Based on the homotrinuclear spin cluster having SU(2)⊗SU(2) symmetry with twofold orbital degeneracy $\tau = 1/2$ and the SU(2) algebraic structures of both $\hat{s}$ and $\hat{\tau}$ subspaces in the external magnetic field, we calculate exactly the non-adiabatic energy levels and the cyclic and non-cyclic non-adiabatic geometric phase of the homotrinuclear spin cluster by making use of the method of algebraic dynamics. The solution will show that the Berry phase is much influenced by the parameters $N = \gamma_s/\gamma_\tau$ ($\gamma_s$ and $\gamma_\tau$ are the magnetic momentums of $\hat{s}$ and $\hat{\tau}$ subspaces, respectively) in addition to $\omega/\Omega$ in a rotating magnetic field. The change of the Berry phase in the basis state of the system is demonstrated from the changing diagram.

Key words: Algebraic Dynamic; Berry Phase; Algebraic Structures.

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