Truncated Painlevé Expansion – A Unified Approach to Exact Solutions and Dromion Interactions of (2+1)-Dimensional Nonlinear Systems

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In this paper, we formulate a method wherein we harness the results of the Painlevé analysis to generate the solutions of the (2+1)-dimensional Ablowitz-Kaup-Newell-Segur system completely in terms of the arbitrary functions. This method is mainly based on the results of the truncated Painlevé expansion. Different types of interactions among dromions are deeply understood both analytically and numerically. Especially, different from the traditional viewpoint, we point out that the soliton (dromion) fission and fusion may be an approximate phenomenon.

Key words: Truncated Painlevé Expansion; Exact Solutions; Dromion Interactions.

1. Introduction

The recent spurt in the study of integrable models in (2+1)-dimensions is mainly attributed to the identification of dromions [1 – 4] which decay exponentially in all directions. These dromions which exist at the point of intersection of two ghost solitons can be driven anywhere in the two-dimensional plane by suitably choosing the boundaries. Can one generate solutions which are more general than the exponentially localized solutions just as the one-dimensional solitons happen to be the special case of the doubly periodic Jacobian elliptic functions? How can they be generated? The primary objective of this paper is to make a nascent contribution in this direction to answer the above questions.

It is known that for higher dimension soliton systems, there are abundant localized excitations and rich interaction phenomena. Especially, the interactions among dromions may be completely elastic in some cases and completely inelastic in some other cases. When the interaction is inelastic, two dromions may exchange some physical quantities or even completely exchange their shapes. Even though the fact that one dromion may split into two [5] and two or more dromions may fuse together to form a single dromion [6] is already known, one does not really know the criterion behind the fission and fusion of dromions. In this paper, we take a typical two-dromion solution of the Ablowitz-Kaup-Newell-Segur (AKNS) system as a simple example to give a clear picture on the dromion interaction.

2. (2+1)-Dimensional AKNS System

The (2+1)-dimensional AKNS system is one of the most important dynamical systems arising in various physical situations [7 – 9] and is given by

$$\begin{align}
&i q_t + q_{xx} + q_{yy} - 2 \lambda q(U + V) = 0, \\
&-i r_t + r_{xx} + r_{yy} - 2 \lambda r(U + V) = 0,
\end{align}$$

(1)

$$V_x = (qr)_y \quad \text{or} \quad V = \int (qr)_y dx + V_2(y,t),$$

(3)

$$U_y = (qr)_x \quad \text{or} \quad U = \int (qr)_x dy + U_2(x,t),$$

(4)

where \( q \) and \( r \) are the complex physical fields, and \( V \) and \( U \) are the potentials. The above equation system (1) – (4) reduces to the Davey-Stewartson (I) equation [10] under the reduction \( r = q^* \). Expanding the physical fields and the potentials in the form of a Laurent series in the neighbourhood of a noncharacteristic manifold \( \phi(x,y,t) = 0 \) and utilizing the results of the Painlevé test admitted by the above equation [11, 12], we obtain the following Bäcklund transformation by truncating at the constant level term:

$$q = \frac{q_0}{\phi} + q_1,$$

(5)
The potentials 
(after substituting the vacuum solutions)
\begin{equation}
V = \frac{V_0}{\phi^2} + \frac{V_1}{\phi} + V_2,
\end{equation}
\begin{equation}
U = \frac{U_0}{\phi^2} + \frac{U_1}{\phi} + U_2.
\end{equation}

