The Bright Soliton Solutions of Two Variable-Coefficient Coupled Nonlinear Schrödinger Equations in Optical Fibers

Deng-Shan Wang[a,c] and Yifang Liu[b]

[a] CEMA and CIAS, Central University of Finance and Economics, Beijing, 100081, China
[b] School of Economics, Central University of Finance and Economics, Beijing, 100081, China
[c] BNLCMP, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

Reprint requests to D.-S. W.; E-mail: wangdsh1980@yahoo.com.cn

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In this paper, with the aid of symbolic computation the bright soliton solutions of two variable-coefficient coupled nonlinear Schrödinger equations are obtained by Hirota's method. Some figures are plotted to illustrate the properties of the obtained solutions. The properties are meaningful for the investigation on the stability of soliton propagation in the optical soliton communications.

Key words: Hirota's Method; Symbolic Computation; Bright Soliton Solution; Coupled Nonlinear Schrödinger Equations.
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1. Introduction

In the recent years, there has been much interest on solitons appearing in optical waveguides in view of their potential applications in a long haul optical communication system or in all-optical ultra-fast switching devices. These solitons arise in two distinct types – the bright and the dark solitons – depending on the sign of the fiber group velocity dispersion term in the nonlinear Schrödinger (NLS) equation which governs the propagation of pulses in a fiber. However, when the coupling of different polarizations and different frequency modes are taken into account, the system is governed by a multi-component NLS equation with additional cross-phase modulation terms. The cross-phase modulation supports various types of soliton configurations such as the bright-bright, the bright-dark, and the dark-dark pairs of solitons [1]. These new types of configuration introduce an important physical concept of the soliton-induced waveguides and light guiding which have been verified experimentally [2] and applied for the compression of bright pulses by dark solitons [3]. Under certain physical conditions [4], the nonlinear coupling term becomes proportional to the total intensity and the resulting vector NLS equation, also known as the Manakov equation, becomes integrable by the inverse scattering method [5].

In many branches of physics, considering the inhomogeneities of media and non-uniformities of boundaries such as for a Bose gas of impenetrable bosons [6], optical fibre communications [7, 8], variable-coefficient nonlinear wave equations can describe real situations more powerfully than their constant coefficient counterparts. Although such variable coefficients often make the studies hard, with the development of symbolic computation [8, 9], it is becoming possible and exercisable for a computer to deal with them.

In this paper, we investigate the variable-coefficient coupled NLS equations which describe two pulses co-propagating in optical fibers. For this purpose, we need to consider the two-component generalization of the single-component propagation equations. We will give the bright soliton solutions of the following two variable-coefficient coupled NLS equations by means of Hirota's method [10 – 12]:

$$\begin{aligned} &\mathrm{i}u_t + ru_{xx} - 2r(|u|^2 - |v|^2)u + (\psi x + \phi)u = 0,\\ &\mathrm{i}v_t - rv_{xx} + 2r(|u|^2 - |v|^2)v - (\psi x + \phi)v = 0, \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\mathrm{i}u_t + ru_{xx} + 2ru\left(|u|^2 + |v|^2\right) + (\psi x + \phi)u = 0,\\ &\mathrm{i}v_t + rv_{xx} + 2rv\left(|u|^2 + |v|^2\right) + (\psi x + \phi)v = 0, \end{aligned} \tag{2}$$

(and their complex conjugates), where u and v are slowly varying envelopes of the two interacting optical modes, the variables x and t are the normalized distance and time, r, ψ, and ϕ, are real functions of t. The

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two variable-coefficient NLS equations are integrable and have isospectral Lax pairs, which can be found in our other work [13, 14].

2. The Soliton Solutions of the Coupled NLS Equations (1) and (2)

In this section, with the aid of symbolic computation, the Hirota's method is applied to investigate the bright soliton solutions of the two variable-coefficient NLS equations (1) and (2).

2.1. Bright Solitons Solutions of (1)

Firstly, we take the Hirota bilinear transformation in the form

$$u=\frac{g}{f},\quad v=\frac{h}{f}, \tag{3}$$

where $g=g(x,t)$, $h=h(x,f)$ are complex functions and $f=f(x,t)$ is a real function.

