

Non-Kinematicity of the Dilation-of-time Relation of Einstein for Time-intervals

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Z. Naturforsch. **55 a**, 563–569 (2000); received September 27, 1999

Light-pulses that are reflected recurrently to one another by two kinematically equivalent dynamically identical inertial systems moving collinearly and irrotationally with uniform relative velocity generate sequences of contiguous time-intervals in both. By means of clocks stationed in the two systems, each time-interval is both measurable locally and calculable non-locally in accord with basic requirements of special relativity theory. Their ratio yields the velocity dependent dilation-of-time relation of Einstein, but an equivalent spatially dependent version of it is obtained as well, because the time-intervals involved are actually determined by the distances that exist between the systems when the reflections occur. As a result, the Einstein relation involves no time-rates of clocks that are actually affected kinematically by the systems containing them.

Key words: Non-kinematic; Time Dilation; Locally-measurable; Non-locally-calculable.

1. Introduction

There appears to be no serious doubt of the mathematical correctness of Einstein's theory of special relativity, introduced almost a century ago [1, 2], many scientists since having elaborated both the theory and the experimental support claimed for it [3, 4]. However, the physical significance of one particular aspect of the theory, that of the time transformation relation first suggested by Lorentz [5, 6, 7] and then derived by Einstein, has long been questioned. According to Einstein, this aspect requires clocks fixed in inertial systems to "run slower" when the systems are moving with uniform velocity in otherwise empty force-free space than when stationary [8]. This aspect of the theory was criticized strongly by Dingle [9] a little more than a half-century later, the critic claiming that it yielded inconsistent results when the roles of two systems regarded as stationary and moving were interchanged, and describing the "twin paradox" that it evoked as contradictory. Although the criticism was refuted soon afterwards as fallacious, by Born [10], the rebuttal was not accepted by Dingle [11]. To date, neither the criticism nor its refutation have been withdrawn, and the paradox still continues to receive attention [12, 13, 14, 15]. Since several alternatives to special relativity theory have been found

recently which are able to account for cases in which time dilation had been claimed to occur [16, 17, 18], the existence of any such kinematic temporal restriction can be contested, as it has been. In fact, without calling upon any new physics and based solely upon the distinction to be made between those time-intervals which are measurable locally and those which are only calculable non-locally in accord with basic requirements of special relativity theory, a new derivation of the Einstein relation is presented here which entails no time-rates whatever that are affected merely because their clocks are stationed in uniformly moving inertial systems.

To show this, a model yielding the Einstein relation is described in Section 2. The model is an elaboration of one known to deal explicitly with possible kinematic influences on moving clocks, [3], p. 245-47. It consists of light-pulses of extremely short duration that are recurrently reflected to one another by two kinematically equivalent dynamically identical inertial systems that move collinearly and irrotationally with uniform velocity relative to each other in otherwise empty force-free space. Basic requirements in accord with special relativity theory are given, pertaining to the time-intervals that elapse between successive light-pulses which are measurable locally by clocks stationed in each of the systems and to their

counterparts which are calculable non-locally by the same clocks.

The ratio of any such pair of corresponding time-intervals, regardless of which system is designated as stationary or moving, is shown in Sect. 3 to yield the Einstein relation expressible entirely in velocity dependent terms, and is then shown to be expressible entirely in spatially dependent terms.

Justification is given in Sect. 4 for regarding the Einstein relation as a general non-kinematic means for converting non-locally calculable time-intervals in uniformly moving inertial systems to the values of their locally measurable counterparts there. Its application to both of the two kinematically equivalent dynamically identical inertial systems involved in its derivation here establishes that the inconsistency which was claimed to occur with special relativity theory does not occur with the present theory. Certain experimental results that are usually regarded as providing perhaps the most direct support for the time dilation ascribed to them by special relativity theory are re-examined and are found instead to comprise support for the absence of any such behavior. A possible limitation of the present theory with respect to the intrinsic statistical nature of all time-dependent quantum processes is noted.

An Appendix establishes that the time-intervals in the Einstein relation which has been derived are not restricted to just those which are produced directly by the recurrent light-pulse reflections of the model used to derive it.