Considering a vacuum solution for the physical fields,
\begin{equation}
q_1 = 0, \quad r_1 = 0.
\end{equation}
The potentials \( V \) and \( U \) can be driven by lower dimensional arbitrary functions of space and time of the form (after substituting the vacuum solutions)
\begin{equation}
V_2 = V_2(y,t), \quad U_2 = U_2(x,t).
\end{equation}
We now substitute the Bäcklund transformation (5)–(8) with the above set of overdetermined equations into (1)–(4) to obtain by collecting the coefficients of \( \phi^{-3} \),
\begin{equation}
\lambda U_0 = \phi_x^2, \quad \lambda V_0 = \phi_y^2, \quad \lambda q_0 r_0 = \phi_x \phi_y.
\end{equation}
Collecting the coefficients of \( \phi^{-2} \), we obtain the following set of equations:
\begin{equation}
-iq_0 \phi_t - 2q_0 \phi_x - q_0 \phi_{xx} - 2q_0 \phi_y - q_0 \phi_{yy} = 0,
\end{equation}
\begin{equation}
iq_0 \phi_t - 2r_0 \phi_x - r_0 \phi_{xx} - 2r_0 \phi_y - r_0 \phi_{yy} = 0,
\end{equation}
\begin{equation}
V_{0x} - V_1 \phi_x = \{ q_0 r_0 \},
\end{equation}
\begin{equation}
U_{0y} - U_1 \phi_x = \{ q_0 r_0 \}.
\end{equation}
Solving the above set of overdetermined equations consistently, we obtain
\begin{equation}
V_1 = \frac{\phi_x \phi_{xx} - \phi_y \phi_{yy}}{\lambda \phi_x},
\end{equation}
\begin{equation}
U_1 = \frac{\phi_x \phi_{xx} - \phi_y \phi_{yy}}{\lambda \phi_y}.
\end{equation}
Now, collecting the coefficients of \( \phi^{-1} \), we obtain the following set of equations:
\begin{equation}
-iq_r + q_{0xx} + q_{0yy} - 2\lambda q_0 [U_2 + V_2] = 0,
\end{equation}
\begin{equation}
iq_r + r_{0xx} + r_{0yy} - 2\lambda r_0 [U_2 + V_2] = 0,
\end{equation}
\begin{equation}
V_{1x} = 0.
\end{equation}
The compatibility of the above equations requires that the manifolds \( \phi \) and \( q_0 \) should evolve as
\begin{equation}
\phi(x,y,t) = \phi_1(x,t) + \phi_2(y,t),
\end{equation}
\begin{equation}
r_0(x,y,t) = q_1(x,t) q_2(y,t).
\end{equation}
From the Painlevé analysis of the (2+1)-dimensional AKNS system, we know that the resonance at \( r = -1 \) represents the arbitrariness of the manifold and this is indeed reflected by (22), while the resonance at \( r = 0 \) represents the arbitrariness of either \( q_0 \) or \( r_0 \) which is in line with (23).

3. Solutions of the (2+1)-Dimensional AKNS Equation and their Interactions

Thus, the physical fields of the (2+1)-dimensional AKNS equation can be explicitly given as
\begin{equation}
q = \frac{q_1(x,t) q_2(y,t)}{\phi_1(x,t) + \phi_2(y,t)},
\end{equation}
\begin{equation}
r = \frac{\phi_{1x} \phi_{2y}}{\phi_1(x,t) + \phi_2(y,t)},
\end{equation}
while the potentials \( U \) and \( V \) can also be solved in terms of the arbitrary functions \( \phi_1(x,t), \phi_2(x,t), q_1(x,t), q_2(y,t) \) and \( c(t) \):
\begin{equation}
U = -\frac{1}{\lambda} \{ \ln[\phi_1(x,t) + \phi_2(y,t)] \}_x + \frac{iq_1(x,t) + q_{1xx}(x,t) - c(t) q_1(x,t)}{2\lambda q_1(x,t)},
\end{equation}
\begin{equation}
V = -\frac{1}{\lambda} \{ \ln[\phi_1(x,t) + \phi_2(y,t)] \}_y + \frac{iq_2(y,t) + q_{2yy}(y,t) + c(t) q_2(y,t)}{2\lambda q_2(y,t)}.
\end{equation}
The presence of the two-dimensional arbitrary functions presents the freedoms to generate a wide class of solutions of the (2+1)-dimensional AKNS equation. From the above, we find that the physical quantity \( "qr" \) takes the form
\begin{equation}
qr = \frac{\phi_{1x} \phi_{2y}}{\lambda (\phi_1(x,t) + \phi_2(y,t))^2}.
\end{equation}
In (1+1)-dimensions, a single soliton can be found from the limiting case of a related periodic solution.
expressed by Jacobi elliptic functions. To generate a one-dromion solution for the quantity $qr$, which can be related to the energy of the system, we now drive the arbitrary function by Jacobi elliptic functions.

