On "Belated Decision in the Hilbert-Einstein Priority Dispute", published by L. Corry, J. Renn, and J. Stachel

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In a paper, published in 1997 by L. Corry, J. Renn, and J. Stachel, it is claimed that the recently discovered printer's proofs of Hilbert's 1915 paper on the general theory of relativity prove that Hilbert did not anticipate Einstein in arriving at the correct form of the gravitational field equations, as it is widely believed, but that only after having seen Einstein's final paper did Hilbert amend his published version with the correct form of the gravitational field equations. However, because a crucial part of the printer's proofs of Hilbert's paper had been cut off by someone, a fact not mentioned in the paper by Corry, Renn, and Stachel, the conclusion drawn by Corry, Renn, and Stachel is untenable and has no probative value. I rather will show that the cut off part of the proofs suggests a crude attempt by some unknown individual to falsify the historical record.

Key words: History of Physics; General Theory of Relativity.

It has been the accepted view that David Hilbert completed the general theory of relativity at least five days before Einstein. And it has been suspected that Einstein arrived at the correct form of the gravitational field equations only after having seen Hilbert's paper, of which Hilbert sent Einstein a copy prior to Hilbert's delivery of his paper to the Goettingen Academy. In an article published in **Science** by Corry, Renn, and Stachel [1], it is claimed that the printer's proofs of Hilbert's paper, recently discovered by Corry in the archives of the Goettingen library, rather prove the opposite, and that Hilbert had amended the published version of his paper with the correct form of the gravitational field equations after he had seen Einstein's final paper. However, Corry, Renn, and Stachel failed to mention even once, that the printer's proofs have been mutilated, with parts of the proofs cut off by someone. The abstract of the paper by Corry, Renn, and Stachel rather makes the statement: "The first set of proofs of Hilbert's paper shows that the theory he originally submitted is not generally covariant and does not include the explicit form of the field equations of general relativity."

The facts are as follows:

1. The upper part of page 8 of the proofs, approximately one third, together with Eq. (17) has been cut off.

2. The text following the cut off part of page 8 refers to the Ricci curvature invariant K and to the metric tensor. This alone shows that the upper part of page 8 with the missing Eq. (17) has to do with the gravitational field equations.

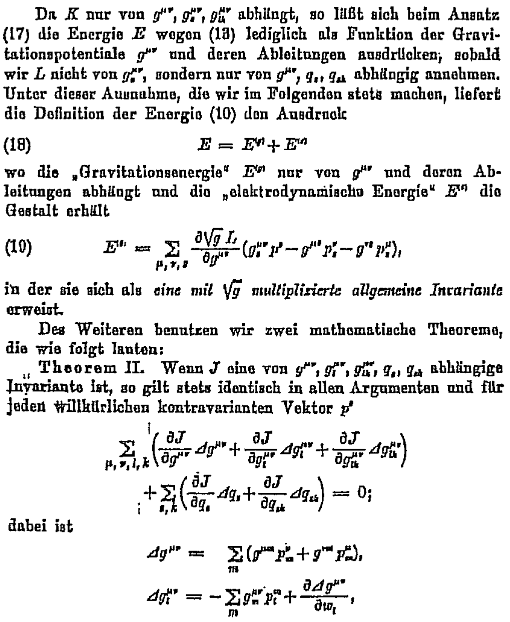
3. In his proofs, and prior to Eq. (26), Hilbert states that with the form of the variational derivative for Eq. (17), the gravitational field equations assume the form given by Eq. (26). But it is the variational derivative for the expression on the l. h. s. of Eq. (26) which contains the trace term, missing in all of Einstein's papers prior to Einstein having seen Hilbert's paper.

Following the widely publicized 1997 paper published in **Science**, Renn and Stachel have been circulating a 113 page long preprint [2], published by the Max Planck Institut für Wissenschaftsgeschichte in Berlin, Germany. In this preprint it is admitted in a footnote on page 17, that the upper part of page 8 of the proofs has been cut off, and it is conjectured that the missing Eq. (17) is the equation

$$H = K + L,$$

where K is the gravitational and L the electromagnetic part of the Lagrangian, as in Hilbert's published version, where the variational derivative automatically leads to the trace term[1]. In his published version [3], Hilbert writes down the variational derivative immediately after Eq. (21) (with Eq. (21) the same as Eq. (26)

[1]Hilbert uses the letter K (as for Gauss's curvature) in K, $K_{\mu\nu}$, and $K^{\mu}_{\nu\lambda\rho}$, instead of R, $R_{\mu\nu}$, and $R^{\mu}_{\nu\lambda\rho}$.



