Radiation and Mass Transfer Effects on the Magnetohydrodynamic Unsteady Flow Induced by a Stretching Sheet

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This investigation deals with the influence of radiation on magnetohydrodynamic (MHD) and mass transfer flow over a porous stretching sheet. Attention has been particularly focused to the unsteadiness. The arising problems of velocity, temperature, and concentration fields are solved by a powerful analytic approach, namely, the homotopy analysis method (HAM). Velocity, temperature, and concentration fields are sketched for various embedded parameters and interpreted. Computations of skin friction coefficients, local Nusselt number, and mass transfer are developed and examined.

\textit{Key words:} Magnetohydrodynamic; Radiation; Concentration Field; Series Solutions.

1. Introduction

The classical problem of boundary layer flow bounded by a stretching surface has been studied extensively for viscous and non-Newtonian fluids. Good lists of relevant references on the topic can be seen in the recent studies \cite{1-10} and several references therein. Examples of stretching flows are found in wire drawing, aerodynamic extrusion of plastic sheets, paper production, crystal growing, etc. Literature survey shows that much attention has been given to the stretching flows in steady situation. Little attention is given to the unsteady flows over a stretching surface \cite{11-15}. Such flows are rarely discussed when interaction of magnetohydrodynamics and radiation is taken into account.

The main purpose of the present paper is to extend the analysis of Ishak et al. \cite{15} in four directions. Firstly, to discuss the MHD effects. Secondly, to describe the influence of radiation. Thirdly, to analyze the interaction of MHD and radiation with mass transfer in chemical reacting fluid. Fourthly, to construct the series solutions by employing the homotopy analysis method \cite{16-30}. The paper is organized as follows: The next section provides the problem of the development. Homotopy analysis solutions are derived in Section 3. Section 4 includes the convergence of the series solution. Sections 5 and 6, respectively, consist of discussion and main points.

2. Mathematical Formulation

Here we examine the unsteady and MHD flow of an incompressible viscous fluid bounded by a porous stretching surface. The fluid is electrically conducting under the influence of a time dependent magnetic field $B(t)$ applied in a direction normal to the stretching surface. The induced magnetic field is negligible under the assumption of a small magnetic Reynolds number. In addition, heat and mass transfer phenomena are considered. We choose the $x$-axis parallel to the porous surface and the $y$-axis normal to it. The boundary layer flow is governed by the following equations:

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(t)}{\rho} u, \\
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_t}{\partial y}, \\
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D \frac{\partial^2 C}{\partial y^2} - R(t) C,
\end{align}

where $u$ and $v$ are the velocity components in the $x$ and $y$-directions, respectively, $\rho$ the fluid density, $\nu$ the kinematic viscosity, $\sigma$ the electrical conductivity, $T$ the temperature, $c_p$ the specific heat, $k$ the thermal conductivity of the fluid, $q_t$ the radiative heat flux, $D$ is
the mass diffusion, $C$ the concentration field, and $R(t)$ represents the reaction rate.

Employing the Rosseland approximation for radiation [31] one has

$$q_t = -\frac{4\sigma^*}{3k} \frac{\partial T^4}{\partial y}, \quad (5)$$

in which $\sigma^*$ is the Stefan-Boltzmann constant and $k^*$ the mean absorption coefficient. We express the term $T^4$ as the linear function of temperature into a Taylor series about $T_w$ by neglecting higher terms, and write

$$T^4 \approx 4T_w^3T - 3T_w^4. \quad (6)$$

From (3), (5), and (6) we have

$$\rho c_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right] = \frac{\partial}{\partial y} \left[ \left( \frac{16\sigma^* T_w^3}{3k^*} + k \right) \frac{\partial T}{\partial y} \right]. \quad (7)$$

