On Diffusion in Some Biological and Economic Systems

E. Ahmed[a,b] and S. Z. Hassan[b]

[a] Mathematics Department, Faculty of Science, Al-Ain, P.O.Box 17551, UAE
[b] Mathematics Department, Faculty of Science, Mansoura 35516, Egypt

Reprint requests to E. A.; E-mail: magd45@yahoo.com

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It is argued that diffusion in biological and economical systems is better modelled by Cattaneo's equation where memory effects are included. Reaction diffusion equations using Cattaneo's system are derived for prisoner's dilemma (PD) and hawk-dove (HD) games. Nonlinear wave solutions are derived for them. As expected the asymptotic solution for the PD case is insufficient. Hence a cellular automata motivated by Cattaneo's system is used to show the existence of cooperation in the case of local game.

Key words: Cattaneo System; Cellular Automata; Non-linear Waves.

1. Cattaneo's Diffusion

The standard diffusion equation depends on the continuity equation and Fick's law

$$j(x,t) = -D\partial c(x,t)/\partial x, \tag{1}$$

where j is the diffusing object (e. g. technology, concept, etc...), c is the distribution function of this object

• *Consider equations as parts of sentences!*

and D is the diffusion constant. The resulting standard diffusion equation is

$$\partial c(x,t)/\partial t = D\partial^2 c(x,t)/\partial x^2. \tag{2}$$

A basic weakness of this equation is that the flux j reacts simultaneously to the gradient of c and consequently an unbounded propagation speed is assumed. This manifests itself in many solutions to (2) e. g.

• *Write (2) and not eqn. (2)!*

$c(x,t) = 1/\sqrt{4\pi Dt}\exp(-x^2/4Dt)$, $c(x,0) = \delta(x)$, i. e. $c(x,t) > 0\,\forall x, \forall t > 0$. This is unrealistic specially in biological and economical systems where it is known that propagation speeds are typically small. To rectify this weakness Cattaneo [1] replaced Fick's law (1) by

• *Write [1] and not ref. [1]!*

[1] G. Cattaneo, Atti. Sem. Mat. Fis. Univ. Modena **3,** 83 (1948).
[2] K. P. Hadeler, in O. Diekman et al. (eds), Mathematics inspired by Biology, Springer, Berlin 1998.
[3] A. Compte and R. Metzler, J. Phys. A **30,** 7277 (1997).

• *Write J. Phys. A* **30,** *and not J. Phys. A,* **30,***!*

[4] J. Hofbauer and K. Sigmund, Evolutionary Games and Population Dynamics, Cambridge U. P., Cambridge 1998.
[5] J. Maynard Smith, Evolution and the Theory of Games, Cambridge U. P., Cambridge 1982.
[6] M. A. Nowak and R. M. May, Int. J. Bifur. and Chaos **3,** 35 (1993).