2. Preliminaries

The following restrictions are adopted throughout:

(i) Any inertial system in uniform motion in force-free space has a space-time frame in terms of which the behavior of all other systems can be described precisely; (ii) Regardless of their intrinsic frequency-constitution, light-pulses received and reflected by any system entail negligibly small time-lapses to do so; (iii) Light-pulses emitted from a source stationed in a uniformly moving inertial system in any direction in otherwise empty force-free space propagate at the same speed; (iv) Time-intervals occurring in any uniformly moving inertial system are locally measurable only by means which are stationed in the immediate vicinity of their occurrence; (v) Clocks stationed in

any inertial systems that move uniformly in force-free space have unchanging time-rates as time passes.

In accord with the foregoing, the model to be considered involves the recurrent reflection of light-pulses to one another by two dynamically identical inertial systems, to be designated in what follows as the A-system and the B-system. They are both restricted to irrotational motion of uniform velocity in which their Cartesian coordinate x -axes coincide with the x -axis of a Cartesian coordinate frame that is fixed in the intervening otherwise empty force-free space. The light-pulses are presumed to be of sufficiently short duration and small spatial cross-section so that their receptions and reflections occur essentially instantaneously at the spatial-coordinate origin of each system. Stationed in the immediate vicinity of each origin are a clock and equipment for timing the receptions and the reflections.

The clock of the A-system renders time-instants denoted by t ; intersystem distances relative to it are rendered in lengths denoted by L , in terms of which intersystem speeds are expressed as v and the speed of light in the intervening otherwise empty force-free space is expressed as c . The clock of the B-system renders time-instants denoted by τ ; intersystem distances relative to it are rendered in lengths denoted by Λ in terms of which intersystem speeds are also expressed as v with the speed of light in the intervening otherwise empty force-free space also being expressed as c . Furthermore, non-locally calculable time-instants have an appended asterisk which those that are locally measurable have not, i.e., non-locally calculable t^* , τ^* contrast with locally measurable t , τ , respectively. All intersystem distances turn out to be non-locally calculable entities, i.e., L^* , Λ^* . In addition, the time-instants and intersystem distances have subscripts indicating the order of their occurrence in the sequence of light-pulse receptions and reflections that take place in the two systems.

After the two systems have each attained and maintained a uniform velocity, the process of interest begins with a light-pulse being sent from the A-system origin to the B-system origin, where it is received and reflected to return to the A-system origin. The process is then repeated many times without delay, each system recording all the time-instants when light-pulses are received and reflected. As a result, a light-pulse leaving the A-system origin at t_{2n} , n a non-negative integer denoting the position of the time-instant in the sequence, reaches the B-system origin at t_{2n+1}^* , where-

upon it is reflected and reaches the A-system origin at t_{2n+2} . Described alternately, a light-pulse leaving the B-system origin at τ_{2n+1} , reaches the A-system origin at τ_{2n+2}^* , whereupon it is reflected and reaches the B-system origin at τ_{2n+3} . Any pair of time-instants with the same subscript but without and with an asterisk denotes values corresponding to locally measurable and non-locally calculable values of the same time-instant.

Upon regarding the A-system as stationary with the B-system as moving, the separation of the two systems being L_n^* at t_{2n+1}^* , it is readily confirmed that the time-instants, time-intervals and intersystem distances relative to the A-system satisfy

$$t_{2n+2} + t_{2n} = 2t_{2n+1}^*, t_{2n+2} - t_{2n} = 2L_n^*/c, \tag{1}$$

$$t_{2n+3}^* - t_{2n+1}^* = (L_{n+1}^* + L_n^*)/c = (L_{n+1}^* - L_n^*)/v,$$

from which straightforward manipulation yields

$$L_n^*/L_{n+1}^* = \frac{1 - v/c}{1 + v/c}. \tag{2}$$

Upon regarding the B-system as stationary with the A-system as moving, the separation of the two systems being A_{n+1}^* at τ_{2n+2}^* , it is readily confirmed that time-instants, time-intervals and intersystem distances relative to the B-system satisfy

$$\tau_{2n+3} + \tau_{2n+1} = 2\tau_{2n+2}^*, \tau_{2n+3} - \tau_{2n+1} = 2A_{n+1}^*/c, \tag{3}$$

$$\tau_{2n+2}^* - \tau_{2n}^* = (A_{n+1}^* + A_n^*)/c = (A_{n+1}^* - A_n^*)/v,$$

from which some further straightforward manipulation yields

$$A_n^*/A_{n+1}^* = \frac{1 - v/c}{1 + v/c}. \tag{4}$$

As a consequence of the fact that any locally measured time-instant can be regarded as the initial one of the subsequent sequence, the two systems are completely interchangeable thereafter and are kinematically equivalent.