Da K nur von $g^{\mu\nu}$, $g^{\mu\nu}_s$, $g^{\mu\nu}_{lk}$ abhängt, so läßt sich beim Ansatz (17) die Energie E wegen (13) lediglich als Funktion der Gravitationspotentiale $g^{\mu\nu}$ und deren Ableitungen ausdrücken; sobald wir L nicht von $g^{\mu\nu}_s$, sondern nur von $g^{\mu\nu}$, q_s, q_{sk} abhängig annehmen. Unter dieser Ausnahme, die wir im Folgenden stets machen, liefert die Definition der Energie (10) den Ausdruck

$$E = E^{(g)} + E^{(e)} \tag{18}$$

wo die „Gravitationsenergie" $E^{(g)}$ nur von $g^{\mu\nu}$ und deren Ableitungen abhängt und die „elektrodynamische Energie" $E^{(e)}$ die Gestalt erhält

$$E^{(e)} = \sum_{\mu,\nu,s} \frac{\partial\sqrt{g}L}{\partial g^{\mu\nu}} (g^{\mu\nu}_s p^s - g^{\mu s} p^\nu_s - g^{\nu s} p^\mu_s), \tag{19}$$

in der sie sich als *eine mit $\sqrt{g}$ multiplizierte allgemeine Invariante* erweist.

Des Weiteren benutzen wir zwei mathematische Theoreme, die wie folgt lauten:

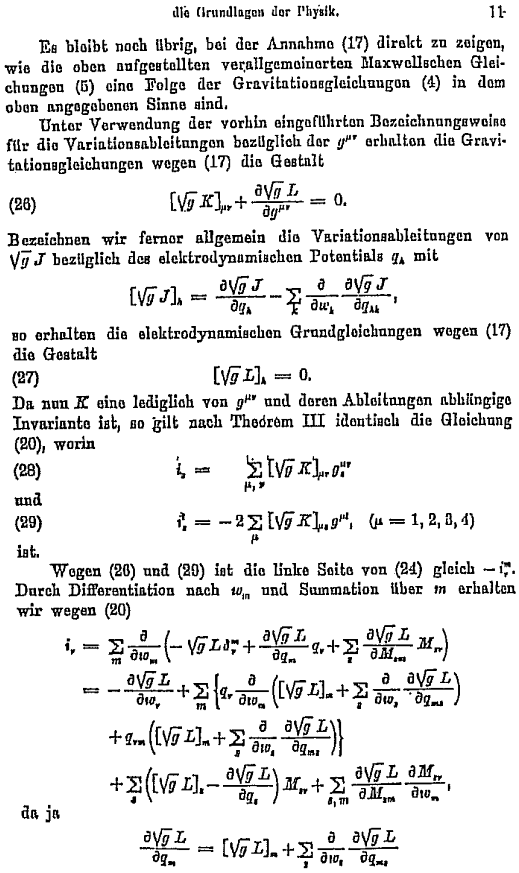
Theorem II. Wenn J eine von $g^{\mu\nu}$, $g^{\mu\nu}_l$, $g^{\mu\nu}_{lk}$, q_s, q_{sk} abhängige Invariante ist, so gilt stets identisch in allen Argumenten und für jeden willkürlichen kontravarianten Vektor p^s

$$\sum_{\mu,\nu,l,k} \left(\frac{\partial J}{\partial g^{\mu\nu}} \Delta g^{\mu\nu} + \frac{\partial J}{\partial g^{\mu\nu}_l} \Delta g^{\mu\nu}_l + \frac{\partial J}{\partial g^{\mu\nu}_{lk}} \Delta g^{\mu\nu}_{lk} \right) + \sum_{s,k} \left(\frac{\partial J}{\partial q_s} \Delta q_s + \frac{\partial J}{\partial q_{sk}} \Delta q_{sk} \right) = 0;$$

dabei ist

$$\Delta g^{\mu\nu} = \sum_m (g^{\mu m} p^\nu_m + g^{\nu m} p^\mu_m),$$

$$\Delta g^{\mu\nu}_l = -\sum_m g^{\mu\nu}_m p^m_l + \frac{\partial \Delta g^{\mu\nu}}{\partial w_l},$$



die Grundlagen der Physik. 11

Es bleibt noch übrig, bei der Annahme (17) direkt zu zeigen, wie die oben aufgestellten verallgemeinerten Maxwellschen Gleichungen (5) eine Folge der Gravitationsgleichungen (4) in dem oben angegebenen Sinne sind.

Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (17) die Gestalt

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial\sqrt{g}L}{\partial g^{\mu\nu}} = 0. \tag{26}$$

Bezeichnen wir ferner allgemein die Variationsableitungen von $\sqrt{g}J$ bezüglich des elektrodynamischen Potentials q_h mit

$$[\sqrt{g}J]_h = \frac{\partial\sqrt{g}J}{\partial q_h} - \sum_k \frac{\partial}{\partial w_k} \frac{\partial\sqrt{g}J}{\partial q_{hk}},$$

so erhalten die elektrodynamischen Grundgleichungen wegen (17) die Gestalt

$$[\sqrt{g}L]_h = 0. \tag{27}$$

Da nun K eine lediglich von $g^{\mu\nu}$ und deren Ableitungen abhängige Invariante ist, so gilt nach Theorem III identisch die Gleichung (20), worin

$$i_s = \sum_{\mu,\nu} [\sqrt{g}K]_{\mu\nu} g^{\mu\nu}_s \tag{28}$$

und

$$i^l_s = -2 \sum_\mu [\sqrt{g}K]_{\mu s} g^{\mu l}, \quad (\mu = 1, 2, 3, 4) \tag{29}$$

ist.

Wegen (26) und (29) ist die linke Seite von (24) gleich $-i^m_\nu$. Durch Differentiation nach w_m und Summation über m erhalten wir wegen (20)

$$\begin{aligned} i_\nu &= \sum_m \frac{\partial}{\partial w_m} \left(-\sqrt{g} L \delta^m_\nu + \frac{\partial\sqrt{g}L}{\partial q_m} q_\nu + \sum_s \frac{\partial\sqrt{g}L}{\partial M_{sm}} M_{s\nu} \right) \\ &= -\frac{\partial\sqrt{g}L}{\partial w_\nu} + \sum_m \left\{ q_\nu \frac{\partial}{\partial w_m} \left([\sqrt{g}L]_m + \sum_s \frac{\partial}{\partial w_s} \frac{\partial\sqrt{g}L}{\partial q_{ms}} \right) \right. \\ &\quad \left. + q_{\nu m} \left([\sqrt{g}L]_m + \sum_s \frac{\partial}{\partial w_s} \frac{\partial\sqrt{g}L}{\partial q_{ms}} \right) \right\} \\ &\quad + \sum_s \left([\sqrt{g}L]_s - \frac{\partial\sqrt{g}L}{\partial q_s} \right) M_{s\nu} + \sum_{s,m} \frac{\partial\sqrt{g}L}{\partial M_{sm}} \frac{\partial M_{s\nu}}{\partial w_m}, \end{aligned}$$

da ja

$$\frac{\partial\sqrt{g}L}{\partial q_m} = [\sqrt{g}L]_m + \sum_s \frac{\partial}{\partial w_s} \frac{\partial\sqrt{g}L}{\partial q_{ms}}$$

Fig. 1. Mutilated page 8 of Hilbert's first proofs, with the Eq. (17) cut off; and page 11 of the proofs, where Eq. (26) is the correct form of the gravitational field equation. Niedersächsische Staats- und Universitätsbibliothek Göttingen, Cod. Ms. d. Hilbert 634, Bl. 23[r] – 29[r].

of the proofs). If the missing Eq. (17) in the proofs is the equation $H = K + L$ as Renn and Stachel believe, the equation for the variational derivative

$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}\Big(K_{\mu\nu} + \frac{1}{2}Kg_{\mu\nu}\Big)$$

would come after Eq. (17) on the missing part of page 8, as in the published version where it comes after Eq. (21), and where it has been given no number. The same is most likely true for this equation in the proofs. But even without writing down the explicit expression for the variational derivative, the equation $H = K + L$, with K the Ricci invariant, is sufficient to obtain the correct gravitational field equation simply by taking the variational derivative of the Langrangian $H = K + L$ in Hilbert's variational principle

$$\delta \int H\sqrt{g}\mathrm{d}\tau = 0,$$

where (apart from surface terms which vanish at ∞)

$$\delta \int K\sqrt{g}\mathrm{d}\tau = \int \Big(K_{\mu\nu} - \frac{1}{2}Kg_{\mu\nu}\Big)\delta g^{\mu\nu}\sqrt{g}\mathrm{d}\tau.$$