Substituting (3) into (1), the Hirota bilinear form of (1) is obtained as

$$\begin{aligned}&\mathrm{i}D_t g\cdot f+rD_x^2 g\cdot f+(\psi x+\phi)gf=0,\\&D_x^2 f\cdot f+2hh^*-2gg^*=0,\\&\mathrm{i}D_t h\cdot f-rD_x^2 h\cdot f-(\psi x+\phi)hf=0,\end{aligned} \tag{4}$$

where the D-operator [10 – 12] is defined by

$$D_x^n D_t^m g(x,t)\cdot f(x,t)\equiv \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^n\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^m g(x,t)f(x',t')|_{x=x',t=t'}.$$

In order to obtain the bright soliton solutions, we proceed in the standard way. Expanding f,g, and h as the series

$$\begin{aligned}&f=1+\delta^2 f_2+\delta^4 f_4+\cdots,\\&g=g_1+\delta^3 g_3+\cdots,\\&h=\delta h_1+\delta^3 h_3+\cdots,\end{aligned} \tag{5}$$

substituting (5) into (4), and comparing the coefficients of the same power of δ yields

$$\begin{aligned}&\mathrm{i}g_{1t}+rg_{1xx}+\big(\psi x+\phi\big)g_1=0,\\&\mathrm{i}\big(g_{1t}f_2-g_1f_{2t}+g_{3t}\big)+r\big(g_{1xx}f_2-2g_{1x}f_{2x}\\&+g_1f_{2xx}+g_{3xx}\big)+\big(\psi x+\phi\big)\big(g_1f_2+g_3\big)=0,\\&\mathrm{i}h_{1t}-rh_{1xx}-(\psi x+\phi)h_1=0,\\&\mathrm{i}\big(h_{1t}f_2-h_1f_{2t}+h_{3t}\big)-r\big(h_{1xx}f_2-2h_{1x}f_{2x}\\&+h_1f_{2xx}+h_{3xx}\big)-\big(\psi x+\phi\big)\big(h_1f_2+h_3\big)=0,\\&f_{2xx}+g_1g_1^*-h_1h_1^*=0,\\&f_{4xx}+f_{2xx}f_2-f_{2x}^2+g_1g_3^*+g_3g_1^*\\&-h_1h_3^*-h_3h_1^*=0,\\&\vdots\end{aligned} \tag{6}$$

If we take

$$\begin{aligned}&g_1=\sum_{i=1}^N\alpha_i(t)\exp(\xi_i),\quad \xi_i=a_i(x,t)+\mathrm{i}b_i(x,t),\\&h_1=\sum_{i=1}^N\beta_i(t)\exp(\eta_i),\quad \eta_i=c_i(x,t)+\mathrm{i}d_i(x,t),\end{aligned}$$

then the N-soliton solution of (1) will be obtained.

To find the one-soliton solution, i. e. for $N=1$, we assume

$$g_1=\alpha(t)\exp(\xi_1),\quad h_1=\beta(t)\exp(\eta_1), \tag{7}$$

where $\xi_1=a_1(x,t)+\mathrm{i}b_1(x,t)$, and $\eta_1=c_1(x,t)+\mathrm{i}d_1(x,t)$ are functions to be determined.

Substituting (7) into (6), one obtains

$$\begin{aligned}&g_1=\alpha\mathrm{e}^{\xi_1},\quad h_1=\beta\mathrm{e}^{\xi_1^*},\\&g_i=h_i=0,\ (i=3,5,\cdots),\\&f_2=\mathrm{e}^{\xi_1+\xi_1^*+\xi_0},\quad f_j=0,\ (j=4,6,\cdots),\\&\mathrm{e}^{\xi_0}=\frac{\beta^2-\alpha^2}{4{C_1}^2},\quad \mathrm{i}=\sqrt{-1},\\&\xi_1=C_1x-2C_1\int\!\!\int r\psi\mathrm{d}t\mathrm{d}t\\&\quad+\mathrm{i}\left[\int\left(C_1^2r-r\left(\int\psi\mathrm{d}t\right)^2+\phi\right)\mathrm{d}t+x\int\psi\mathrm{d}t\right],\\&\xi_1^*=C_1x-2C_1\int\!\!\int r\psi\mathrm{d}t\mathrm{d}t\\&\quad-\mathrm{i}\left[\int\left(C_1^2r-r\left(\int\psi\mathrm{d}t\right)^2+\phi\right)\mathrm{d}t+x\int\psi\mathrm{d}t\right],\end{aligned} \tag{8}$$

where C_1,α,β are arbitrary real constants.