The subjected boundary conditions are

$$u = U_w, \quad v = V_w, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \quad (8)$$

$$u \rightarrow 0, \quad T \rightarrow T_w, \quad C \rightarrow C_w \quad \text{as} \quad y \rightarrow \infty. \quad (9)$$

$$V_w = -\sqrt{\frac{\nu U_w}{x}} \frac{f(0)}{\eta} \quad (10)$$

represents the mass transfer at the surface with $V_w > 0$ for injection and $V_w < 0$ for suction. We further assume the stretching velocity $U_w(x,t)$, surface temperature $T_w(x,t)$, and concentration at the surface $C_w(x,t)$ in the following forms:

$$U_w(x,t) = \frac{ax}{1 - ct}, \quad T_w(x,t) = T_m + \frac{bx}{1 - ct}, \quad (11)$$

$$C_w(x,t) = C_w + \frac{ex}{1 - ct},$$

in which $a, b, e$, and $c$ are constants with $a > 0, b \geq 0, e \geq 0,$ and $c \geq 0$ with $ct < 1$. We choose a time dependent magnetic field [32–36] $B(t) = B_0(1 - ct)^{-1}$ and a time dependent reaction rate $R(t) = R_0(1 - ct)^{-1}$ with $B_0$ and $R_0$ as the uniform magnetic field and reaction rate, respectively.

We introduce

$$\eta = \sqrt{\frac{U_w}{V_w}}, \quad \psi = \sqrt{\nu U_w f(\eta)},$$

$$\theta(\eta) = \frac{T - T_m}{T_w - T_m}, \quad \phi(\eta) = \frac{C - C_w}{C_w - C_m}. \quad (12)$$

and the velocity components

$$u = \frac{\partial \psi}{\partial \eta}, \quad v = -\frac{\partial \psi}{\partial \eta}, \quad (13)$$

where $\psi$ is a stream function. The continuity equation is identically satisfied and the resulting problems for $f, \theta$, and $\phi$ become

$$f'' + f f'' - f^2 - A \left( f' + \frac{1}{2} \eta f'' \right) - M^2 f' = 0, \quad (14)$$

$$\frac{1}{Pr} \left( 1 + \frac{4}{3} R_d \right) \theta'' + f \theta' - \theta f' - A \left( \theta + \frac{1}{2} \eta \theta' \right) = 0, \quad (15)$$

$$\frac{1}{Sc} \phi'' + f \phi' - \phi f' - \gamma \phi - A \left( \phi + \frac{1}{2} \eta \phi' \right) = 0, \quad (16)$$

$$f(0) = S, \quad f'(0) = 1, \quad \theta(0) = 1,$$

$$\phi(0) = 1, \quad f'(-1) \rightarrow 0, \quad \theta(-1) \rightarrow 0, \quad (17)$$

$$\phi(\eta) \rightarrow 0, \quad \eta \rightarrow \infty,$$

with $f(0) = S$ which for $S < 0$ corresponds to suction case and $S > 0$ implies injection. Here $A = c/a$ is an unsteadiness parameter and for $A = 0$ the problem reduce to the steady state situation. The Hartman number $M$, the Prandtl number $Pr$, the radiation parameter $R_d$, the Schmidt number $Sc$ and the chemical reaction parameter $\gamma$ are, respectively, given by

$$M^2 = \frac{\sigma B_0^2}{\rho a}, \quad Pr = \frac{\mu c_p}{k}, \quad R_d = \frac{4\sigma^* T_w^3}{k^* k}, \quad (18)$$

$$Sc = \frac{V}{D}, \quad \gamma = \frac{R_0}{a},$$

and the prime denotes the derivative with respect to $\eta$.

Expressions of the skin friction coefficient $C_i$, local Nusselt number $Nu_{x}$, and the surface mass transfer $\phi'$ at the wall are defined as

$$C_i = \frac{\tau_w}{\rho U_w c/2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_m)}, \quad (19)$$

$$\phi'(0) = \left( \frac{\partial \phi}{\partial \eta} \right)_{\eta=0} \leq 0,$$

where the skin friction $\tau_w$ and the heat transfer $q_w$ from the plate are

$$\tau_w = \mu \left( \frac{\partial u}{\partial \eta} \right)_{\eta=0}, \quad (20)$$

$$q_w = -\left[ k + \frac{16\sigma^* T_w^3}{3k} \right] \left( \frac{\partial T}{\partial y} \right)_{\eta=0}.$$
In terms of dimensionless variables we have
\[
\frac{1}{2}C_i \text{Re}^{-1/2} = f''(0), \\
\text{Nu}_e \text{Re}^{-1/2} \left( \frac{4}{4 + 3 R_d} \right) = -\theta'(0). \tag{21}
\]