For instance, if we choose

$$\phi_1(x,t) = a_0 + a_1 \text{sn}(k_1 x - \omega_1 t, m_1), \quad (29)$$

$$\phi_2(y,t) = a_2 \text{sn}(k_2 y - \omega_2 t, m_2), \quad |a_0| > |a_1| + |a_2|, \quad (30)$$

we obtain the one dromion of the AKNS system in terms of the Jacobian elliptic function. Figure 1a is a snapshot of the periodic solution (28) under the above choice of arbitrary functions with the following parameters:

$$a_0 = 8, \quad a_1 = a_2 = k_1 = k_2 = \lambda = 1,$$

$$\omega_1 = 2, \quad \omega_2 = 0, \quad m_1 = 0.2, \quad m_2 = 0.3 \quad (31)$$

at time $t = 0$. Figure 1b is a contour plot of Figure 1a. When $m_1$ and $m_2$ approach unity, we get a dromion lattice solution given by Fig. 1c for the same parametric choice except that $m_1 = 0.99$, $m_2 = 0.9995$. Finally, when $m_1 = m_2 = 1$, the dromion lattice tends to a single dromion which is described by (28) with

$$\phi_1(x,t) = a_0 + a_1 \tanh(k_1 x - \omega_1 t), \quad (32)$$

$$\phi_2(y,t) = a_2 \tanh(k_2 y - \omega_2 t), \quad |a_0| > |a_1| + |a_2|, \quad (33)$$

and is shown in Fig. 1d at $t = 0$. Thus, we find that the exponentially localized dromions found earlier by Boiti et al. [1] appear only as a special case of the solutions driven by Jacobian elliptic functions.

The above analysis can be easily extended to generate multiple periodic wave solutions unlike (1+1)-dimensions where multiple soliton solutions can not be obtained from a limiting case of multiple periodic wave solutions expressed by means of the Jacobi elliptic functions. However, one can find many kinds of multiple periodic solutions in (2+1)-dimensions which are the generalizations of the different types of multidromions.
Fig. 2. Approximate dromion fission at times: (a) $t = -3$; (b) $t = 0$; (c) $t = 0.7$; (d) $t = 2$; (e) $t = 3$. (f) The tiny dromion at time $t = -3$ which is not observed in Fig. 1a as its amplitude is much smaller than that of the bigger one.

For instance, the choice

\[ \phi_1(x,t) = a_0 + \sum_{i=1}^{N} a_i \text{sn}^2(k_i x - \omega_i t, m_i), \]
\[ \text{ci} > 0, \quad \text{di} > 0, \quad (34) \]

\[ \phi_2(y,t) = a_3 \sum_{i=1}^{M} b_i \text{sn}^2(K_i y - \Omega_i t, \mu_i), \]
\[ |a_0| > \sum_{i=1}^{N} |a_i| + \sum_{i=1}^{M} |b_i| \]
\[ (35) \]

generates $(M + N)$ periodic wave interaction solutions which are the generalization of an $M \times N$ dromion solution.