So by setting $\delta=1$ the one-bright soliton of (1) is obtained as

$$u=\frac{\alpha\mathrm{e}^{\xi_1}}{1+\mathrm{e}^{\xi_1+\xi_1^*+\xi_0}},\quad v=\frac{\beta\mathrm{e}^{\xi_1^*}}{1+\mathrm{e}^{\xi_1+\xi_1^*+\xi_0}}, \tag{9}$$

with ξ_1,ξ_1^*, and ξ_0 satisfy (8).

For $N=2$, we have

$$\begin{aligned}&g_1=\alpha_1(t)\exp(\xi_1)+\alpha_2(t)\exp(\xi_2),\\&h_1=\beta_1(t)\exp(\eta_1)+\beta_2(t)\exp(\eta_2),\end{aligned} \tag{10}$$

where $\xi_i = a_i(x,t) + \mathrm{i}b_i(x,t)$, and $\eta_i = c_i(x,t) + \mathrm{i}d_i(x,t)$ $(i = 1,2)$ are functions to be determined.

With the aid of symbolic computation, substituting (10) into (6), and solving the final equations, the two-soliton solution of (1) is obtained as follows (we have set $\delta = 1$):

$$u = \left[\alpha_1 e^{\xi_1} + \alpha_2 e^{\xi_2} + e^{\xi_1+\xi_1^*+\xi_2+\xi_1^0} + e^{\xi_2+\xi_2^*+\xi_1+\xi_2^0}\right] \cdot \Big[1 + e^{\xi_1+\xi_1^*+R_1} + e^{\xi_2+\xi_2^*+R_2} + e^{\xi_1^*+\xi_2+R_3} + e^{\xi_1+\xi_2^*+R_3} + e^{\xi_1+\xi_1^*+\xi_2+\xi_2^*+R_4}\Big]^{-1}, \tag{11}$$

$$v = \left[\beta_1 e^{\xi_1^*} + \beta_2 e^{\xi_2^*} + e^{\xi_1^*+\xi_1+\xi_2^*+\eta_1^0} + e^{\xi_2^*+\xi_2+\xi_1^*+\eta_2^0}\right] \cdot \Big[1 + e^{\xi_1+\xi_1^*+R_1} + e^{\xi_2+\xi_2^*+R_2} + e^{\xi_1^*+\xi_2+R_3} + e^{\xi_1+\xi_2^*+R_3} + e^{\xi_1+\xi_1^*+\xi_2+\xi_2^*+R_4}\Big]^{-1}, \tag{12}$$

where

$$\xi_j = C_j x - 2C_j \int\!\!\int r\psi\,\mathrm{d}t\,\mathrm{d}t + \mathrm{i}\left[\int\left(C_j^2 r - r\left(\int\psi\,\mathrm{d}t\right)^2 + \phi\right)\mathrm{d}t + x\int\psi\,\mathrm{d}t\right],$$

$$\xi_j^* = C_j x - 2C_j \int\!\!\int r\psi\,\mathrm{d}t\,\mathrm{d}t - \mathrm{i}\left[\int\left(C_j^2 r - r\left(\int\psi\,\mathrm{d}t\right)^2 + \phi\right)\mathrm{d}t + x\int\psi\,\mathrm{d}t\right],$$

$$e^{\xi_1^0} = \frac{C_2 - C_1}{4C_1^2 (C_2 + C_1)^2}\Big[\alpha_2 C_2 \left(\beta_1^2 - \alpha_1^2\right) + \alpha_2 C_1 \left(\alpha_1^2 + \beta_1^2\right) - 2C_1\alpha_1\beta_1\beta_2\Big],$$

$$e^{\xi_2^0} = \frac{C_2 - C_1}{4C_2^2 (C_2 + C_1)^2}\Big[-C_2\alpha_1 \left(\alpha_2^2 + \beta_2^2\right) - \alpha_1 C_1 \left(\beta_2^2 - \alpha_2^2\right) + 2C_2\alpha_2\beta_1\beta_2\Big],$$

$$e^{\eta_1^0} = \frac{C_2 - C_1}{4C_1^2 (C_2 + C_1)^2}\Big[\beta_2 C_2 \left(\beta_1^2 - \alpha_1^2\right) - \beta_2 C_1 \left(\alpha_1^2 + \beta_1^2\right) + 2\,\alpha_1\alpha_2\beta_1 C_1\Big],$$

