

Piecewise-Truncated Parametric Iteration Method: a Promising Analytical Method for Solving Abel Differential Equations

Jajar Saberi-Nadjafi and Asghar Ghorbani

Department of Applied Mathematics, School of Mathematical Sciences,
Ferdowsi University of Mashhad, Mashhad, Iran

Reprint requests to S. N.; Fax: (+ 98) 511 8828606; E-mail: najafi141@gmail.com or
A. G.; E-mail: as_gh56@yahoo.com

Z. Naturforsch. **65a**, 529 – 539 (2010); received March 31, 2009 / revised October 26, 2009

This paper deals with the analytical approximate solution of Abel differential equations (ADEs) of the type $du/dt = \sum_{k=0}^m g_k(t)u^k$ by proposing a new modified version of the parametric iteration method (PIM). The modified algorithm analytically approximates the solution of ADEs in a sequence of subintervals which is continuous everywhere. The local convergence and the stability of the algorithm are discussed in details. Also we show how to characterize the stability function and the region on which the algorithm is presented. Some examples are given to demonstrate the efficiency and accuracy of this algorithm. Comparison with numerical Runge-Kutta methods (RK) shows that the modified algorithm presented in this paper has the advantage of giving an analytical form of the solution, which is not possible in the purely numerical RK techniques. Moreover, the approximations obtained by the new algorithm converge faster than the numerical RK4 methods, as will be shown. The obtained results reveal that the present algorithm is a promising iterative analytical method for solving ADEs. Furthermore, the proposed algorithm provides us with an easy way to modify the rate and region convergence of the solution. Most promising, however, it seems that the newly developed technique can be further implemented easily to solve other nonlinear ordinary differential equations (ODEs) of physical interests.

Key words: Piecewise-Truncated Parametric Iteration Method; Truncated Parametric Iteration Method; Parametric Iteration Method; Abel Differential Equations; Runge-Kutta Methods

PACS numbers: 02.30.Hq, 02.60.Lj