It is known that localized excitations in higher dimensions undergo both elastic and inelastic collisions. Two localized excitations may exchange their physical quantities such as the energy and the momentum. Two solitons (or solitary waves) may fuse together to form one soliton, and one soliton may split into two solitons. To bring out a deeper understanding on the interaction of dromions, we take a two-dromion solution of the AKNS system with the following choices of the arbi-
is about a smaller dromion does really exist with the amplitude phenomenon. dromion fission may be considered only as an approx-
ture 2f shows that before the interaction (at $t = -3.5$) we can recover the smaller one. Fig-
relatively too small as to observe it explicitly. If we
system. Actually, before the interaction, there are two
\[
\phi_1(x,t) = a_0 + a_1 \tanh(k_1 x - \omega_1 t) + a_2 \tanh(k_2 x + \omega_2 t),
\]
\[
\phi_2(y,t) = a_3 \tanh(k_3 y - \omega_3 t),
\]
\[
|a_0| > |a_1| + |a_2| + |a_3|.
\]
Before we give a detailed analytical analysis of the in-
interaction of dromions, we first discuss them numeri-
Figure 2 shows the evolution of the two-dromion
\[
a_0 = 24, \quad a_1 = 20, \quad a_2 = a_3 = k_1 = k_2 = k_3 = 1, \]
\[
\omega_1 = 2, \quad \omega_2 = -2, \quad \lambda = 1
\]
at times $t = -3, 0, 0.7, 2$ and $3$, respectively.
From Figs. 2a and 2b, we observe only one dromion. From Figs. 2c – 2e, we find that one dromion splits into two. This phenomenon is known as dromion (or soli-
ton) fission. However, this observation may not be ex-
correct at least for the (2+1)-dimensional AKNS
An important function in the dromion fission is the interaction of dromions. While one is explicitly visible, the other is
\[
\phi_1(x,t) = a_0 + a_1 \text{sech}(k_1 x - \omega_1 t) + a_2 \text{sech}(k_2 x + \omega_2 t),
\]
Different choices of the constant parameters in (36) and (37) may lead to different inelastic collision phe-
omena. Figure 3 shows the inelastic pursuant dromion interaction and illustrates how they evolve in time ex-
changing energy among themselves, while Fig. 2 dis-
plays a head on collision of dromions.
To bring out the exchange interaction [13] where
the interacting dromions completely exchange their
shapes, we again consider the two-dromion solu-
tion (28) with (36) and (37) and choose the parameters as
\[
a_0 = 24, \quad a_1 = a_2 = a_3 = k_1 = k_2 = k_3 = 1, \]
\[
\omega_1 = 2, \quad \omega_2 = -2, \quad \lambda = 1
\]
From Figs. 4a and 4c, one finds that the left moving
dromion after the interaction ($t = 2.5$) possesses the
shape of the right moving dromion before the interaction ($t = -2.5$) and vice versa. This conclusion can be strictly proved analytically later.
Figure 5 brings out the fusion of two dromions. Be-
fore the interaction (Fig. 5a), there are two explicit
dromions. After the interaction (Fig. 5c), we observe
only one dromion and we call this phenomenon as
dromion (or soliton) fusion. It must be emphasized
that the concept of fusion is again an approximate
phenomenon like fission with reference to the (2+1)-
dimensional AKNS system as the tiny dromion with
amplitude $\sim 0.0005$ can be made visible by search-
ing far away from the domain of the bigger dromion
(Fig. 5f).
The interaction discussed so far is inelastic in nature. To bring out the multi-dromion elastic interaction, we choose the arbitrary functions as
\[
\phi_1(x,t) = a_0 + a_1 \text{sech}(k_1 x - \omega_1 t) + a_2 \text{sech}(k_2 x + \omega_2 t),
\]
Fig. 4. The exchange dromion interaction at times: (a) $t = -2.5$; (b) $t = 0$; (c) $t = 2.5$.