3. Homotopy Analysis Solutions

The velocity \( f(\eta) \), the temperature \( \theta(\eta) \), and the concentration fields \( \phi(\eta) \) can be expressed by the base functions
\[
\{ \eta^k \exp(-n\eta) | k \geq 0, n \geq 0 \} \tag{22}
\]
in the form
\[
f(\eta) = a_{0,0}^0 + \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp(-n\eta), \tag{23}
f(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta), \tag{24}
\phi(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} c_{m,n}^k \eta^k \exp(-n\eta), \tag{25}
\]
where \( a_{m,n}^k, b_{m,n}^k, \) and \( c_{m,n}^k \) are the coefficients. Based on the rule of solution expressions and the boundary conditions (17), one can choose the initial guesses \( f_0, \theta_0, \) and \( \phi_0 \) of \( f(\eta), \theta(\eta), \) and \( \phi(\eta) \) as
\[
f_0(\eta) = 1 + S - \exp(-\eta), \tag{26}
\theta_0(\eta) = \exp(-\eta), \tag{27}
\phi_0(\eta) = \exp(-\eta), \tag{28}
\]
and the auxiliary linear operators are expressed by the following equations:
\[
\mathcal{L}_f = \frac{d^3 f}{d\eta^3} - \frac{d f}{d\eta}, \tag{29}
\mathcal{L}_\theta = \frac{d^2 \theta}{d\eta^2} - \theta, \tag{30}
\mathcal{L}_\phi = \frac{d^2 \phi}{d\eta^2} - \phi. \tag{31}
\]
Note that the above operators possess the following properties:
\[
\mathcal{L}_f [C_1 + C_2 \exp(\eta) + C_3 \exp(-\eta)] = 0, \tag{32}
\mathcal{L}_\theta [C_4 \exp(\eta) + C_5 \exp(-\eta)] = 0, \tag{33}
\mathcal{L}_\phi [C_6 \exp(\eta) + C_7 \exp(-\eta)] = 0, \tag{34}
\]
where \( C_i (i = 1 - 7) \) are arbitrary constants.

If \( p \in [0, 1] \) is the embedding parameter and \( h_f, h_\theta, \) and \( h_\phi \) indicate the non-zero auxiliary parameters, respectively, then the zeroth-order deformation problems are
\[
(1 - p) \mathcal{L}_f [\hat{f}(\eta, p) - f_0(\eta)] = p h_f N_f [\hat{f}(\eta, p)], \tag{35}
(1 - p) \mathcal{L}_\theta [\hat{\theta}(\eta, p) - \theta_0(\eta)] = p h_\theta N_\theta [\hat{\theta}(\eta, p)], \tag{36}
(1 - p) \mathcal{L}_\phi [\hat{\phi}(\eta, p) - \phi_0(\eta)] = p h_\phi N_\phi [\hat{\phi}(\eta, p)], \tag{37}
\]
with the boundary conditions
\[
\hat{f}(\eta; p)|_{\eta=0} = S, \quad \left. \frac{\partial \hat{f}(\eta; p)}{\partial \eta} \right|_{\eta=0} = 0, \tag{38}
\hat{\theta}(\eta; p)|_{\eta=0} = 1, \quad \left. \frac{\partial \hat{\theta}(\eta; p)}{\partial \eta} \right|_{\eta=0} = 0, \tag{39}
\hat{\phi}(\eta; p)|_{\eta=0} = 1, \quad \left. \frac{\partial \hat{\phi}(\eta; p)}{\partial \eta} \right|_{\eta=0} = 0, \tag{40}
\]
and the nonlinear operators \( N_f, N_\theta, \) and \( N_\phi \) are
\[
N_f [\hat{f}(\eta; p)] = \frac{\partial^3 \hat{f}(\eta, p)}{\partial \eta^3} + \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \frac{\partial^2 \hat{f}(\eta, p)}{\partial \eta^2} - \left( \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \right)^2 - M^2 \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \tag{41}
\]
\[
N_\theta [\hat{\theta}(\eta; p), \hat{f}(\eta; p)] = \left( 1 + \frac{4}{3 R_d} \right) \frac{\partial^2 \hat{\theta}(\eta; p)}{\partial \eta^2} + Pr \left( \hat{f}(\eta, p) \frac{\partial \hat{\theta}(\eta; p)}{\partial \eta} - \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \hat{\theta}(\eta; p) \right) \tag{42}
\]
\[
N_\phi [\hat{\phi}(\eta; p), \hat{f}(\eta; p)] = \left( 1 + \frac{2}{3 R_d} \right) \frac{\partial^2 \hat{\phi}(\eta; p)}{\partial \eta^2} + Sc \left( \hat{f}(\eta, p) \frac{\partial \hat{\phi}(\eta; p)}{\partial \eta} - \frac{\partial \hat{f}(\eta, p)}{\partial \eta} \hat{\phi}(\eta; p) - \gamma \hat{\phi}(\eta; p) \right) \tag{43}
\]
For $p = 0$ and $p = 1$, we have
\begin{align}
\dot{f}(\eta; 0) &= f_0(\eta), \\
\dot{f}(\eta; 1) &= f(\eta), \\
\dot{\theta}(\eta; 0) &= \theta_0(\eta), \\
\dot{\theta}(\eta; 1) &= \theta(\eta), \\
\dot{\phi}(\eta; 0) &= \phi_0(\eta), \\
\dot{\phi}(\eta; 1) &= \phi(\eta).
\end{align}
(44) - (49)