$$e^{\eta_2^0} = \frac{C_2 - C_1}{4C_2^2 (C_2 + C_1)^2}\Big[\beta_1 C_2 \left(\alpha_2^2 + \beta_2^2\right) - \beta_1 C_1 \left(\beta_2^2 - \alpha_2^2\right) - 2C_2\alpha_1\alpha_2\beta_2\Big],$$

$$e^{R_1} = \frac{\beta_1^2 - \alpha_1^2}{4C_1^2}, \quad e^{R_2} = \frac{\beta_2^2 - \alpha_2^2}{4C_2^2},$$

$$e^{R_3} = \frac{\beta_1\beta_2 - \alpha_1\alpha_2}{(C_1 + C_2)^2}, \quad j = 1,2,$$

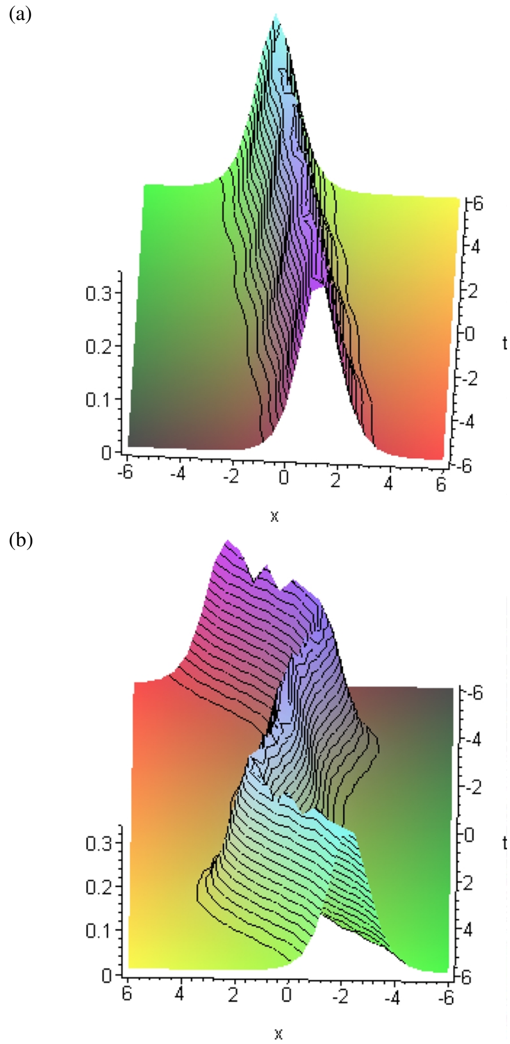

Fig. 1. Plots of one-soliton solution $|u|^2$ for (1) denoted by (9) with different parameters: (a): $\alpha = 1$, $C_1 = 1$, $\beta = 2$, $r = \cos(t)$, $\psi = 0.2\sin(t)$, and $\phi = \cos(t)$; (b): $\alpha = 1$, $r = t$, $\psi = 0.2\cos(t)$, $C_1 = 1$, $\beta = 2$, and $\phi = t^2$.

$$e^{R_4} = \frac{(C_2 - C_1)^2}{16C_2^2 C_1^2 (C_2 + C_1)^4}\Big[C_2(\beta_2 + \alpha_2)(\beta_1 - \alpha_1) - C_1(\beta_2 - \alpha_2)(\beta_1 + \alpha_1)\Big]\Big[C_2(\beta_2 - \alpha_2)(\beta_1 + \alpha_1) - C_1(\beta_2 + \alpha_2)(\beta_1 - \alpha_1)\Big],$$

and C_i, α_i, β_i $(i = 1,2)$ are arbitrary real constants.

The two-solitons are bright two-solitons with six parameters which have elastic collision properties with specific choices of parameters.

For $N = 3$, by using the same procedures three-soliton solutions of (1) can also be obtained.