Mentioning the mutilation of Hilbert's proofs in a footnote of an unpublished preprint can not excuse Corry, Reno, and Stachel for having failed to mention this mutilation in their **Science** article which with the title "Belated Decision in the Hilbert-Einstein Priority Dispute", claims to prove Einstein's priority. With Hilbert's definition of the variational derivative

$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}\Big(K_{\mu\nu} - \frac{1}{2}Kg_{\mu\nu}\Big)$$

the gravitational field equations appear both in the proofs (there as Eq. (26)) and in the published version (there as Eq. (21)) in the abbreviated form[2]

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial \sqrt{g}L}{\partial g^{\mu\nu}} = 0,$$

which is the same as

$$K_{\mu\nu} - \frac{1}{2}Kg_{\mu\nu} = \text{const}\ T_{\mu\nu},$$

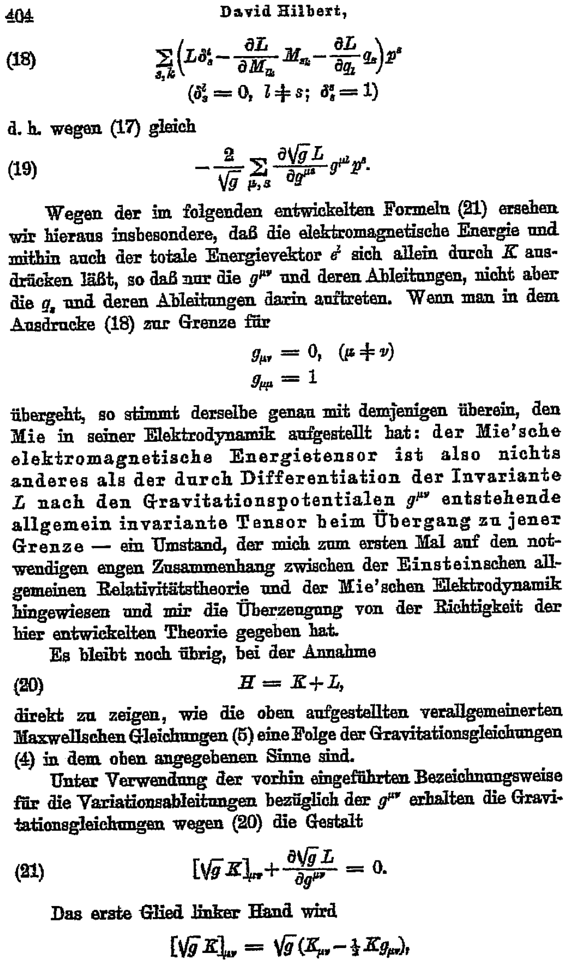
except that Hilbert uses for the matter part the Lagrangian of Mie's theory.

Further proof of Hilbert's priority is supported by the chronology of the papers by Einstein and Hilbert [4]:

1. Nov. 4, 1915, Einstein submits the still incorrect equations to the Prussian Academy.
2. Nov. 11, 1915, Einstein again submits the incorrect equations to the Prussian Academy.
3. Nov. 18, 1915, Einstein acknowledges having received in advance a copy of Hilbert's paper to be delivered by Hilbert to the Goettingen Academy on Nov. 20, 1915, and Einstein writes Hilbert that he had obtained the same equations in the last weeks, even though only one week before, on Nov. 11, 1915, he still had the wrong equations.
4. Nov. 20, 1915, Hilbert presents his equations to the Goettingen Academy, but someone had later cut off critical parts of Hilbert's page proofs.
5. Nov. 25, 1915, Einstein submits the correct equations to the Prussian Academy.