Expanding $\hat{f}(\eta; p)$, $\hat{\theta}(\eta; p)$, and $\hat{\phi}(\eta; p)$ in Taylor's theorem with respect to an embedding parameter $p$, one has
\begin{align}
\hat{f}(\eta; p) &= f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, \\
\hat{\theta}(\eta; p) &= \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)p^m, \\
\hat{\phi}(\eta; p) &= \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)p^m.
\end{align}
(47) - (49)

The auxiliary parameters are so properly chosen that the series (47)–(49) converge at $p = 1$, then we have
\begin{align}
f(\eta) &= f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \\
\theta(\eta) &= \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta), \\
\phi(\eta) &= \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta).
\end{align}
(51) - (53)

The $m$th-order deformation problems are
\begin{align}
L_f[f_m(\eta) - \lambda_m f_{m-1}(\eta)] &= \partial_y R_m^f(\eta), \\
L_\theta[\theta_m(\eta) - \lambda_m \theta_{m-1}(\eta)] &= \partial_y R_m^\theta(\eta), \\
L_\phi[\phi_m(\eta) - \lambda_m \phi_{m-1}(\eta)] &= \partial_y R_m^\phi(\eta), \\
f_m(0) &= 0, f'_m(0) = 0, f'_m(\infty) = 0, \\
\theta_m(0) &= 0, \theta_m(\infty) = 0, \phi_m(0) = 0, \phi_m(\infty) = 0,
\end{align}
(54) - (56)

The general solutions of (54)–(57) are
\begin{align}
f_m(\eta) &= f_m^*(\eta) + C_1 + C_2 \exp(\eta) + C_3 \exp(-\eta), \\
\theta_m(\eta) &= \theta_m^*(\eta) + C_4 \exp(\eta) + C_5 \exp(-\eta),
\end{align}
(61)

where $f_m^*(\eta)$, $\theta_m^*(\eta)$, and $\phi_m^*(\eta)$ denote the special solutions and
\begin{align}
C_2 &= C_4 = C_6 = 0, \\
C_1 &= -C_3 - f_m^*(0), \\
C_3 &= \frac{\partial f_m^*(\eta)}{\partial \eta} |_{\eta=0}, \\
C_5 &= -\theta_m^*(0), \\
C_7 &= -\phi_m^*(0).
\end{align}
(64)

Note that (54)–(56) can be solved by Mathematica one after the other in the order $m = 1, 2, 3, \ldots$

### 4. Convergence of the Homotopy Solutions

The analytical series solutions (51)–(53) contain the non-zero auxiliary parameters $h_f$, $h_\theta$, and $h_\phi$ which can adjust and control the convergence of the series solutions. In order to see the range of admissible values of $h_f$, $h_\theta$, and $h_\phi$ of the functions $f''(\eta)$, $\theta'(\eta)$, and $\phi'(\eta)$ the $h_f$, $h_\theta$, and $h_\phi$-curves are displayed for 25th-order of approximations. It is obvious from Figure 1 that the range for the admissible values of $h_f$, $h_\theta$, and $h_\phi$ are $-0.8 \leq h_f \leq -0.3$, $-1.5 \leq h_\theta \leq -0.3$, and $-2.0 \leq h_\phi \leq -1.0$. 

