

# Nonlinear Ion Acoustic Waves in a Magnetized Dusty Plasma in the Presence of Nonthermal Electrons

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The Kadomtsev-Petviashvili (KP) equation is derived for weakly nonlinear ion acoustic waves in a magnetized dusty plasma in the presence of nonthermal electrons. Soliton solutions are obtained in both the one-dimensional and two-dimensional framework. For the one-dimensional soliton solution the ‘tanh’ method is considered while the two-dimensional solution is obtained by a method introduced by S. V. Manacov et al., Phys. Lett. A **63**, 205 (1977). It is found that in case of the one-dimensional solution, both compressive and rarefactive solitary waves exist which could be obtained depending on the ratio of the electron and ion density. It is also seen that the nonthermal distribution of electrons has some significant effect in the shape of both the one-dimensional and two-dimensional solitary wave.

*Key words:* Ion Acoustic Wave; Dusty Plasma; Nonthermal Electron; Kadomtsev-Petviashvili Equation.

## 1. Introduction

Plasmas and dust are ubiquitous in the universe. Dust has importance in space plasma, astrophysical plasmas, laboratory plasmas and the environment. The presence of a dusty plasma in cometary tails, asteroid zones, planetary rings, interstellar medium, earth’s ionosphere and magnetosphere makes this subject increasingly important [1–7]. It also plays vital roles in other fields like low-temperature physics, radio frequency plasma discharge [8], coating and etching of thin films [9], plasma crystals [10, 11].

The waves in dusty plasmas were studied in different modes like the dust acoustic (DA) mode [12, 13], dust ion acoustic (DIA) mode [14, 15] dust Bernstein-Greene-Kruskal (DBGK) mode [16], dust lattice (DL) mode [17], Shukla-Varma mode [18], dust-drift mode [19] by many investigators. Dust acoustic waves (DAW) and dust ion acoustic waves (DIAW) were also observed experimentally [20, 21]. Recently electromagnetic modes and electrostatic modes in magnetized dusty plasma were studied [22, 25]. Also a number of theoretical studies on DIA soliton [26, 27], DA soliton [28, 29] and DL soliton [30] were done with low-frequency dust-associated electrostatic and electromagnetic waves.

Recently several authors studied solitary waves in a plasma considering a non-thermal distribution for the electrons applied in space and astrophysical plasmas. Cairns et al. [31] used a nonthermal distribution of electrons to study the ion acoustic solitary structures observed by the FREJA satellite. Singh and Lakhina [32] studied the effect of a nonthermal electron distribution on nonlinear electron acoustic waves in an unmagnetized three-component plasma consisting of nonthermal electrons, cold electrons and ions. They have shown that the inclusion of nonthermal electrons will change the properties as well as the regime of existence of solitons. Sahu and Roychoudhury [33] studied the relativistic effects on electron acoustic solitary waves (EASW) in an unmagnetized three-component plasma consisting of nonthermal hot electrons, cold relativistic electrons and relativistic ions. They have shown also the role of  $\alpha$  (the nonthermal parameter) on the formation of EASW. Mendoza-Briceno et al. [34] considered a hot nonthermal dusty plasma, consisting of fast ions and negatively charged hot dust grains to study arbitrary amplitude DA solitary waves. To study DA solitary waves and double layers El-Labany and El-Taibany [35] also considered nonthermally distributed electrons. The magnetic field was taken along the  $z$ -axis. Recently Choi et al. [36] have studied the nonlinear ion acoustic soli-

tary wave in a magnetized dusty plasma, propagating obliquely to an external magnetic field. Using the Sagdeev pseudopotential technique they found compressive and rarefactive ion acoustic solitary waves as well as kink-type double layer, in addition to conventional hump-type solitary waves. Using the reductive perturbation technique (RPT) Mamun and Shukla [37] have studied linear and nonlinear dusty hydromagnetic waves in a magnetized dust-ion plasma in the framework of the Korteweg de-Vries (KdV) equation. However in most of the earlier mentioned works, nonlinear waves were studied in an one-dimensional geometry. Recently, Duan [38] studied dust acoustic waves in an unmagnetized plasma in two dimensional geometry in the framework of the Kadomtsev-Petviashvili (KP) equation. He compared his results with those obtained by Mamun and Shukla [39] in case of a magnetized dusty plasma and concluded that the magnetized dusty plasma and the unmagnetized dusty plasma are different mainly in two-dimensional long wavelength perturbations.