In summary: Einstein's letter of Nov. 18, 1915 to Hilbert proves that Hilbert had the correct equations before Einstein. Einstein's claim that he had the correct equations weeks earlier is contradicted by Einstein's paper to the Prussian Academy of Nov. 11, 1915, not weeks, but just one week earlier. Since Einstein still believed his erroneous equations were correct as late as Nov. 18, 1915, it is clear that Hilbert, who had the correct equations before Nov. 18, 1915, had arrived at them before Einstein.

The question remains how much credit shall go to Einstein and how much to Hilbert. A close examination of the historical record, leading to the discovery of the correct field equations in 1915, shows that Einstein



404 David Hilbert,

$$\sum_{s,k}\left(L\delta^l_s - \frac{\partial L}{\partial M_{lk}} M_{sk} - \frac{\partial L}{\partial q_l} q_s\right)p^s \qquad (18)$$
$$(\delta^l_s = 0,\ l \neq s;\ \delta^s_s = 1)$$

d. h. wegen (17) gleich

$$-\frac{2}{\sqrt{g}}\sum_{\mu,s}\frac{\partial\sqrt{g}L}{\partial g^{\mu s}}g^{\mu l}p^s. \qquad (19)$$

Wegen der im folgenden entwickelten Formeln (21) ersehen wir hieraus insbesondere, daß die elektromagnetische Energie und mithin auch der totale Energievektor e^l sich allein durch K ausdrücken läßt, so daß nur die $g^{\mu\nu}$ und deren Ableitungen, nicht aber die q_s und deren Ableitungen darin auftreten. Wenn man in dem Ausdrucke (18) zur Grenze für

$$g_{\mu\nu} = 0, \quad (\mu \neq \nu)$$
$$g_{\mu\mu} = 1$$

übergeht, so stimmt derselbe genau mit demjenigen überein, den Mie in seiner Elektrodynamik aufgestellt hat: der Mie'sche elektromagnetische Energietensor ist also nichts anderes als der durch Differentiation der Invariante L nach den Gravitationspotentialen $g^{\mu\nu}$ entstehende allgemein invariante Tensor beim Übergang zu jener Grenze — ein Umstand, der mich zum ersten Mal auf den notwendigen engen Zusammenhang zwischen der Einsteinschen allgemeinen Relativitätstheorie und der Mie'schen Elektrodynamik hingewiesen und mir die Überzeugung von der Richtigkeit der hier entwickelten Theorie gegeben hat.

Es bleibt noch übrig, bei der Annahme

$$H = K + L, \qquad (20)$$

direkt zu zeigen, wie die oben aufgestellten verallgemeinerten Maxwellschen Gleichungen (5) eine Folge der Gravitationsgleichungen (4) in dem oben angegebenen Sinne sind.

Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (20) die Gestalt

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial\sqrt{g}L}{\partial g^{\mu\nu}} = 0. \qquad (21)$$

Das erste Glied linker Hand wird

$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}(K_{\mu\nu} - \tfrac{1}{2}Kg_{\mu\nu}),$$

Fig. 2. Page 404 of Hilbert's published version where the equation (21) is the gravitational field equation, and where Hilbert's abbreviation $[\sqrt{g}K]_{\mu\nu} = \sqrt{g}(K_{\mu\nu} - \frac{1}{2}Kg_{\mu\nu})$ follows in the text Eq. (21). D. Hilbert, Kgl. Ges. d. Wiss. Nachrichten, Math.-phys. Klasse, 1915, Heft 3.

first recognized that the gravitational field must be described by the 10 components of the metric tensor g_{ik} for the four-dimensional Minkowski space-time, but it was Grossmann, not Einstein, who in a groundbreaking paper [5] for the first time in the history of physics named the contracted Riemann tensor R_{ik} for the solution of the gravitational problem sought by Einstein. Since R_{ik} is linear in the 2nd derivatives of the metric tensor g_{ik}, Grossmann was wondering if in the limit of static weak fields R_{ik} reduces to the Laplace operator, that is to $\nabla^2 g_{44} = 0$, the vacuum field equation for g_{44}, but this amounts to making the hypothesis that

$$R_{ik} = 0$$

[2] I express my thanks to the Niedersaechsische Staats- und Universitaetsbibliothek Goettingen for their permission to reproduce parts of the proofs and of the paper by D. Hilbert: Die Grundlagen der Physik (Erste Mitteilung).

must be the correct vacuum field equation. Grossmann however, incorrectly concluded that in the limit of weak static fields R_{ik} cannot be reduced to $\nabla^2 g_{44} = 0$.