\begin{align}
R_m^f(\eta) &= f_{m-1}'' - M^2 f_{m-1}' - A \left[ f_{m-1}' + \frac{1}{2} \eta f_{m-1}' \right] \\
+ \sum_{k=0}^{m-1} \left[ f_{m-1-k} f_k' - f_{m-1-k} f'_k \right], \\
R_m^\theta(\eta) &= \left( 1 + \frac{4}{3} R_\eta \right) \theta_{m-1}' - AP \left[ \theta_{m-1} + \frac{1}{2} \eta \theta_{m-1}' \right] \\
+ \sum_{k=0}^{m-1} \left[ f_{m-1-k} \theta_k' - \theta_{m-1-k} f'_k \right], \\
R_m^\phi(\eta) &= \phi_{m-1}' - Sc \gamma \phi_{m-1} - AS \left[ \phi_{m-1} + \frac{1}{2} \eta \phi_{m-1}' \right] \\
+ \sum_{k=0}^{m-1} \left[ f_{m-1-k} \phi_k' - \phi_{m-1-k} f'_k \right],
\end{align}
Table 1. Convergence of HAM solution for different order of approximations.

<table>
<thead>
<tr>
<th>Order of approximation</th>
<th>$-f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.46875</td>
<td>0.83542</td>
<td>1.93750</td>
</tr>
<tr>
<td>5</td>
<td>1.78492</td>
<td>0.73571</td>
<td>1.80378</td>
</tr>
<tr>
<td>10</td>
<td>1.80191</td>
<td>0.72477</td>
<td>1.80242</td>
</tr>
<tr>
<td>15</td>
<td>1.80242</td>
<td>0.72338</td>
<td>1.80242</td>
</tr>
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<td>20</td>
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<tr>
<td>30</td>
<td>1.80242</td>
<td>0.72309</td>
<td>1.80242</td>
</tr>
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</table>

Fig. 1. $\bar{h}$-curves for 25th-order of approximations.

The results are obtained for $-1.4 \leq \bar{h}_f \leq -0.1$. It is found from our computations that the series given by (51)–(53) converge in the whole region of $\eta$ when $\bar{h}_f = -0.6$ and $\bar{h}_\theta = -1 = \bar{h}_\phi$.

Table 1 shows the convergence of the homotopy solutions for different order of approximations as $A = 0.5$, $M = 1.0$, $S = 0.5 = Pr$, $R_d = 0.2$, $Sc = \gamma = 1.0$.

5. Discussion of the Results

This section deals with the variations of Hartman number $M$, unsteadiness parameter $A$, the suction parameter $S$, the Prandtl number $Pr$, radiation parameter $R_d$, the Schmidt number $Sc$, and the chemical reaction parameter $\gamma$ on the velocity $f'$, the concentration $\phi$, and the temperature fields $\theta$. Figures 2–4 represent the variations of $A$, $M$, and $S$ on $f'$. Figure 2 describes the
The effect of $A$ on $f'$. It is noticed that $f'$ decreases when $A$ increases. Figures 3 and 4 show the effects of $M$ and $S$ on $f'$, respectively. Obviously $f'$ is a decreasing function of $M$ and $S$.

Figures 5 – 9 depict the influences of $A$, $M$, $S$, $Pr$, and $R_d$ on $\theta$. Figure 5 indicates that $\theta$ decreases as $A$ increases. Figure 6 gives the behaviour of $M$ on $\theta$. The temperature profile increases as $M$ increases. Figure 7 elucidates the influence of $S$ on $\theta$. The temperature field $\theta$ decreases when $S$ increases. It is observed that $\theta$ decreases when $Pr$ increases (Fig. 8). Figure 9 describes the effects of $R_d$ on $\theta$. Here $\theta$ increases as $R_d$ increases.

