Numerical Solution of a Boussinesq Type Equation Using Fourier Spectral Methods

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Efficient numerical methods for solving nonlinear wave equations and studying the propagation and stability properties of their solitary waves (solitons) are applied to a Boussinesq type equation in one space dimension. These methods use a pseudospectral (Fourier transform) treatment of the space dependence, together with (a) finite differences, or (b) a fourth-order Runge-Kutta scheme (RK4), for the time evolution. Our schemes follow very accurately single solitons, which are given by simple closed formulas and are known to be stable for all allowed velocities. However, as a parameter of the problem tends to the critical value $b = 0.5$, where the velocity of the exact soliton vanishes, our solutions destabilize due to numerical errors, producing two small solitons in the place of the exact one. On the other hand, when we study the interaction of two such solitons, starting far apart from each other, we find in the b_1, b_2 parameter plane a curve beyond which the solution becomes unstable by exponential blow-up of the amplitudes, independently of our space and time discretization. We claim that this is due to a dynamical resonance rather than the accumulation of numerical errors, in agreement with theoretical predictions. Our implementation relies on the fast Fourier transform (FFT) algorithm and no major differences are observed, when either scheme (a) or (b) is used for the evolution of time.

Key words: Fourier Spectral Method; Fast Fourier Transform; Boussinesq Equation; Nonlinear Waves.