Following in Grossmann's footsteps, Einstein conjectured up to November 11, 1915, that the correct field equation would have to be

$$R_{ik} = -\kappa T_{ik},$$

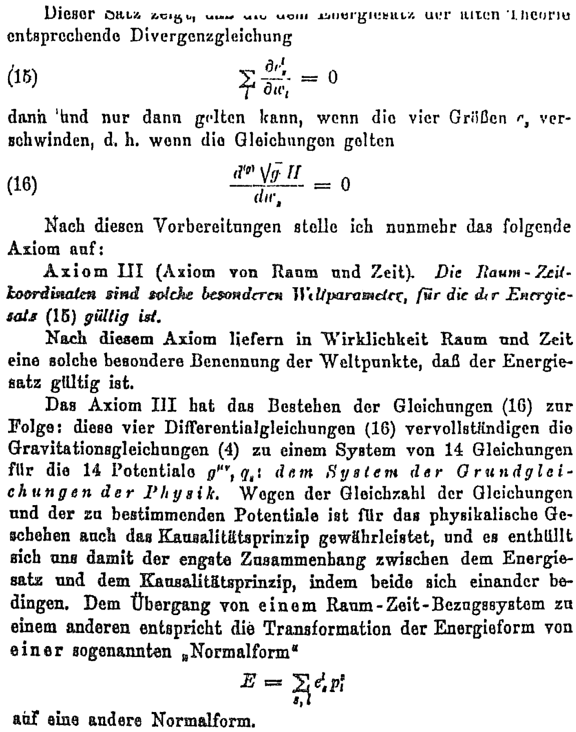
but because this equation does not satisfy the condition $T^i_{k;i} = 0$, except for the case of electromagnetic radiation, Einstein incorrectly believed that matter must in some way be described by electromagnetic radiation [6]. Nevertheless, with the geodetic equation for the motion in the field of a spherical mass and the ad hoc assumption that for weak fields the vacuum solution obtained from $R_{ik} = 0$ should match Newton's solution, Einstein was able to derive the perihelion motion and the deflection of light[3].

In spite of this remarkable success, Einstein was not able to obtain the correct field equation in the presence of matter. Because Grossmann was unable to figure it out for Einstein, Einstein sought the help of the famous Goettingen mathematicians Felix Klein and David Hilbert. Inspired by Mie's theory, Hilbert was already working on a unified field theory of gravity and electromagnetism. Even though Hilbert sent Einstein a copy of his not yet published paper, which Einstein received on Nov. 18, 1915, one can not prove that Einstein corrected his incorrect field equation after having seen Hilbert's paper, because it cannot be excluded that in the week following Nov. 11, 1915 Einstein had finally and independently arrived at the same solution. My analysis of Hilbert's mutilated proofs therefore cannot prove that Einstein copied from Hilbert. It proves less, which is that it cannot be proved that Einstein could not have copied from Hilbert. But it proves that Hilbert had not copied from Einstein, as it has been insinuated following the paper by Corry, Renn, and Stachel.

I must also disagree in at least one point with Sauer [7], who otherwise comes to similar conclusions. He too notices the cut off part of Hilbert's page proofs, and he too believes that the equation

$$H = K + L$$

[3] The equation for the perihelion motion, however, was already known and derived by Gerber with a potential similar to the potential used in Weber's electrodynamics (see [10]).


Dieser Satz zeigt, daß die dem Energiesatz der alten Theorie entsprechende Divergenzgleichung

(15) $$\sum_l \frac{\partial e^l_s}{\partial w_l} = 0$$

dann und nur dann gelten kann, wenn die vier Größen e_s verschwinden, d. h. wenn die Gleichungen gelten

(16) $$\frac{d^{(\sigma)} \sqrt{g}\, H}{d w_s} = 0$$

Nach diesen Vorbereitungen stelle ich nunmehr das folgende Axiom auf:

Axiom III (Axiom von Raum und Zeit). *Die Raum-Zeitkoordinaten sind solche besonderen Weltparameter, für die der Energiesatz* (15) *gültig ist.*

Nach diesem Axiom liefern in Wirklichkeit Raum und Zeit eine solche besondere Benennung der Weltpunkte, daß der Energiesatz gültig ist.

