

# Resonances of a Nonlinear Single-Degree-of-Freedom System with Time Delay in Linear Feedback Control

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The primary and subharmonic resonances of a nonlinear single-degree-of-freedom system under feedback control with a time delay are studied by means of an asymptotic perturbation technique. Both external (forcing) and parametric excitations are included. By means of the averaging method and multiple scales method, two slow-flow equations for the amplitude and phase of the primary and subharmonic resonances and all other parameters are obtained. The steady state (fixed points) corresponding to a periodic motion of the starting system is investigated and frequency-response curves are shown. The stability of the fixed points is examined using the variational method. The effect of the feedback gains, the time-delay, the coefficient of cubic term, and the coefficients of external and parametric excitations on the steady-state responses are investigated and the results are presented as plots of the steady-state response amplitude versus the detuning parameter. The results obtained by two methods are in excellent agreement.

*Key words:* Nonlinear Single-Degree-of-Freedom; Resonances; Time Delay; Perturbation Methods; Fixed Points; Stability.