Group Invariant Solution for a Liquid Film on the Surface of a Sphere

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When a circular jet of liquid strikes a sphere normal to the tangent plane at a point, a liquid film is formed on the surface of the sphere. This is a new problem. The flow in the liquid film is studied by means of boundary layer theory for laminar flow. The governing equations are Prandtl's momentum boundary layer equation and the continuity equation. To solve the problem completely a conserved quantity is required as well as boundary conditions. The conserved quantity for the film of liquid on the sphere is derived with the help of a conserved vector. Two conservation laws for the system have been obtained by Naeem and Naz (Int. J. Nonlin. Sci. **7**, 149 (2009)), and one of these is used to derive the conserved quantity for the liquid film. A stream function is introduced which reduces the system to a single third-order partial differential equation. The group invariant solution for this partial differential equation is constructed by considering a linear combination of its Lie point symmetries. The velocity profile of the liquid film on the sphere is investigated.

Key words: Liquid Film; Blasius Boundary Layer; Group Invariant; Solution; Conserved Quantity; Free Surface.