In the present paper we derive the KP equation for weakly nonlinear ion acoustic waves in a three-component dusty plasma subjected to an external magnetic field. In our model the plasma consists of ions, negatively charged, massive dust grains, and nonthermally distributed electrons. The dust dynamics is not taken into account and the charges of the dust grains are assumed to be constant.

The organization of the paper is as follows. In Section 2 basic equations are written. The KP equation is derived in Section 3. Solutions for the KP equation, results and a discussion are given in Section 4, and Section 5 is kept for conclusions.

## 2. Basic Equations

The basic equations are as follows:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i v_i) = 0, \quad (1)$$

$$\frac{\partial v_i}{\partial t} + (v_i \cdot \nabla) v_i = -\frac{e \nabla \phi}{m_i} + \frac{e B_0}{m_i c} v_i \times e_z, \quad (2)$$

$$\nabla^2 \phi = -4\pi [-en_e + en_i - ez_d n_d], \quad (3)$$

where  $n_e$ ,  $n_i$ , and  $n_d$  are the densities of electrons, ions and dust, respectively,  $v_i$  and  $m_i$  are the velocity and mass of ions, and  $\phi$  is the plasma potential.  $z_d$  is the dust charge number, so that the charge of the

dust is given by  $q_d = -ez_d$ , where  $e$  is the elementary charge. As electrons are assumed to be nonthermally distributed, to model the electron distribution with a population of fast particles, we choose the distribution function after Cairns *et al.* [31]

$$f_{0h}(v) = \frac{n_{0h} \left(1 + \frac{\alpha v^4}{v_{th}^4}\right)}{\sqrt{2\pi v_{th}^2 (1 + 3\alpha)}} e^{-\frac{v^2}{2v_{th}^2}},$$

where  $n_{0h}$  is the hot electron density,  $v_{th}$  is the thermal speed of the hot electrons, and  $\alpha$  is a parameter that determines the population of energetic nonthermal electrons.  $\alpha$  essentially measures the deviation of  $f_{0h}(v)$  given in the above equation from the Maxwellian case.

The density of electrons is given by

$$n_e = n_{e0} \left[ (1 - \beta_1 \phi + \beta_1 \phi^2) \exp\left(\frac{e\phi}{T_e}\right) \right]. \quad (4)$$

We assume that the wave is propagating in the  $xz$ -plane. After normalization the system reduces to

$$\frac{\partial n}{\partial t} + \frac{\partial (nv_x)}{\partial x} + \frac{\partial (nv_z)}{\partial z} = 0, \quad (5)$$

$$\frac{\partial v_x}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_x = -\frac{\partial \phi}{\partial x} + v_y, \quad (6)$$

$$\frac{\partial v_y}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_y = -v_x, \quad (7)$$

$$\frac{\partial v_z}{\partial t} + \left( v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z} \right) v_z = -\frac{\partial \phi}{\partial z}, \quad (8)$$

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \phi = \beta [(1 - \beta_1 \phi + \beta_1 \phi^2) e^\phi - \delta_1 n + \delta_2], \quad (9)$$

where  $\beta = \frac{r_g^2}{\lambda_e^2}$ ,  $\delta_1 = \frac{n_{i0}}{n_{e0}}$ ,  $\delta_2 = \frac{n_d z_d}{n_{e0}}$ , and  $r_g = \frac{C_s}{\Omega}$  is the ion gyroradius, and  $\lambda_e = \left( \frac{T_e}{4\pi n_{e0} e^2} \right)^{1/2}$  is the electron Debye length. The normalizations are as follows:  $\Omega t \rightarrow t$ ,  $(C_s/\Omega) \nabla \rightarrow \nabla$ ,  $v_i/C_s \rightarrow v$ ,  $n_i/n_{i0} \rightarrow n$ ,  $e\phi/T_e \rightarrow \phi$ , where  $C_s = (T_e/m_i)^{1/2}$  is the ion acoustic velocity,  $\Omega = \frac{eB_0}{m_i c}$  is the ion gyrofrequency.  $n_{e0}, n_{i0}$  are the electron and ion densities, respectively, in the unperturbed state. To obtain the dispersion relation for low-frequency waves we write the dependent variables as a sum of equilibrium and perturbed parts. Writing

$n = 1 + \bar{n}$ ,  $v_x = \bar{v}_x$ ,  $v_z = \bar{v}_z$ ,  $v_y = \bar{v}_y$ ,  $\phi = \bar{\phi}$ , (5)–(9) can be written as

$$\frac{\partial \bar{n}}{\partial t} + \frac{\partial \bar{v}_x}{\partial x} + \frac{\partial \bar{v}_z}{\partial z} = 0, \tag{10}$$

$$\frac{\partial \bar{v}_x}{\partial t} = -\frac{\partial \bar{\phi}}{\partial x} + \bar{v}_y, \tag{11}$$

$$\frac{\partial \bar{v}_y}{\partial t} = -\bar{v}_x, \tag{12}$$

$$\frac{\partial \bar{v}_z}{\partial t} = -\frac{\partial \bar{\phi}}{\partial z}, \tag{13}$$