Figures 10 – 15 are plotted for the effects of $A$, $M$, $S$, $Sc$, and $\gamma$ on the concentration field $\phi$. It is seen from Figure 10 that $\phi$ decreases as the unsteadiness parameter increases. Figure 11 depicts the concentration field $\phi$ for various values of $M$. Here $\phi$ increases for large $M$. Figure 12 shows the variation of $S$ on the concentration field $\phi$. Clearly, $\phi$ is a decreasing function of $S$ and the concentration boundary layer thickness also decreases when $S$ increases. The variation of Schmidt number $Sc$ on $\phi$ is shown in Figure 13. The concentration field $\phi$ decreases by increasing $Sc$. The concentration boundary layer thickness also decreases for large values of $Sc$. Figure 14 displays the
influence of the destructive chemical reaction parameter ($\gamma > 0$) on the concentration profile $\phi$. It is obvious that the fluid concentration decreases with an increase in the destructive chemical reaction parameter. Figure 15 illustrates the effect of the generative chemical reaction parameter ($\gamma < 0$) on the concentration profile $\phi$. This figure illustrates that the concentration field $\phi$ has an opposite behaviour for $\gamma < 0$ when compared with the case of the destructive chemical reaction parameter ($\gamma > 0$).
Table 1. Values of skin friction coefficient $\frac{1}{2}C_f \text{Re}_x^{1/2}$ for the parameters $A, M$, and $S$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$M$</th>
<th>$S$</th>
<th>$\frac{1}{2}C_f \text{Re}_x^{1/2}$</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>1.2</td>
<td>0.5</td>
<td>1.831929</td>
</tr>
<tr>
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<td>0.7</td>
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<td>1.5</td>
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<tr>
<td>0.3</td>
<td>0</td>
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<td></td>
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<td></td>
<td>1.0</td>
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<td>1.5</td>
<td></td>
<td>2</td>
<td>1.27268</td>
</tr>
<tr>
<td>2</td>
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<td>1.631209</td>
</tr>
</tbody>
</table>

Table 2. Values of skin friction coefficient $\frac{1}{2}C_f \text{Re}_x^{1/2}$ for the parameters $A, M$, and $S$. Table 2 includes the values of the skin friction coefficient $\frac{1}{2}C_f \text{Re}_x^{1/2}$. It is noticed that the magnitude of the skin friction coefficient increases for large values of $A, M$, and $S$. Table 3 depicts the variation of the heat transfer characteristic at the wall $-\theta'(0)$ when $M = R_d = 0$. From this table one can see that the HAM solution is in good agreement with an exact solution [15]. Table 4 presents the values of $-\theta'(0)$ for some values of $A, M,$ and $R_d$ when $Pr = 0.5 = S$. Table 5 consists of the surface mass transfer $-\psi'(0)$ for some values of $A, M, S, Sc,$ and $\gamma$. It is apparent from this table that the magnitude of $-\phi'(0)$ increases for large values of $A$ and $S$, and decreases for large values of $M$. The magnitude of $-\phi'(0)$ increases when $Sc$ and $\gamma$ increases. 

6. Conclusions

This article presents the series solution for the unsteady two dimensional flow bounded by a stretching surface. Emphasis in this study is given to the unsteadiness, radiation, MHD, and mass transfer effects. The salient features of present analysis are reproduced below.

- The variations of $M, A,$ and $S$ on $f'$ are qualitatively similar.
- Effects of $A, Pr,$ and $S$ on $\theta$ are similar.
- The behaviours of $M$ and $R_d$ on $\theta$ are opposite to that of $A, Pr,$ and $S$. 

Table 4. Values of $-\theta'(0)$ for some values of $A, M,$ and $R_d$ when $Pr = 0.5 = S$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$M$</th>
<th>$R_d$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
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Table 5. Values of mass transfer $-\psi'(0)$ for some values of $A, M, S,$ and $\gamma$.

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Variation of $M$ on $\theta$ and $\phi$ is similar whereas reverse trend is noted for $f'$. 
- Effects of $S$ and $A$ on $f'$, $\theta$, and $\phi$ are similar in the qualitative sense. 
- The variation of $Sc$ on $\phi$ is similar to that of $\gamma > 0$ and is opposite to $\gamma < 0$. 
- Variations of $A$ on the magnitudes of skin friction coefficients and local Nusselt number and mass transfer are similar. 
- Effects of $M$ on the magnitudes of mass transfer and local Nusselt number is same but is different for the skin friction coefficient.

Acknowledgements

We are grateful to the referees for their fruitful comments and suggestions.