Fig. 5. The approximate fusion interaction of dromions at times: (a) $t = -3$; (b) $t = 0$; (c) $t = 3$. (d) A tiny dromion after the interaction at $t = 3$. 
dromions (with positive and negative amplitude) given analytically by carrying out the asymptotic analysis of while for the two-dipole-type-dromion solution (28) with (40) and (41), we obtain

\[ \phi_2(y,t) = a_3 \tanh(k_3y - \omega_0 t). \]  

(41)

Figure 6 shows elastic interaction of the dipole-type dromions at times: (a) \( t = -3 \); (b) \( t = 0 \); (c) \( t = 3 \).

The above numerical results can also be proved analytically by carrying out the asymptotic analysis of

\[
qr^+ \equiv qr_{t \to +\infty} = \frac{a_1 k_1 a_3 k_3 \text{sech}^2(k_1 x - \omega_1 t) \text{sech}^2(k_3 y)}{(a_0 + a_2 + a_1 \tanh(k_1 x - \omega_1 t) + a_3 \tanh(k_3 y))^2} + \frac{a_2 k_2 a_3 k_3 \text{sech}^2(k_2 x - \omega_2 t) \text{sech}^2(k_3 y)}{(a_0 + a_1 + a_2 \tanh(k_2 x - \omega_2 t) + a_3 \tanh(k_3 y))^2},
\]

(43)

\[
qr^- \equiv qr_{t \to -\infty} = \frac{a_1 k_1 a_3 k_3 \text{sech}^2(k_1 x - \omega_1 t) \text{sech}^2(k_3 y)}{(a_0 - a_2 + a_1 \tanh(k_1 x - \omega_1 t) + a_3 \tanh(k_3 y))^2} + \frac{a_2 k_2 a_3 k_3 \text{sech}^2(k_2 x - \omega_2 t) \text{sech}^2(k_3 y)}{(a_0 - a_1 + a_2 \tanh(k_2 x - \omega_2 t) + a_3 \tanh(k_3 y))^2},
\]

(44)

while for the two-dipole-type-dromion solution (28) with (40) and (41), we obtain

\[
qr^\pm \equiv qr_{t \to \pm\infty} = \frac{a_1 k_1 a_3 k_3 \text{sech}^2(k_1 x - \omega_1 t) \tanh(k_1 x - \omega_1 t) \text{sech}^2(k_3 y)}{(a_0 + a_1 \text{sech}(k_1 x - \omega_1 t) + a_3 \tanh(k_3 y))^2} + \frac{a_2 k_2 a_3 k_3 \text{sech}^2(k_2 x - \omega_2 t) \tanh(k_2 x - \omega_2 t) \text{sech}^2(k_3 y)}{(a_0 + a_2 \text{sech}(k_2 x - \omega_2 t) + a_3 \tanh(k_3 y))^2},
\]

(45)
For the dipole-dromion interaction, the elastic interaction occurs by virtue of the asymptotic behavior of (45).

For the two-dromion solution (28) with (36) and (37), we first write down the amplitudes of the dromions before and after interaction. Before the interaction, the amplitude for the faster moving dromion (we assume right moving is faster than left moving) is given by

$$A_{1-} = \frac{|a_1a_3k_1k_3|}{(a_0 - a_2)^2},$$

(46)

while for the slower moving dromion, we have the amplitude given by

$$A_{2-} = \frac{|a_2a_3k_2k_3|}{(a_0 + a_1)^2},$$

(47)

After the interaction, the amplitudes are

$$A_{1+} = \frac{|a_1a_3k_1k_3|}{(a_0 + a_1)^2},$$

(48)

for the faster moving dromion and

$$A_{2+} = \frac{|a_2a_3k_2k_3|}{(a_0 - a_2)^2},$$

(49)

for the slower moving dromion, respectively.