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \bar{\phi} = \beta [\beta_2 \bar{\phi} - \delta_1 \bar{n}], \tag{14}$$

where  $\beta_2 = (1 - \beta_1)$ . We assume that the perturbation is of the form  $e^{i(k_x x + k_z z - \omega t)}$ , where  $k_x$  and  $k_z$  are the wave numbers in  $x$  and  $z$  directions, respectively, and  $\omega$  is the ion acoustic wave frequency ( $\omega \ll \Omega$ ) obtained as

$$\omega = k_z \left[ \frac{\beta_2}{\delta_1} + \left( 1 + \frac{1}{\beta \delta_1} \right) k_x^2 + \left( 1 + \frac{1}{\beta \delta_1} \right) k_z^2 \right]^{-1/2}. \tag{15}$$

### 3. The KP Equation

To obtain the KP equation we use the reductive perturbation technique. The stretching of the independent variables is as follows [38]:

$$X = \varepsilon^2 x, \tag{16}$$

$$\xi = \varepsilon(z - Vt), \tag{17}$$

$$\tau = \varepsilon^3 t, \tag{18}$$

where  $V$  is the phase velocity of the ion acoustic wave, and  $\varepsilon$  is a small parameter measuring the strength of the nonlinearity. The dependent variables are expanded as

$$n = 1 + \varepsilon^2 n_1 + \varepsilon^4 n_2 + \dots, \tag{19}$$

$$v_x = \varepsilon^3 v_{x1} + \varepsilon^5 v_{x2} + \dots, \tag{20}$$

$$v_y = \varepsilon^3 v_{y1} + \varepsilon^5 v_{y2} + \dots, \tag{21}$$

$$v_z = \varepsilon^2 v_{z1} + \varepsilon^4 v_{z2} + \dots, \tag{22}$$

$$\phi = \varepsilon^2 \phi_1 + \varepsilon^4 \phi_2 + \dots. \tag{23}$$

Substituting the expansions in (5)–(9) and equating the coefficients of different powers of  $\varepsilon$ , we get

$$n_1 = \frac{1}{V} v_{z1}, \tag{24}$$

$$\phi_1 = \frac{\delta_1}{1 - \beta_1} n_1, \tag{25}$$

$$v_{y1} = 0 = v_{y2}, \tag{26}$$

$$v_{x1} = 0 = v_{x2}, \tag{27}$$

$$\frac{\partial v_{y1}}{\partial \xi} = 0, \tag{28}$$

$$V \frac{\partial v_{x1}}{\partial \xi} = \frac{\partial \phi_1}{\partial X}, \tag{29}$$

$$v_{z1} = \frac{1}{V} \phi_1, \tag{30}$$

$$\frac{\partial n_1}{\partial \tau} - V \frac{\partial n_2}{\partial \xi} + \frac{\partial v_{x1}}{\partial X} + \frac{\partial v_{z2}}{\partial \xi} + \frac{\partial(n_1 v_{z1})}{\partial \xi} = 0, \tag{31}$$

$$\frac{\partial v_{z1}}{\partial \tau} - V \frac{\partial v_{z2}}{\partial \xi} + v_{z1} \frac{\partial v_{z1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial \xi}, \tag{32}$$

$$\frac{\partial v_{x1}}{\partial \tau} - V \frac{\partial v_{x2}}{\partial \xi} + v_{z1} \frac{\partial v_{x1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial X}, \tag{33}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \beta [-\beta_1 \phi_2 + \phi_2 + \phi_1^2/2 - \delta_1 n_2]. \tag{34}$$

From (24), (25) and (30), we get

$$V^2 = \frac{\delta_1}{1 - \beta_1}. \tag{35}$$

From the relations (24)–(34), we obtain the KP equation as

$$\frac{\partial \left[ \frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} \right]}{\partial \xi} + C \frac{\partial^2 \phi_1}{\partial X^2} = 0, \tag{36}$$

where

$$A = \frac{-V^2 + 3 - 3\beta_1}{2V(1 - \beta_1)}, \quad B = \frac{V}{2\beta(1 - \beta_1)}, \quad C = \frac{V}{2}. \tag{37}$$

