Stability of Plasmas Held by Radiation Pressure

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An extension of a simplified model of Brillouin scattering
instability to arbitrarily inhomogeneous media is presented.
It is shown that light pressure, if treated self-consistently,
always drives the plasma unstable.

Most of the studies of stability of plasmas in the
presence of electromagnetic waves are concerned
with wave decay or parametric effects, such as
Brillouin scattering, in homogeneous or slightly
inhomogeneous plasmas [1]. In contrast this short
contribution is devoted to the stability investigation
of arbitrarily inhomogeneous plasmas held by light
pressure, the importance of which was pointed out
years ago [2]. The Brillouin type instability is due
to the effect that a density perturbation of twice
the local light wavelength modifies the electro-
magnetic wave in such a way that its radiation
pressure tends to increase the original perturbation
[3]. Since the frequency shift of the scattered
electromagnetic wave is small (ωacoustic ≪ ωlight),
it is a good approximation to use the same index
of refraction for the incident and scattered waves,
at least far from the critical density. The essential
features of possible instabilities are thus preserved
if a simplified response of the plasma to the light as
described in the following equations [4] is assum ed:

\[ \frac{\partial}{\partial t} \rho + u \frac{\partial \rho}{\partial x} = - s^2 \frac{\partial \rho}{\partial x} - \mu \frac{\partial}{\partial x} \left< EE^* \right>, \]  
\[ \frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x} (\rho u) = 0, \]  
\[ \frac{\partial^2 E}{\partial x^2} + \chi_0^2 n^2 E = 0, \]  
where \( n^2 = 1 - q(x)/\rho_c \). The symbols have the
usual meaning [4] (u is the x component of velocity,
\( \rho_c \) is the critical density, E is the electric field,
\( \chi_0 \) is the vacuum wave number, s is the sound velocity).
The equations are valid if the fluid is 1-dimensional
and isothermal, behaves hydrodynamically, and
dissipation can be neglected.

We are interested in a time evolution of u which is
much longer than the period of the light wave,
and so it is justified to replace the averages on the
rapid time scale in Eq. (1) by the x dependent
amplitudes \( E(x) \). Furthermore, \( \chi_0^2 n^2 \) is real and
allows us to consider only real \( E(x) \) so that we
replace \( \left< EE^* \right> \) by \( E^2 \).

If we assume a static equilibrium for the zero
order, Eqs. (1) to (3) reduce to

\[ s^2 \frac{\partial}{\partial x} \rho_0 + \mu \frac{\partial}{\partial x} \left( E^2 \right) = 0, \]  
\[ \frac{\partial^2 E_0}{\partial x^2} + \chi_0^2 \left( 1 - \frac{\rho_0(x)}{\rho_c} \right) E_0 = 0, \quad \rho_0 < \rho_c. \]  
The linearized equations around \( \rho_0, E_0 \) are then

\[ \rho_0 \frac{\partial u}{\partial t} = - s^2 \frac{\partial q_1}{\partial x} - \mu \frac{\partial E_0^2}{\partial x} - 2 \mu \frac{\partial}{\partial x} (E_0 E_1), \]  
\[ \frac{\partial}{\partial t} q_1 + \frac{\partial}{\partial x} (\rho_0 u) = 0, \]  
\[ \frac{\partial^2 E_1}{\partial x^2} + \chi_0^2 \left( 1 - \frac{\rho_0}{\rho_c} \right) E_1 = \chi_0^2 E_0 \frac{q_1}{\rho_c}, \]  
\[ E_1 \] can be expressed in terms of Green's function
\( G(x, x') \)

\[ E_1 = \chi_0^2 \int_0^b \frac{d}{d} \left( G(x, x') E_0(x') \right) \frac{q_1(x')}{\rho_c} \, dx'. \]  
By taking the time derivative of Eq. (6) and using
Eq. (9) the system (6) — (8) reduces to

\[ \frac{\partial^2 u}{\partial t^2} = s^2 \frac{\partial^2 \rho_0}{\partial x^2} (\rho_0 u) + \mu \frac{\partial}{\partial x} \left( \frac{\partial (E_0^2)}{\partial x} (\rho_0 u) \right) \]  
\[ + 2 \mu \chi_0^2 \rho_0 \frac{\partial}{\partial x} \left( E_0 \left[ G(x, x') E_0(x') \right] \frac{\partial}{\partial x'} (\rho_0 u) \, dx' \right), \]

or

\[ \rho_0 \ddot{u} + F \dot{u} = 0, \]  
where \( F \) is the integro-differential operator on \( u \) of the
R.H.S. of Equation (10). The reader can easily
check that the differential part of \( F \) is symmetric
for vanishing \( u \) at the boundaries. The integral
operator is also symmetric because Green's function
inverts the symmetric operator on the L.H.S. of
Eq. (8) and then has to be symmetric with respect to interchange of \(x\) and \(x'\). This property of Eq. (11) allows a necessary and sufficient condition of stability to be derived in the form of an energy principle as known from Ref. [5].

Let \(\delta W = \int_a^b u F u \, dx\), i.e.

\[
\delta W = s^2 \int_a^b [(\alpha u)_x]^2 \frac{dx}{q} + 2 \mu \frac{\alpha_0^2}{q} \int_a^b \int_a^b G(x, x') E_0(x') (\alpha u)_x' E_0(x) (\alpha_0 u)_x \, dx' \, dx.
\]  

(12)

If \(\delta W > 0\) for all \(u\) vanishing at \(x = a\) and \(x = b\), the system is stable. If for any test function \(u\), vanishing at \(a\) and \(b\), \(\delta W < 0\) holds, the system is unstable.

This means that without the self-consistent reaction of the plasma to the light, i.e. \(E' = 0\), the equilibrium is stable. Let us investigate this self-consistent response by analyzing the properties of Green’s function \(G(x, x')\) of Equation (8). The operator

\[
L \equiv \frac{\partial^2}{\partial x^2} + \alpha_0^2 \left(1 - \frac{\alpha_0}{\alpha_c}\right)
\]

has a null eigenvalue if

\[
\alpha_0 (b - a) \left(1 - \frac{\alpha_0}{\alpha_c}\right)^{1/2} = \alpha_{\text{crit}} \approx 1
\]

where the bar indicates an average in \(x\). When this condition is barely satisfied, Green’s function becomes very large and can change sign. This is easy to understand from the fact that if \(L\) has the eigenvalue \(\lambda\), \(L^{-1}\) has the eigenvalue \(\lambda^{-1}\). So \(G(x, x')\) can be negative and large if

\[
\alpha_0 (b - a) \lesssim \left(1 - \frac{\alpha_0}{\alpha_c}\right)^{-1/2}.
\]

This means that the double integral in Expression (12) can be made negative and large enough to yield \(\delta W < 0\) for a properly chosen test function.

In conclusion, we can say that the plasma is unstable if the ponderomotive force is perturbed self-consistently, no matter how large the modulation of the light and the plasma inhomogeneity are.

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