By (2) and (4) the intersystem distances satisfy

$$A_m^*/A_{m+1}^* = L_n^*/L_{n+1}^* \tag{5}$$

for all non-negative integers m and n . As a result, any quantities that depend solely on the velocity ratio v/c

can be expressed alternatively to depend solely on the intersystem distance ratios A_n^*/A_{n+1}^* or L_n^*/L_{n+1}^* .

3. The Einstein Relation

After some straightforward manipulation eliminating the non-locally calculable distances from (1), it follows that

$$\frac{t_{2n+2} - t_{2n}}{1 - v/c} = t_{2n+3}^* - t_{2n+1}^* = \frac{t_{2n+4} - t_{2n+2}}{1 + v/c} \tag{6}$$

of which the time-intervals locally measurable in the stationary-regarded A-system (unasterisked) are related to adjacent non-locally calculable ones occurring in the moving-regarded B-system (asterisked). It further follows from (3), after some more straightforward manipulation eliminating the calculable distances, that

$$\frac{\tau_{2n+3} - \tau_{2n+1}}{1 - v/c} = \tau_{2n+4}^* - \tau_{2n+2}^* = \frac{\tau_{2n+5} - \tau_{2n+3}}{1 + v/c} \tag{7}$$

of which the time-intervals locally measurable in the stationary-regarded B-system (unasterisked) are related to adjacent non-locally calculable ones occurring in the moving-regarded A-system (asterisked). As a result, we obtain

$$\frac{t_{2n+3}^* - t_{2n+1}^*}{\tau_{2n+3} - \tau_{2n+1}} \cdot \frac{\tau_{2n+4}^* - \tau_{2n+2}^*}{t_{2n+4} - t_{2n+2}} = \frac{1}{1 - (v/c)^2}. \tag{8}$$

Each ratio of this equation is that of a non-locally calculable time-interval occurring in the appropriate system divided by its locally measurable counterpart actually occurring there. On decreasing n by unity in (7), combining the result with (6) and comparing with (8), we obtain

$$\frac{\tau_{2n+2}^* - \tau_{2n}^*}{t_{2n+2} - t_{2n}} = \frac{\tau_{2n+4}^* - \tau_{2n+2}^*}{t_{2n+4} - t_{2n+2}}, \tag{9}$$

so that all locally measurable time-intervals occurring in the A-system have the same ratio with their non-locally calculable counterparts, thereby indicating that any locally measurable time-interval in the A-system has a unique non-locally calculable counterpart there, and vice versa. Similarly, but now increasing n by unity in (6), combining the result with (7) and comparing the result with (8), we obtain

$$\frac{t_{2n+3}^* - t_{2n+1}^*}{\tau_{2n+3} - \tau_{2n+1}} = \frac{t_{2n+5}^* - t_{2n+3}^*}{\tau_{2n+5} - \tau_{2n+3}}, \tag{10}$$

so that all locally measurable time-intervals occurring in the B-system have the same ratio with their non-locally calculable counterparts, thereby indicating that any locally measurable time-interval in the B-system has a unique non-locally calculable counterpart there, and vice versa. As a result, it further follows that

$$\frac{\tau_{2n+2}^* - \tau_{2n}^*}{t_{2n+2} - t_{2n}} \cdot \frac{\tau_{2m+3}^* - \tau_{2m+1}^*}{t_{2m+3} - t_{2m+1}} = \frac{1}{1 - (v/c)^2}. \quad (11)$$

for all non-negative integers n and m .

It is to be noted that each time-interval ratio in (11) refers to a different system. Because of the evident symmetry of the model, and the resulting kinematic equivalence of the two systems which allows them to be interchanged, the two ratios must have the same value. Then, designating typical locally measurable time-intervals by Δt or $\Delta\tau$, depending on the system involved, and their comparable non-locally calculable counterparts by Δt^* or $\Delta\tau^*$, we finally obtain from (11) that

$$\frac{\Delta\tau^*}{\Delta t} \equiv \frac{\Delta t^*}{\Delta\tau} = \frac{1}{\sqrt{1 - (v/c)^2}}. \quad (12)$$

the celebrated velocity dependent dilation-of-time relation of Einstein, which we have sought. It further follows after some straightforward manipulation of (2) that

$$\frac{\Delta\tau^*}{\Delta t} \equiv \frac{\Delta t^*}{\Delta\tau} = \sqrt{(1 + L_2^*/L_1^*)^2 / 4(L_2^*/L_1^*)}. \quad (13)$$

the version of the relation that is solely spatially dependent, which we also have sought. Emphasizing its intrinsic non-kinematic nature, it is to be noted that the same relation holds without change for both systems.