## 4. Results and Discussion

### 4.1. Traditional Solution by the ‘tanh’ Method

To get a travelling solitary wave solution let us

define the variable

$$\chi = \alpha(l\xi + mX - U\tau),$$

where  $l$  and  $m$  are the direction cosines of the angles made by the wave propagation with the  $z$ -axis and  $x$ -axis, respectively,  $U$  is the velocity of the wave, and  $\alpha$  is a constant. Considering  $\psi(X) = \phi_1(\xi, X, \tau)$ , (36) reduces to

$$B\alpha^2 l^4 \frac{d^2 \psi}{d\chi^2} + (Cm^2 - lU)\psi + \frac{Al^2 \psi^2}{2} = 0. \quad (38)$$

Now substituting  $Y = \tanh(\chi)$ , (38) transforms to

$$B\alpha^2 l^4 (1 - Y^2)^2 \frac{d^2 S}{dY^2} - 2B\alpha^2 l^4 Y(1 - Y^2) \frac{dS}{dY} + (Cm^2 - lU)S + \frac{Al^2}{2} S^2 = 0, \quad (39)$$

where  $S(Y) = \psi(\chi)$ .

Now (39) can be solved by using the so-called ‘tanh’ method. The solitary wave solution of (39) is given by

$$S(Y) = S_0 \operatorname{sech}^2 \left( \frac{\chi'}{\alpha_1} \right), \quad (40)$$

where  $\chi' = l\xi + mX - U\tau$ ,  $S_0 = \frac{12B\alpha^2 l^2}{A}$  is the amplitude of the solitary waves, and  $\alpha_1 = 2 \left( \frac{Bl^4}{lU - Cm^2} \right)^{1/2}$  is the width of the solitary waves. Putting the values of  $B$  and  $A$  in the expression of  $S_0$  we get  $S_0 = \frac{12V^2 \alpha^2 l^2}{\beta(-V^2 + 3 - 3\beta_1)}$ .

It is seen from the expression of  $S_0$  that, if  $V^2 > 3 - 3\beta_1$ , the value of  $S_0$  is negative, and hence the solitary wave is rarefactive. Again if  $V^2 < 3 - 3\beta_1$ , then the value of  $S_0$  is positive, and so the solitary wave corresponds to a compressive solitary wave. If  $V^2 = 3 - 3\beta_1$ , the value of  $S_0$  is infinite, and so no soliton solution exists. Similarly, putting the values of  $B$  and  $C$  in the expression of  $\alpha_1$ , we get  $\alpha_1 =$

$$2 \left( \frac{l^4 \sqrt{\delta_1}}{2\beta(1-\beta_1) \left[ lU(\sqrt{1-\beta_1}) - \frac{m^2 \sqrt{\delta_1}}{2} \right]} \right)^{1/2}.$$

It is seen from the expression of  $\alpha_1$  that, if  $\beta_1 > 1$ , then the width of the solitary wave will become complex, and so  $\beta_1$  should always be less than 1.

From the expressions of  $S_0$  and  $\alpha_1$  it is seen, that the amplitude and width of the solitary wave depend on  $\beta_1$ , the nonthermal parameter, on  $l$ , the direction cosines

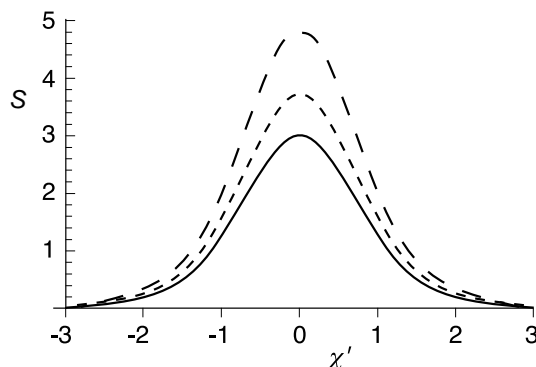


Fig. 1. Plot of  $S(Y)$  vs.  $\chi'$  for different values of  $\beta_1$ , viz.  $\beta_1 = 0$  (solid line), 0.001 (dotted line), 0.1 (dashed line). The other parameters are  $\delta_1 = 1.5$ ,  $\alpha = 0.5$ ,  $\beta = 1$ .