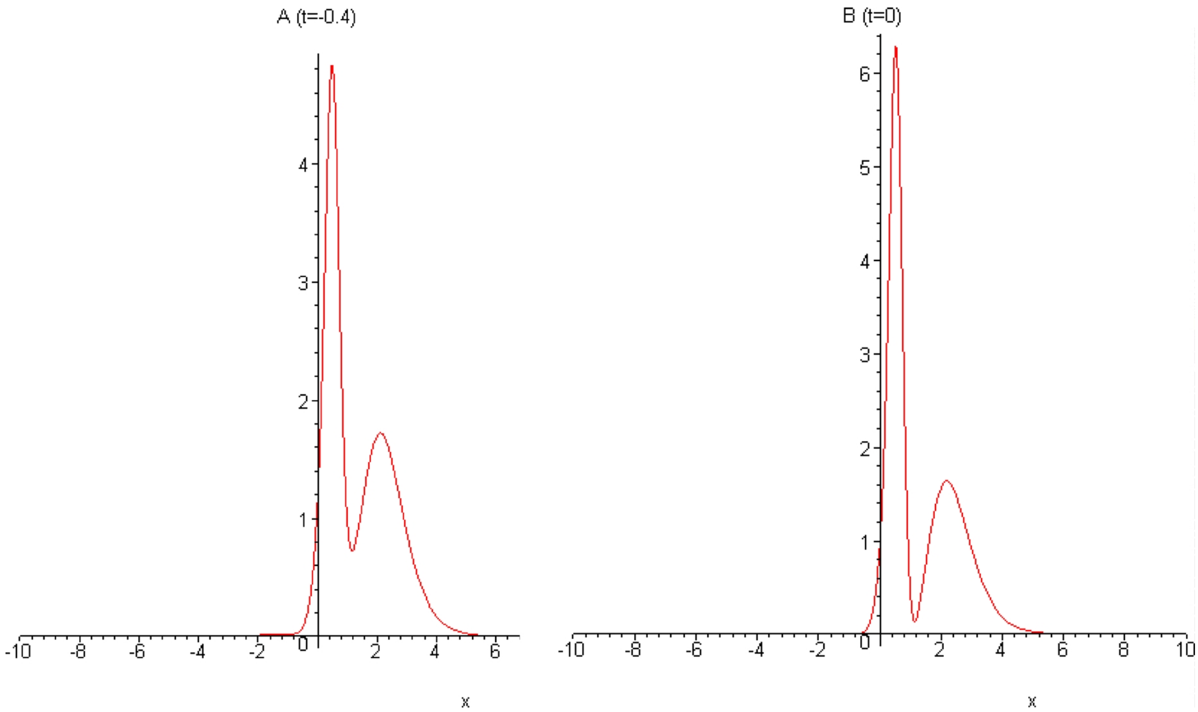

Fig. 2. Evolution plots of the two-soliton solution $|u|^2$ of equation (1) at $t = -0.4$ and $t = 0$ with parameters mentioned in the text $(*)$.

In what follows, we will analyze the properties of the obtained solutions by investigating their figures.

Figure 1 shows the shape and motion of the one-bright soliton solution $|u|^2$ of equation (1) given by expression (9) with (8) when for the parameters suitable values are chosen.

Figures 2–4 provide the evolution plots of the two-soliton solution $|u|^2$ of equation (1) with the following parameters at time $t = -0.4$, $t = 0$, $t = 0.8$, $t = 1.6$, $t = 2.4$, and $t = 3.2$, respectively:

$$\begin{aligned}&\alpha_2 = 2,\ \beta_1 = 3,\ \beta_2 = 4,\ C_1 = 1,\ C_2 = 2,\ \alpha_1 = 1,\\ &\psi = \cos(t), \phi = \cos(t),\ \text{and}\ \ r = \sin(t).\end{aligned} \tag{$*$}$$

3. Bright Solitons Solutions of (2)

Substituting (3) into (2), the bilinear form of (2) is obtained as

$$\begin{aligned}&\mathrm{i}D_t g\cdot f + rD_x^2 g\cdot f + (\psi x+\phi)gf = 0,\\ &D_x^2 f\cdot f - 2hh^* - 2gg^* = 0,\\ &\mathrm{i}D_t h\cdot f + rD_x^2 h\cdot f + (\psi x+\phi)hf = 0.\end{aligned} \tag{13}$$

By means of the same procedures as in Section 2, the bright solitons solutions of (2) can be constructed. In what follows, we only list the final results.