4. Discussion

While it might be argued that the spatially dependent relation (13) relies implicitly on the relative motion of the two systems when they engage in receiving and reflecting light-pulses, it could also be argued that the velocity dependent relation (12) relies implicitly on the actual spatial separations of the systems when they receive and reflect them. The essential result of the foregoing analysis is that the relation can be either velocity dependent or spatially dependent without implying any actual dilation-of-time occurring in clocks that may be stationed in the systems.

In special relativity theory, non-locally calculable time-intervals in uniformly moving inertial systems and their locally measurable counterparts occurring there are assumed to be identical. This assumed identification appears to have arisen because of an interpretation made by Einstein himself ([1] p. 904; [2] p. 49), with the result that a “peculiar consequence” of the Einstein relation was that clocks stationed in uniformly moving inertial systems should have slower time-rates than they would have when stationed in non-moving inertial systems. In special relativity theory, as a result, the Einstein relation is regarded to be a general kinematic restriction whereby the intrinsic time-rates of all uniformly moving clocks are slower than their stationary counterparts.

In the present theory, by contrast, non-locally calculable time-intervals in uniformly moving inertial systems and their locally measurable counterparts occurring there are required to differ. Together, they imply the Einstein relation, which is simply a means of converting all non-locally calculable time-intervals to the precise values of their locally measurable counterparts, without the motion of the systems affecting the intrinsic time-rates of any clocks stationed within them.

Although a distinction between non-locally calculable time-intervals and their locally measurable counterparts has been noted in the literature ([19], [3] p. 224), it appears not to have been taken into account adequately. For example, a traditional derivation of the dilation-of-time relation involves the emission and reflection of light in a non-symmetrical path that is locally perpendicular to the motion of a uniformly moving source-and-mirror combination [20, 21]. It is then assumed that the path that is calculated by a stationary observer is the same as that determined by an observer moving together with the source-and-mirror combination. Dilation-of-time is then regarded to actually occur in the moving system, which cannot be the case according to the present theory. It is evident that the essential difference in the two theories lies neither in the physics nor in the mathematics involved in them but rather in their viewpoints concerning time-intervals that are determined by means of non-local calculations and their relation to the physics involved. Which viewpoint is appropriate to characterize the physics can be determined only from the physics itself.

In this regard, it is to be emphasized that the present theory avoids both the earlier criticism which was

made of time dilation assumed by special relativity theory [9] and the rebuttal of it which was given later [10]. In view of the symmetry of the model that has been employed here and the restrictions that have been adopted in determining its consequences, the Einstein relation is equally applicable without any modification to the two kinematically equivalent systems involved in it. When they are in relative motion, any non-locally calculable time-interval in either system that may be regarded as moving must exceed its locally measurable counterpart there. This causes no difficulty at all for the present theory, since it is not only entirely consistent for non-locally calculable time-intervals in each system to be larger than their locally measurable counterparts there, but they must be so. In special relativity theory, however, the locally measurable time-intervals in the moving system are to be identified with values obtainable in the other system, which is to be regarded as stationary. Since either system can be so regarded according to special relativity theory, the clock in each system would then be required to run slower than the clock in the other, an inconsistency that had been noted and criticized [13]. As already indicated, this inconsistency does not arise with the present theory and, with the ensuing absence of time dilation following from it, neither can the twin-paradox arise.

Furthermore, although results are known that have been considered to provide perhaps the most direct experimental support for the time dilation which has been presumed by special relativity theory ([22], [23], [3] p. 259, [4] pp. 180-83), such a conclusion turns out to be entirely unjustified according to the present theory. Thus, for any non-locally calculable time-interval and its locally measurable counterpart to equal each other, their ratio evidently would have to equal unity. In the velocity dependent Einstein relation, this would be so if and only if the relative velocity of the two inertial systems was to vanish. Similarly, in the spatially dependent version, this would be so if and only if the successive separations between the two inertial systems when reflections took place were to be the same, and remain so with the passage of time. Because neither of these requirements is fulfilled generally, the equality of non-locally calculable time-intervals and their locally measurable counterparts cannot be generally attained, and neither can time dilation.

