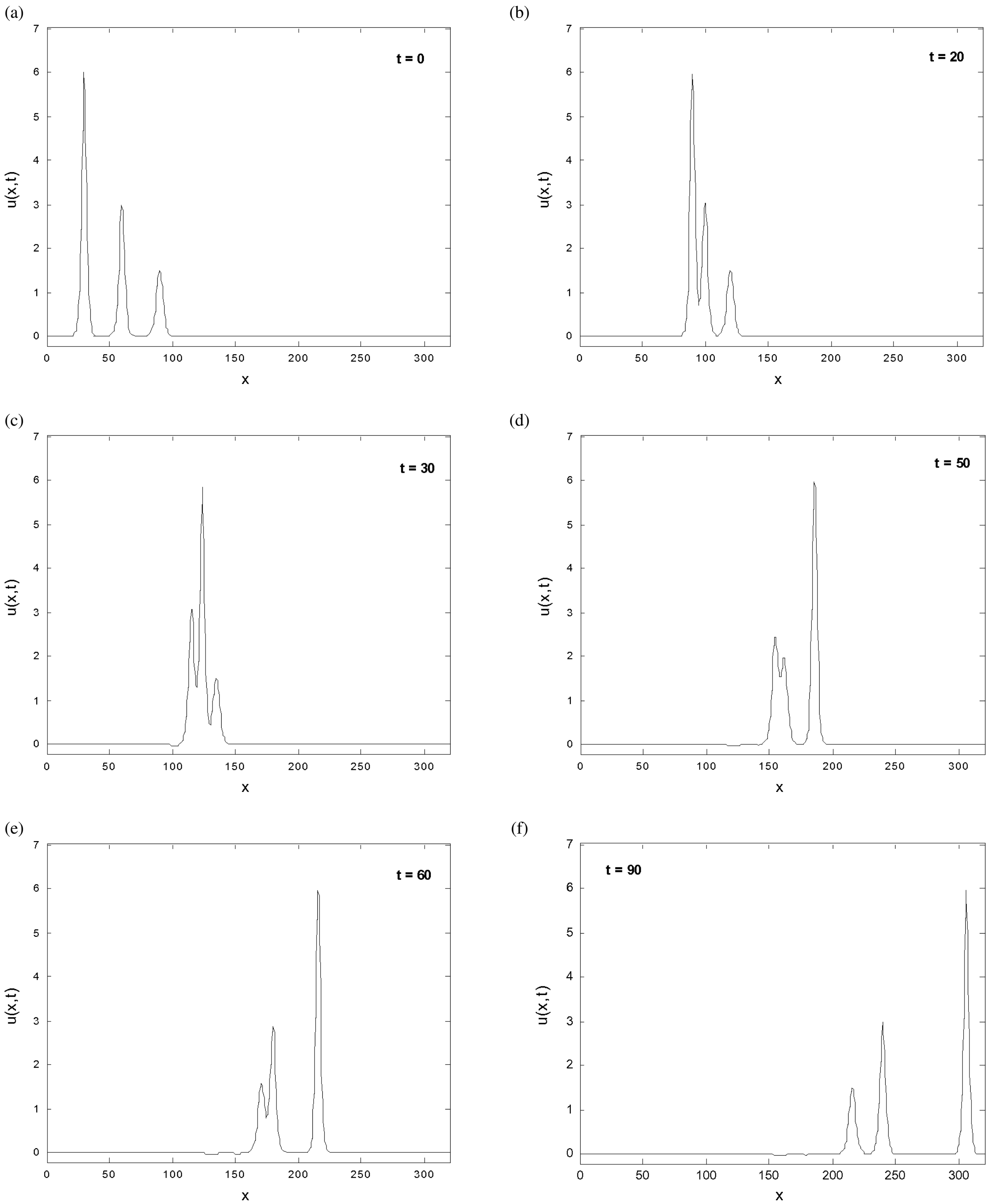

Fig. 4. Fourier spectral solution of interaction of three positive solitary waves of the RLW equation using FLF scheme with $\Delta t = 0.001$ and $N = 512$.

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