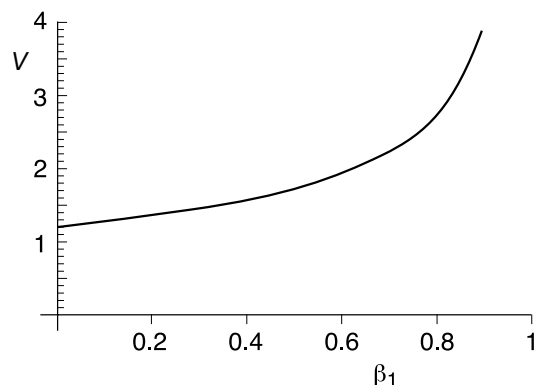


Fig. 2. Plot of  $V$ , the soliton velocity, vs.  $\beta_1$ . The other parameters are the same as in Figure 1.

of the angles made by the wave propagation with the  $z$ -axis, on  $\delta_1$ , the ratio of the initial electron density to the initial ion density, and on the angle of the wave propagation to the direction of the magnetic field. In spite of nonthermal distribution of electrons and the magnetic field both play important roles in describing the behaviour of nonlinear waves. For  $v^2 = 3 - 3\beta_1$ ,  $S_0$  is infinite and, hence, the soliton solution ceases to exist. Then one can obtain a modified KP equation whose solution has been discussed in some detail in [40] for a simplified model.

To see the effect of  $\beta_1$  on the speed and shape of the solitary wave Fig. 1 is drawn.  $S$  is plotted vs.  $\chi'$  for different values of  $\beta_1$ , viz.  $\beta_1 = 0$  (solid line), 0.001 (dotted line), 0.1 (dashed line). The other parameters are  $\delta_1 = 1.5$ ,  $\alpha = 0.5$ ,  $\beta = 1$ . From the figure it is seen that  $\beta_1$  has a significant effect on the width and amplitude of the solitary waves.

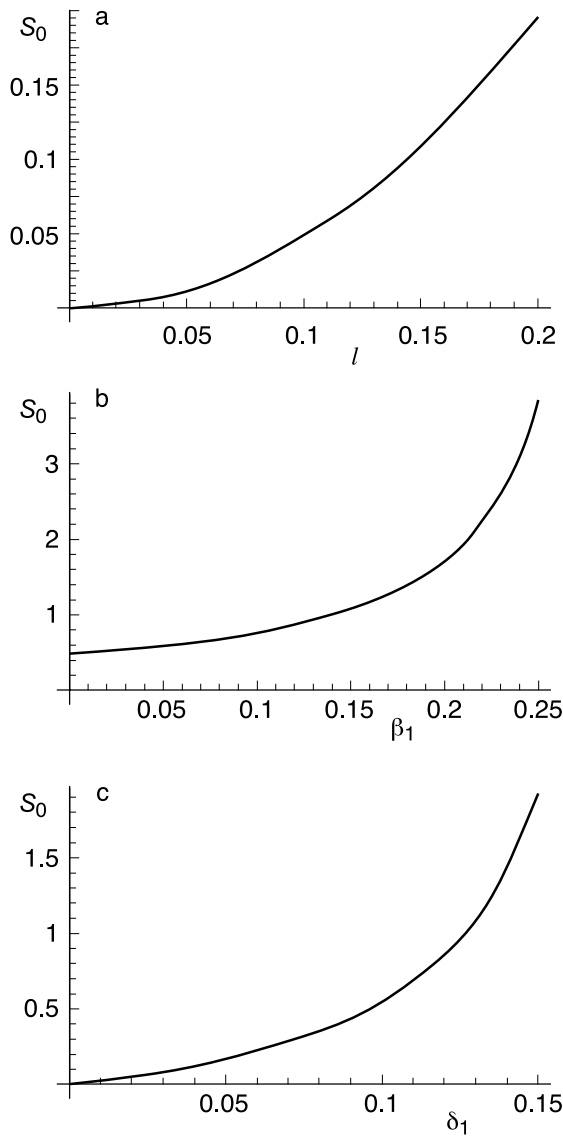


Fig. 3. (a)  $S_0$  plotted against  $l$ . The other parameters are  $\alpha = 0.5$ ,  $\beta = 1$ ,  $\beta_1 = 0.1$ , and  $\delta_1 = 1.5$ . (b)  $S_0$  plotted against  $\beta_1$ .  $l = 0.4$  and the other parameters are the same as in Figure 3a. (c)  $S_0$  plotted against  $\delta_1$ .  $\beta_1 = 0.751$ ,  $l = 0.4$  and the other parameters are the same as in Figure 3a.

Figure 2 shows the the plot of  $V$ , the soliton velocity, vs.  $\beta_1$ . The other parameters are the same as those in Figure 1. From the figure it is seen that the nonthermal distribution of electrons has a significant effect on the speed of the solitary wave.