The one-bright soliton solution of (2) is as follows:

$$u = \frac{\alpha \mathrm{e}^{\xi_1}}{1+\mathrm{e}^{\xi_1+\xi_1^*+\xi_0}}, \quad v = \frac{\beta \mathrm{e}^{\xi_1}}{1+\mathrm{e}^{\xi_1+\xi_1^*+\xi_0}}, \tag{14}$$

where

$$\mathrm{e}^{\xi_0} = \frac{\alpha^2+\beta^2}{4C_1^2}, \quad \mathrm{i} = \sqrt{-1},$$

$$\begin{aligned}\xi_1 &= C_1 x - 2C_1\int\!\!\int r\psi \mathrm{d}t\mathrm{d}t\\ &\quad + \mathrm{i}\left[\int\left(C_1^2 r - r\left(\int\psi\mathrm{d}t\right)^2+\phi\right)\mathrm{d}t + x\int\psi\mathrm{d}t\right],\\ \xi_1^* &= C_1 x - 2C_1\int\!\!\int r\psi \mathrm{d}t\mathrm{d}t\\ &\quad - \mathrm{i}\left[\int\left(C_1^2 r - r\left(\int\psi\mathrm{d}t\right)^2+\phi\right)\mathrm{d}t + x\int\psi\mathrm{d}t\right],\end{aligned}$$

and C_1, α, β are arbitrary constants.

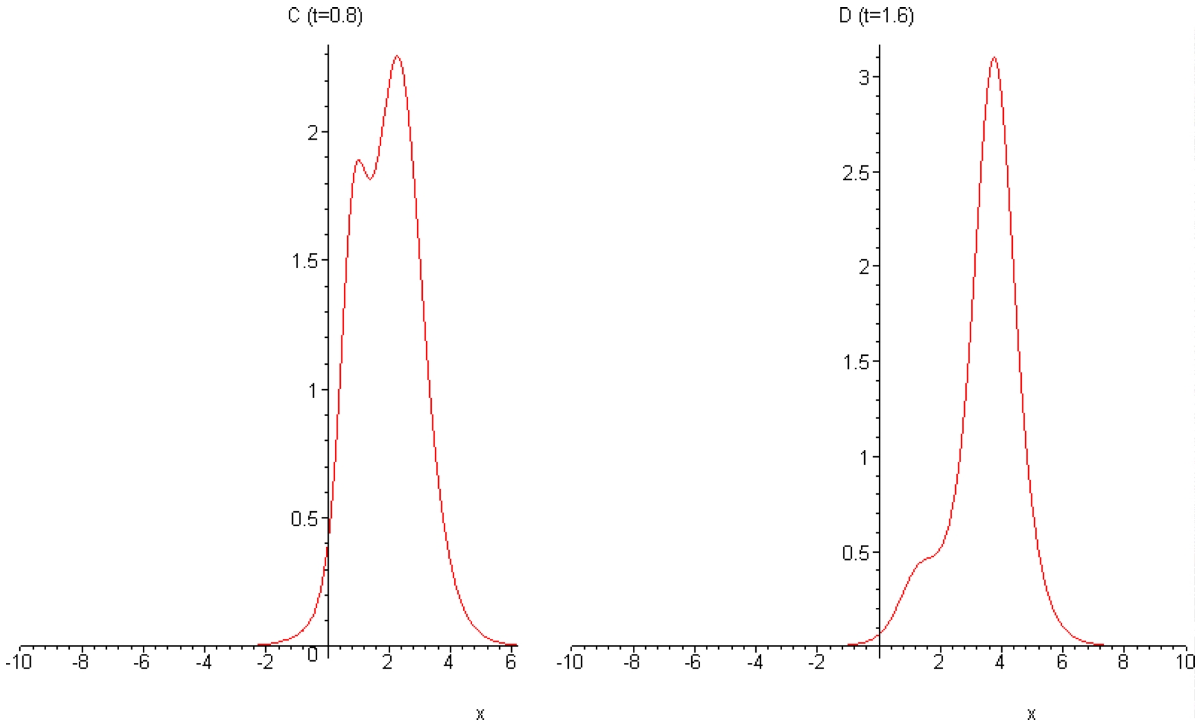

Fig. 3. Evolution plots of the two-soliton solution $|u|^2$ of equation (1) at $t = 0.8$ and $t = 1.6$ with parameters mentioned in the text $(*)$.

