

Symmetry Reductions and Exact Solutions of the (2+1)-Dimensional Navier-Stokes Equations

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By means of the classical symmetry method, we investigate the (2+1)-dimensional Navier-Stokes equations. The symmetry group of Navier-Stokes equations is studied and its corresponding group invariant solutions are constructed. Ignoring the discussion of the infinite-dimensional subalgebra, we construct an optimal system of one-dimensional group invariant solutions. Furthermore, using the associated vector fields of the obtained symmetry, we give out the reductions by one-dimensional and two-dimensional subalgebras, and some explicit solutions of Navier-Stokes equations are obtained. For three interesting solutions, the figures are given out to show their properties: the solution of stationary wave of fluid (real part) appears as a balance between fluid advection (nonlinear term) and friction parameterized as a horizontal harmonic diffusion of momentum.

Key words: Navier-Stokes Equations; Classical Lie Symmetry Method; Optimal System; Explicit Solution.