To see the effect of  $l$ ,  $\beta_1$  and  $\delta_1$  on the amplitude of the solitary waves Figs. 3a, b and c are drawn. In

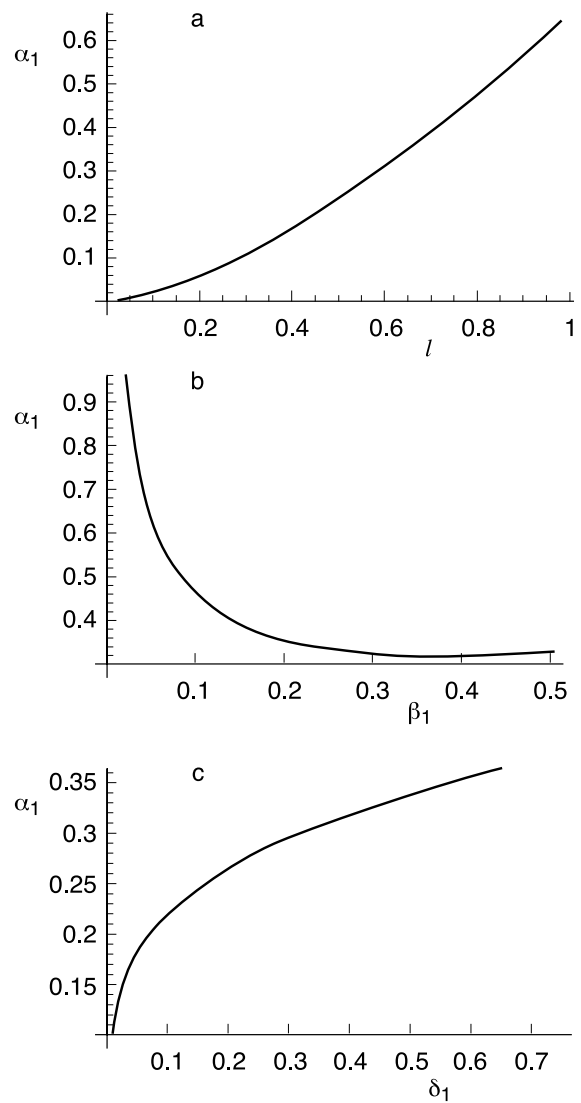


Fig. 4. (a)  $\alpha_1$  plotted against  $l$ . The other parameters are  $U = 1$ ,  $\beta = 1$ ,  $\beta_1 = 0.1$ , and  $\delta_1 = 1.5$ . (b)  $\alpha_1$  plotted against  $\beta_1$ . The other parameters are  $U = 1$ ,  $\beta = 1$ ,  $l = 0.4$ , and  $\delta_1 = 1.5$ . (c)  $\alpha$  plotted against  $\delta$ . The other parameters are  $U = 1$ ,  $\beta = 1$ ,  $l = 0.4$ ,  $\beta_1 = 0.1$ .

Fig. 3a  $S_0$  is plotted against  $l$ , the other parameters are  $\alpha = 0.5$ ,  $\beta = 1$ ,  $\beta_1 = 0.1$ , and  $\delta_1 = 1.5$ . From this figure it is seen that the amplitude of the solitary wave increases with the increase of  $\beta_1$ . In Fig. 3b  $S_0$  is plotted against  $\beta_1$  for  $l = 0.4$ . The other parameters are the same as those in Fig. 3a. Here also  $S_0$  increases as  $\beta_1$  increases. In Fig. 3c  $S_0$  is plotted against  $\delta_1$  for  $\beta_1 = 0.751$  and  $l = 0.4$ . The other parameters are the same as those in Figure 3a. Here

it is seen that  $S_0$  also increases with the increase of  $\delta_1$ .

From Figs. 3a–c it is seen that  $l$ ,  $\beta_1$  and  $\delta_1$  have significant effects on the amplitude of the solitary waves.

To show the effect of  $l$ ,  $\beta_1$  and  $\delta_1$  on the width of the solitary waves Figs. 4a, b and c are drawn. In Fig. 4a  $\alpha_1$  is plotted against  $l$ . The other parameters are  $U = 1$ ,  $\beta = 1$ ,  $\beta_1 = 0.1$ , and  $\delta_1 = 1.5$ . From this figure it is seen that the width of the solitary waves increases as  $l$  increases. In Fig. 4b  $\alpha_1$  is plotted against  $\beta_1$ , the other parameters are  $U = 1$ ,  $\beta = 1$ ,  $l = 0.4$ , and  $\delta_1 = 1.5$ . Here it is seen that  $\alpha_1$  decreases with the increase of  $\beta_1$ . In Fig. 4c  $\alpha_1$  is plotted against  $\delta_1$ , the other parameters are  $U = 1$ ,  $\beta = 1$ ,  $l = 0.4$ ,  $\beta_1 = 0.1$  and  $\delta_1 = 1.5$ . From this figure it is seen that the width of the solitary waves increases as  $\delta_1$  increases.