Two-soliton solution of (2) is as follows:

$$u = \left[\alpha_1 e^{\xi_1} + \alpha_2 e^{\xi_2} + e^{\xi_1+\xi_1^*+\xi_2+\xi_1^0} + e^{\xi_2+\xi_2^*+\xi_1+\xi_2^0}\right] \cdot \left[1 + e^{\xi_1+\xi_1^*+R_1} + e^{\xi_2+\xi_2^*+R_2} + e^{\xi_1^*+\xi_2+R_3} + e^{\xi_1+\xi_2^*+R_3} + e^{\xi_1+\xi_1^*+\xi_2+\xi_2^*+R_4}\right]^{-1},$$

$$v = \left[\beta_1 e^{\xi_1} + \beta_2 e^{\xi_2} + e^{\xi_1+\xi_1^*+\xi_2+\eta_1^0} + e^{\xi_2+\xi_2^*+\xi_1+\eta_2^0}\right] \cdot \left[1 + e^{\xi_1+\xi_1^*+R_1} + e^{\xi_2+\xi_2^*+R_2} + e^{\xi_1^*+\xi_2+R_3} + e^{\xi_1+\xi_2^*+R_3} + e^{\xi_1+\xi_1^*+\xi_2+\xi_2^*+R_4}\right]^{-1},$$

where

$$\xi_j = C_j x - 2C_j \int\int r\psi \,\mathrm{d}t\,\mathrm{d}t + \mathrm{i}\left[\int\left(C_j^2 r - r\left(\int \psi \,\mathrm{d}t\right)^2 + \phi\right)\mathrm{d}t + x\int \psi \,\mathrm{d}t\right],$$

$$\xi_j^* = C_j x - 2C_j \int\int r\psi \,\mathrm{d}t\,\mathrm{d}t - \mathrm{i}\left[\int\left(C_j^2 r - r\left(\int \psi \,\mathrm{d}t\right)^2 + \phi\right)\mathrm{d}t + x\int \psi \,\mathrm{d}t\right],$$

$$e^{\xi_1^0} = \frac{C_1 - C_2}{4C_1^2 (C_1 + C_2)^2}\left[\alpha_2 C_1 \left(\alpha_1^2 - \beta_1^2\right) - \alpha_2 C_2 \left(\alpha_1^2 + \beta_1^2\right) + 2C_1 \alpha_1 \beta_1 \beta_2\right],$$

$$e^{\xi_2^0} = \frac{C_1 - C_2}{4C_2^2 (C_1 + C_2)^2}\left[\alpha_1 C_1 \left(\alpha_2^2 + \beta_2^2\right) - \alpha_1 C_2 \left(\alpha_2^2 - \beta_2^2\right) - 2\,\alpha_2 \beta_1 \beta_2 C_2\right],$$

$$e^{\eta_1^0} = \frac{C_2 - C_1}{4C_1^2 (C_1 + C_2)^2}\left[C_1 \beta_2 \left(\alpha_1^2 - \beta_1^2\right) + C_2 \beta_2 \left(\alpha_1^2 + \beta_1^2\right) - 2C_1 \alpha_1 \alpha_2 \beta_1\right],$$

$$e^{\eta_2^0} = \frac{C_2 - C_1}{4C_2^2 (C_1 + C_2)^2}\left[-C_1 \beta_1 \left(\alpha_2^2 + \beta_2^2\right) - C_2 \beta_1 \left(\alpha_2^2 - \beta_2^2\right) + 2\,\alpha_1 \alpha_2 \beta_2 C_2\right],$$

$$e^{R_1} = \frac{\alpha_1^2 + \beta_1^2}{4C_1^2}, \quad e^{R_2} = \frac{\alpha_2^2 + \beta_2^2}{4C_2^2},$$

$$e^{R_3} = \frac{\alpha_1 \alpha_2 + \beta_1 \beta_2}{(C_1 + C_2)^2}, \quad j = 1, 2,$$