Das Axiom III hat das Bestehen der Gleichungen (16) zur Folge: diese vier Differentialgleichungen (16) vervollständigen die Gravitationsgleichungen (4) zu einem System von 14 Gleichungen für die 14 Potentiale $g^{\mu\nu}, q_s$: *dem System der Grundgleichungen der Physik.* Wegen der Gleichzahl der Gleichungen und der zu bestimmenden Potentiale ist für das physikalische Geschehen auch das Kausalitätsprinzip gewährleistet, und es enthüllt sich uns damit der engste Zusammenhang zwischen dem Energiesatz und dem Kausalitätsprinzip, indem beide sich einander bedingen. Dem Übergang von einem Raum-Zeit-Bezugssystem zu einem anderen entspricht die Transformation der Energieform von einer sogenannten „Normalform"

$$E = \sum_{s,l} e^l_s p^s_l$$

auf eine andere Normalform.

Fig. 3. Backside of page 8, which is page 7 of Hilbert's first proofs, showing the slightly curved cut passing through a sentence.

must have be in the cut off part. But his statement: "One possible reason for Hilbert's cutting out this piece would be that he wanted to paste it into some other manuscript in order to avoid the pains of copying the equations by hand," is not very credible for such a simple equation. Instead of assuming that the cut off piece also contained Hilbert's definition for the variational derivative, he rather believes that it contained the expression of the curvature invariant K in terms of the tensor $K_{\mu\nu}$ which, as Sauer correctly says, is sufficient to arrive at the trace term missing in the equation by Einstein and Grossmann.

Inspecting the back of page 8, which is page 7, one can see that the cut is not straight, but rather slightly curved in passing through a sentence on page 7. This raises the suspicion that it was not done with scissors, but with a razor blade or pocket knife, possibly in the special collection – reading room of the Goettingen library, with the intent to erase the long held view that Hilbert had the correct final form of the field equation before Einstein, a view held by many physicists, including celebrity physicist Steven Hawking [8]. As C. J. Bjerknes [9] has pointed out to me, the fact that the cut passes through a sentence on page 7 and not on page 8, suggests that it was intended for page 8, giving further support for the hypothesis of a forgery with the purpose to suggest that Hilbert had copied from Einstein. In science as in history, forgeries are nothing new. Examples are the Constitutum Constantini, the vineland map, the Piltdown man hoax, and most recently the burial box of James, the brother of Jesus. Sauer's conjecture that Hilbert had cut off the upper one third of page 8 to paste it into one of his other manuscripts to save him the time to rewrite the equations of this upper part, is in view of my analysis of the content of the cut off part highly improbable. Hilbert uses both in the proofs and in the published version the short hand bracket notation for the variational derivative, but only the published version has the definition equation for the bracket notation. This is strong evidence that the proofs must have contained this definition equation as well, and this equation must have been in the cut off part of page 8. The remaining space in the cut off part of page 8 is probably too small to have contained the explicit expression of the curvature invariant (requiring two lines) as it is believed by Sauer, but even if true, would not change my conclusion.

In summary, one can say that the general theory of relativity is the creation of three men:

1. Einstein, who by the analogy with Gauss's theory of curved surfaces, concluded that the gravitational field must be expressed by the 10 components of the metric tensor of a curved four-dimensional Minkowski space-time.

2. Grossmann, who identified the contracted Riemann tensor as the key for the solution of the problem posed by Einstein.

3. Hilbert, for having completed the mathematical structure of the theory with his variational principle for the curvature scalar in four space-time dimensions.

Acknowledgement

The author expresses his thanks to C. J. Bjerknes for his critical reading of the manuscript and improving the text at several places.

Final Comment

A previous version of this paper was on Nov. 21, 2002 submitted to **Science**, in response to the article by Corry, Renn, and Stachel published in **Science**. Normally, such a criticism paper, together with the reply of those criticized, would be published in the Journal where the article to be criticized had appeared, but **Science** refused to publish my criticism paper, with the argument that my paper was allegedly of low priority for **Science**. However, I had given a preprint of my paper to C.J. Bjerknes, a historian of science, who had quoted my findings in his book "Anticipation of Einstein in the General Theory of Relativity," [10]. His book is quoted in a preprint by Logunov, Mestvirishvili, and Petrov [11], coming to the same conclusion. I express my thanks to Mr. Bjerknes, who had provided me with a preprint of their work.

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