From Figs. 4a–c it is seen that  $l$ ,  $\beta_1$  and  $\delta_1$  have significant effects on the width of the solitary waves.

#### 4.2. New Two-Dimensional Soliton

To obtain the two-dimensional soliton solution we follow the method by Manacov *et al.* [41]. We transform the variables as follows:

$$\phi_1 = \frac{2V_1}{A} f(\zeta, \rho), \tag{41}$$

$$\text{where } V_1 \ll V \text{ and } \rho = \frac{V_1}{\sqrt{BC}} X, \tag{42}$$

$$\zeta = \sqrt{\frac{V_1}{B}} (\xi - V_1 \tau). \tag{43}$$

Using the transformations given by (41)–(43), (36) reduces to

$$\frac{\partial^2 f^2}{\partial \zeta^2} = \frac{\partial^2 f}{\partial \zeta^2} - \frac{\partial^2 f}{\partial \rho^2} - \frac{\partial^4 f}{\partial \zeta^4}. \tag{44}$$

Equation (44) is very similar to the one obtained by Petviashvili [42]. Exploiting the result obtained in [41], we obtain a two-dimensional soliton solution of (44) given by

$$f = 12(3 - \rho^2 - \zeta^2)(3 - \rho^2 + \zeta^2)^{-2}. \tag{45}$$

Figure 5 shows the two-dimensional soliton solution of  $f$  vs. the scaled variables  $\rho$  and  $\zeta$  where  $-15 < \rho < 15$  and  $-6 < \zeta < 6$ .

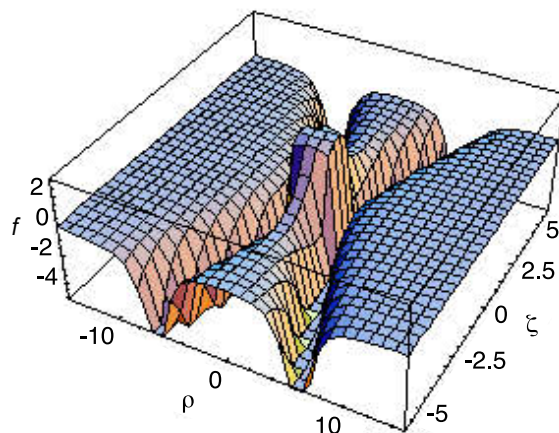


Fig. 5. Two-dimensional soliton solution of  $f$  plotted against the scaled variables  $\rho$  and  $\zeta$ .

#### 5. Conclusions

We have investigated nonlinear ion acoustic waves in a magnetized dusty plasma in the presence of non-thermal electrons. Using the RPT the KP equation was derived. Both one- and two-dimensional soliton solutions were studied. The one-dimensional soliton solution was obtained using the tanh method and the two-dimensional soliton solution was obtained using the technique derived in [41]. From the one-dimensional soliton solution, it was shown that the amplitude and width of the soliton solution depends on  $\beta_1$ ,  $\delta_1$ ,  $l$ . It was also shown that there exist conditions  $V^2 > \leq 3 - 3\beta_1$  on which the one-dimensional soliton solution exists or not. The effect of all these parameters on the amplitude and width of the solitary waves were discussed extensively. It was seen that in case of the one-dimensional soliton, if  $n_{i0}/n_{e0}$ , the ratio of equilibrium ion and electron density, and  $\beta_1$  satisfy the relation  $v^2 > 3 - 3\beta_1$ , then the soliton is stable. The two-dimensional solution was also obtained in [41]. It was also seen that a nonthermal distribution of electron has also a significant effect on solitary waves in plasmas.