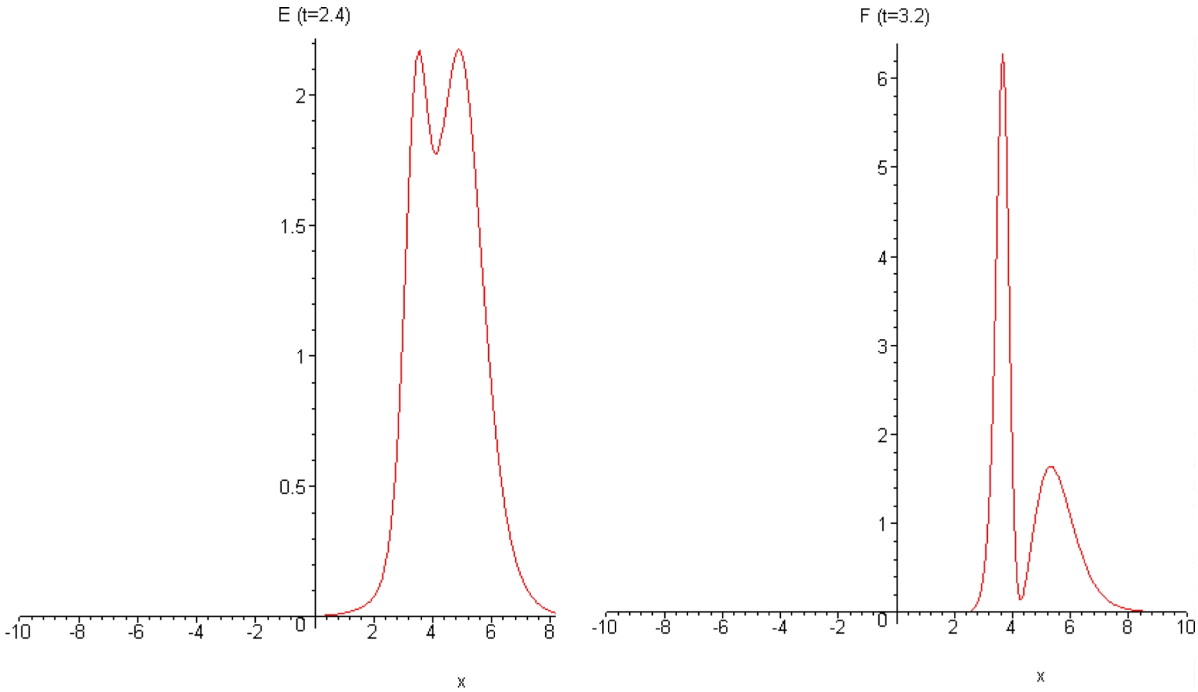

Fig. 4. Evolution plots of the two-soliton solution $|u|^2$ of equation (1) at $t = 2.4$ and $t = 3.2$ with parameters mentioned in the text $(*)$.

$$e^{R_4} = \frac{(C_2 - C_1)^2}{16C_1^2C_2^2(C_1+C_2)^4}\Big[(C_1^2+C_2^2)(\alpha_1^2+\beta_1^2)(\alpha_2^2 + \beta_2^2) - 2C_1C_2\Big((\beta_1\beta_2+\alpha_1\alpha_2)^2 - (\alpha_1\beta_2-\alpha_2\beta_1)^2\Big)\Big],$$

and C_i, α_i, β_i $(i = 1,2)$ are arbitrary real constants.

In this section, using Hirota's method the one- and two-bright soliton solutions of the coupled NLS equations (1) and (2) have been derived successfully. From the obtained solutions one can find that the one-soliton solution of (1) is very different from that of (2) formally. The main difference is the form of the field v. As is shown in (9) and (14), field v of (1) is of the form $v = \beta e^{\xi_1^*}/[1+e^{\xi_1+\xi_1^*+\xi_0}]$, but that of (2) is $v = \beta e^{\xi_1}/[1+e^{\xi_1+\xi_1^*+\xi_0}]$. This is because equation (1) is a focusing-defocusing equation but (2) is a focusing-focusing equation. In addition, by using the same procedures three-soliton and N-soliton solutions of (1) and (2) can be obtained as well.

4. Summary and Discussion

In conclusion, the one- and two-bright soliton solutions of two variable-coefficient coupled nonlinear Schrödinger equations are derived by means of Hirota's method. Some figures are plotted to illustrate the properties of the obtained solutions. The properties are meaningful for the investigation on the stability of soliton propagation in the optical soliton communications. The further questions to equations (1) and (2) are whether they have double Wronskian solutions, how to construct them, and whether they have other types of single soliton solutions. One useful method is the Wronskian technique [15]. It is known that the classical nonlinear Schrödinger equation has a double Wronskian solution. So the author think that equations (1) and (2) may have double Wronskian solutions. In addition, the physical applications of (1)-(2) and their soliton solutions are under investigation in the future.

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