From the expression for the amplitudes, we observe that the “approximate” fission phenomenon is related to

$$\frac{A_{1-}}{A_{2-}} = \frac{a_1k_1(a_0 + a_1)^2}{a_2k_2(a_0 - a_2)^2} \gg 1, \text{ or } \frac{A_{1-}}{A_{2-}} \ll 1,$$

(50)

while the “approximate” fusion phenomenon will be observed when

$$\frac{A_{1+}}{A_{2+}} = \frac{a_1k_1(a_0 - a_1)^2}{a_2k_2(a_0 + a_2)^2} \gg 1, \text{ or } \frac{A_{1+}}{A_{2+}} \ll 1.$$  

(51)

For the dipole-dromion interaction, the error plots of $d1 \equiv 10^9|qr - qr_−|$ and $d2 \equiv 10^9|qr - qr_+|$ at times: (a) $t = -3.5$ and (b) $t = 3.5$ related to Figure 3.

Figures 2 and 5 correspond to the cases (50) and (51), respectively.

If the conditions

$$A_{1+} = A_{2-}, \quad A_{2+} = A_{1-}, \quad k_1 = k_2$$

(52)

are satisfied, then we obtain the exchange interaction shown in Figure 4.

To see the accuracy of the approximate expressions, we plot down the quantities

$$d1 \equiv 10^9|qr - qr_−| \text{ and } d2 \equiv 10^9|qr - qr_+|$$

in Fig. 7 for the inelastic pursuant collision corresponding to Figure 3. Figure 7 shows that the errors between the exact solution and asymptotic expressions are only about $10^{-9}$.

From (24) – (27), we find that two more arbitrary functions $q_1(x,t)$ and $q_2(y,t)$ have been included in exact solutions unlike the solutions obtained from the multilinear variable separation approach [12]. These arbitrary functions have no effect on the quantity $qr$ while their effect on the potentials will have to be investigated. Figure 8 displays the structures of the potential $V$ for the choice of $q_1$ and $c(t)$ as

$$q_1 = 4 + \tanh(x + 2t), \quad c(t) = e^{-t}.$$  

(53)

while all the other parameters and arbitrary functions are the same as in the case of Figure 1d. Figures 8a – 8c show the evolution of the real parts of $V$ while Fig. 8d exhibits the structure of the imaginary parts of $V$. From Fig. 8, we observe that the arbitrary function $q_1$ may generate new line solitons for the potential $V$.

### 4. Summary and Discussion

In this paper, we have formulated a new method to construct the solutions of the (2+1)-dimensional

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For the two-dromion solution (28) with (36) and (37), we first write down the amplitudes of the dromions before and after interaction. Before the interaction, the amplitude for the faster moving dromion is given by

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Fig. 8a

Fig. 8b

Fig. 8c

Fig. 8d

Fig. 8. The time evolution of the real part of the potential $V$ given by (27) with (32), (33) and (53) at times: (a) $t = -3$; (b) $t = 0$; (c) $t = 3$. (d) The corresponding structure of the imaginary part of the potential $V$ at $t = 0$.

AKNS system by suitably harnessing the results of the Painlevé analysis. This method which is more elegant and straightforward gives us an unprecedented possibility of constructing a wide class of solutions of (2+1)-dimensional soliton equations.

We have obtained abundant localized exact solutions and studied interaction properties among different types of localized excitations. We also observed that the dromion interactions may be elastic or inelastic. When the interaction is inelastic, two dromions may exchange their physical quantities partially and may completely exchange their shapes. Contrary to the traditional viewpoint, we emphasized that the concept of fission (or fusion) of dromions may be an approximate phenomenon at least with reference to the (2+1)-dimensional AKNS system.

We have also obtained multiple periodic wave solutions which may degenerate to multiple dromions just as one soliton can be obtained as the limiting case of Jacobi elliptic function in (1+1)-dimensions.

The investigation of the other well-known (2+1)-dimensional soliton equations using the Painlevé truncation method is under progress and the results will be published later.

Acknowledgement

The work was supported by the National Natural Science Foundations of China (No. 90203001, No. 10475055 and No. 90503006).