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- [1] M. Horanyi and D. A. Mendis, *J. Astrophys.* **294**, 357 (1985).
- [2] M. Horanyi and D. A. Mendis, *J. Astrophys.* **307**, 800 (1986).
- [3] C. K. Goertz, *Rev. Geophys.* **27**, 271 (1989).
- [4] T. G. Northrop, *Phys. Scr.* **75**, 475 (1992).
- [5] D. A. Mendis and M. Rosenberg, *IEEE Trans. Plasma Sci.* **20**, 929 (1992).
- [6] D. A. Mendis and M. Rosenberg, *Ann. Rev. Astron. Astrophys.* **32**, 419 (1994).
- [7] F. Verheest, *Space Sci. Rev.* **77**, 267 (1996).
- [8] J. H. Chu, J. B. Du, and I. Lin, *J. Phys. D* **27**, 296 (1994).
- [9] G. S. Selwyn, *Jpn. J. Appl. Phys. Part 1* **32**, 3068 (1993).
- [10] H. Thomas, G. E. Morfill, and V. Dammell, *Phys. Rev. Lett.* **73**, 652 (1994).
- [11] Y. Hayashi and K. Tachibana, *Jpn. J. Appl. Phys. Part 2* **33**, L804 (1994).
- [12] N. N. Rao, P. K. Shukla, and M. Y. Yu, *Planet Space Sci.* **38**, 543 (1990).
- [13] A. A. Mamun, *Astrophys. Space Sci.* **268**, 443 (1999).
- [14] P. K. Shukla and V. P. Slin, *Phys. Scr.* **45**, 508 (1992).
- [15] R. L. Merlino, A. Barkan, C. Thompson, and N. D'Angelo, *Phys. Plasmas* **5**, 1607 (1998).
- [16] M. Tribeche and T. H. Zerguini, *Phys. Plasmas* **11**, 4115 (2004).
- [17] F. Melandso, *Phys. Plasmas* **3**, 3890 (1996).
- [18] P. K. Shukla and R. K. Varma, *Phys. Fluids B* **5**, 236 (1993).
- [19] P. K. Shukla, M. Y. Yu, and R. Bharuthram, *J. Geophys. Res.* **96**, 21343 (1992).
- [20] A. Barkan, R. L. Merlino, and N. D'Angelo, *Phys. Plasmas* **2**, 3563 (1997).
- [21] Y. Nakamura and A. Sarma, *Phys. Plasmas* **8**, 3921 (2001).
- [22] P. K. Shukla and V. P. Slin, *Phys. Scr.* **45**, 508 (1992).
- [23] P. Meuris and F. X. Verheest, *Phys. Lett. A* **219**, 299 (1992).
- [24] M. Mahmood, M. Ansar, M. Arshad, P. H. Sakanaka, and G. X. Murtaza, *Phys. Plasmas* **9**, 3794, (1994).
- [25] O. A. Pokhotelov, O. G. Onishchenko, P. K. Shukla, and I. Stenflo, *Geophys. Res.* **104**, 19797 (1999).
- [26] A. A. Mamun, *Phys. Ser.* **58**, 505 (1998).
- [27] A. A. Mamun and P. K. Shukla, *Phys. Plasmas* **10**, 4341 (2003).
- [28] A. A. Mamun, R. A. Cairns, and P. K. Shukla, *Phys. Plasmas* **3**, 2610 (1996).
- [29] R. Roychoudhury and S. Maitra, *Phys. Plasmas* **9**, 4160 (2002).
- [30] B. Farokhi, P. K. Shukla, N. L. Tsintsadze, and D. D. Tskhakaya, *Phys. Lett. A* **264**, 318 (1999).
- [31] R. A. Cairns, A. A. Mamun, R. Bingham, R. Bostrom, R. O. Dendy, C. M. C. Nairn, and P. K. Shukla, *Geophys. Res. Lett.* **22**, 2709 (1995).
- [32] S. V. Singh and G. S. Lakhina, *Nonlinear Process Geophys.* **11**, 275 (2004).
- [33] B. Sahu and R. Roychoudhury, *Phys. Plasmas* **13**, 072302 (2006).
- [34] C. A. Mendoza-Briceno, S. M. Russel, and A. A. Mamun, *Planet Space Sci.* **48**, 599 (2000).
- [35] S. K. El-Labany and W. F. El-Taibany, *Phys. Plasmas* **11**, 926, (2003).
- [36] C. R. Choi, C. M. Ryu, N. C. Lee, and D. Y. Lee, *Phys. Plasmas* **12**, 022304 (2005).
- [37] A. A. Mamun and P. K. Shukla, *Phys. Plasmas* **9**, 268 (2002).
- [38] W. S. Duan, *Phys. Plasmas* **8**, 3583 (2001).
- [39] A. A. Mamun and P. K. Shukla, *Phys. Scr.* **107**, T98 (2002).
- [40] G. C. Das, J. Sarma, and C. Oberoj, *Phys. Plasmas* **4**, 2095, (1997).
- [41] S. V. Manakov, V. E. Zakharov, I. A. Bordag, A. R. Ths, and V. B. Mateev, *Phys. Lett.* **63A**, 205 (1977).
- [42] B. B. Kadomastev, *Plasma Physics*, Mir Publishers, Moscow, p. 126, 1981.