A Direct Transformation Method and its Application to Variable Coefficient Nonlinear Equations of Schrödinger Type

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In this paper, the generalized variable coefficient nonlinear Schrödinger (NLS) equation and the cubic-quintic nonlinear Schrödinger (CQNLS) equation with variable coefficients are directly reduced to simple and solvable ordinary differential equations by means of a direct transformation method. Taking advantage of the known solutions of the obtained ordinary differential equations, families of exact nontravelling wave solutions for the two equations have been constructed. The characteristic feature of the direct transformation method is, that without much extra effort, we circumvent the integration by directly reducing the variable coefficient nonlinear evolution equations to the known ordinary differential equations. Another advantage of the method is that it is independent of the integrability of the given nonlinear equation. The method used here can be applied to reduce other variable coefficient nonlinear evolution equations.

Key words: Direct Transformation Method; Generalized Variable Coefficient Nonlinear Schrödinger (NLS) Equation; Cubic-quintic Nonlinear Schrödinger (CQNLS) Equation with Variable Coefficients; Nontravelling Wave Solutions.