In support of this, the data pertaining to the foregoing experiments were presented only graphically ([16], p. 348 gives a criticism of the graphical presen-

tation) and depict three separate time-intervals during which the time-differences between each of four cesium atomic clocks stationed in an airplane and a reference one stationed on the earth's surface were measured directly and accumulated. During the two intervening time-lapses, when the clocks were airborne on trips circumnavigating the earth, one eastward and one westward, they were not so dealt with. From the recognition that the relative time-rates of the atomic clocks could differ from each other to an extent that depended on the pair of clocks involved, the average of the individual accumulated time-differences recorded in each time-lapse was determined and also displayed graphically. Considerations of special relativity would then require that average to be disjoint from one time-lapse to another, and such was inferred by the experimenters.

However, in order to obtain an independent analysis of the data, the time-difference graphs were enlarged (by the present writer) to enable a reasonably precise determination to be made of their actual values and individual accumulated time-differences were then obtained at ten equally elapsed time-instants. It then turned out that they could be fitted by a least-squares procedure so that, within estimated uncertainties of the data analyzed, they yielded a linear combination constituting a weighted average of some sort which proved to be essentially constant throughout the entire course of time involved in the experiment. It would thus appear, contrary to the conclusion reached by the original experimenters, that no significant deviations had actually occurred in the composite behavior of the clocks when they were airborne from that which prevailed when they were stationary. This result is what was to be expected from the present theory (see in this connection a similar conclusion reached in [12]). It would therefore seem to be worthwhile to undertake another experiment along similar lines to test this conclusion, but one in which the clocks are chosen beforehand to have essentially uniform time-rates individually, or together, and in which all time-differences are to be accumulated when they are airborne as well as when they are grounded.

A possibly important limitation of the conclusions reached here involves the assumption of negligibly small time-lapses for light-pulses to be received and reflected. This seems difficult to justify rigorously if the reception and reflection of light-pulses were to be regarded in quantum-mechanical terms. As pointed out by Dirac for the resonance scattering of particles

involving their absorption and subsequent re-emission [24], such processes involving light comprised of photons would also seem to require some non-zero time-lapses to occur. These could be particularly important when the reflection processes occur between nearly coincident rapidly moving systems. Perhaps a more serious limitation of all time-dependent quantum processes which have been used to test the dilation-of-time relation (see, for example [4], pp. 195-200) may be their intrinsic statistical nature.

Appendix

It may seem that the present theory can deal only with time-intervals that are generated directly by the recurrent reflection of light-pulses in the model. However, it is readily extended to include all locally measurable time-intervals and their non-locally calculable counterparts that can arise. This proves to be possible because equality of the time-interval ratios in (9) - (10) enables (11) to be extended to

$$\frac{\sum_n f(n)(\tau_{2n+2}^* - \tau_{2n}^*)}{\sum_n f(n)(t_{2n+2} - t_{2n})} \cdot \frac{\sum_m g(m)(t_{2m+3}^* - t_{2m+1}^*)}{\sum_m g(m)(\tau_{2m+3} - \tau_{2m+1})} = \frac{1}{1 - (v/c)^2}, \quad (\text{A1})$$

where $f(n)$ and $g(m)$ are arbitrary finite real functions of their arguments. They can be chosen in a variety

of ways such that virtually any non-locally calculable time-interval can be obtained together with its locally measurable counterpart. As a result, the present theory is not restricted to just the time-intervals directly produced by the recurrent light-pulse reflections.

To show this entails no difficulty. We suppose that the $f(n)$'s and the $g(m)$'s can be chosen so that the two numerators of the foregoing time-interval ratios each equal the same non-locally calculable time-interval of interest, say $\Delta\tau^*$, viz.,

$$\sum_n f(n)(\tau_{2n+2}^* - \tau_{2n}^*) = \sum_m g(m)(t_{2m+3}^* - t_{2m+1}^*) \equiv \Delta\tau^*. \quad (\text{A2})$$

Because of uniqueness of the ratios in (9) - (10), we can conclude that there is a locally measurable time-interval counterpart to which it corresponds, say Δt . Thereupon, it follows that

$$\sum_n f(n)(\tau_{2n+2} - \tau_{2n}) \cdot \sum_m g(m)(t_{2m+3} - t_{2m+1}) \equiv (\Delta t)^2, \quad (\text{A3})$$

which then leads to (12), but with the time-intervals now not restricted to just those generated directly by the light-pulse reflections.

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