Kinetic Modulational Instability of Upper-hybrid Turbulence

P. K. Shukla, M. Y. Yu, and S. G. Tagare

Institut für Theoretische Physik der Ruhr-Universität Bochum, 4630 Bochum

(Z. Naturforsch. 32a, 335—336 [1977]; received January 25, 1977)

It is shown that a plasma containing randomly distributed non-interacting upper-hybrid waves can become unstable against ion quasi-modes. The growth rate of the instability is presented.

In previous papers 1, 2, it has been shown that the upper-hybrid turbulence consisting of an ensemble of random-phased upper-hybrid waves can become modulationally unstable with respect to low-frequency ion-cyclotron, lower-hybrid 1, and adiabatic perturbations 2. The latter is valid for the quasi-static regime.

In this letter, we consider the problem of modulation of upper-hybrid turbulence by low-frequency perturbations \( \Omega, \varphi \). A general dispersion relation which is valid for the quasi-static \( |\Omega /q v_{TI}| \ll 1 \), the inertial \( |\Omega /q v_{TI}| > 1 \), and the transitional \( |\Omega /q v_{TI}| \sim 1 \) regimes is obtained. Specifically, we shall be concerned with the last regime, namely the problem of coupling of upper-hybrid turbulence with ion quasi-modes. For the latter type of perturbations, the two fluid description fails and one should retain the Vlasov description.

In what follows, the high-frequency upper-hybrid waves shall be allowed to have a small component \( E_z \) along the external magnetic field \( B_0 \), so that they may couple with the ion quasi-modes. The dynamics of the upper-hybrid turbulence is governed by a wave kinetic equation 1, 3. The change of the upper-hybrid turbulence distribution \( \bar{N}_k \) in the presence of ion quasi-mode perturbations \( \bar{n}_e \) is given by

\[
\bar{N}_k = \frac{\omega_k}{2n_0} q \cdot \frac{\partial N_k^0}{\partial k} / \Omega - q \cdot v_e,
\]

where \( N_k^0 = \langle E_k^0 \rangle / 4\pi \omega_k \) is the unperturbed distribution, and

\[
\omega_k^2 = \omega_{pe}^2 + \omega_{ce}^2 + 3 k^2 \frac{v_{Te}}{\omega_{pe}} \frac{\omega_{pe}}{\omega_{ce}^2} / (\omega_{pe}^2 - 3 \omega_{ce}^2)
\]

is the characteristic frequency of the turbulence. Here \( v_{Te}, \omega_{pe} \), and \( \omega_{ce} \) are respectively the thermal velocity, plasma and gyrofrequencies of the electrons. The group velocity of the high-frequency waves is given by

\[
v_g = \hat{x} [3k \lambda_e v_{Te} \omega_{pe} / \omega_k (1 - 3 \omega_{ce}^2 / \omega_{pe}^2)] ,
\]

where \( \lambda_e = v_{Te} / \omega_{pe} \) is the electron Debye length. We note that for \( 3 \omega_{ce}^2 \ll \omega_{pe}^2 \), the upper-hybrid modes have positive group dispersion, whereas for \( \omega_{pe}^2 \ll 3 \omega_{ce} \), the modes have negative group dispersion. In the following, we consider only the upper-hybrid waves with positive group dispersion, namely \( 3 \omega_{ce}^2 \ll \omega_{pe}^2 \).

The charge density perturbation \( \bar{n}_e \) in the presence of the modified distribution (1) is obtained from the electron Vlasov equation. We find

\[
4\pi \bar{n}_e = -q^2 \chi_e (\Phi + \Phi_{pe})
\]

where \( \Phi \) is the ambipolar potential, and the ponderomotive potential \( \Phi_{pe} \) is given by

\[
\Phi_{pe} = -\sum_k \frac{n_k}{n_0} \frac{q^2}{\omega_e} \frac{\omega_k^2}{\omega_{pe}^2} - \omega_{ce}^2 \]

In (3), \( \chi_e = (q \lambda_e)^{-2} G' (\Omega /q v_{TI}) \) is the electron susceptibility, and \( G \) is the plasma dispersion function. The electrons are assumed to be highly magnetized.

From the linearized ion Vlasov equation, we obtain

\[
4\pi \bar{n}_i = -q^2 \chi_i \Phi,
\]

where \( \chi_i = (q \lambda_i)^{-2} G' (\Omega /q v_{TI}) \) is the ion susceptibility, and ions are assumed to be unmagnetized. The ponderomotive potential \( \Phi_{pl} \) of the ions is smaller than that of the electrons by a factor \( m_e /m_i \) and is therefore neglected.

Combining (3) and (5), and using Poisson's equation \( q^2 \varphi = 4\pi (\bar{n}_i + \bar{n}_e) \), one gets

\[
\Phi = -\chi_e \Phi_{pe} / \epsilon,
\]

where \( \epsilon = 1 + \chi_e + \chi_i \). Inserting (6) into (3), we get

\[
\bar{n}_e = \frac{q^2 \chi_e (1 + \chi_i)}{4\pi n_0 \epsilon} \Phi_{pe}.
\]

Combining (1) and (7), we obtain the dispersion relation

\[
1 = \frac{L}{16\pi n_0} \frac{q^2 \chi_e (1 + \chi_i)}{\epsilon} \frac{\omega_{pe}^2 + \omega_{ce}^2}{\omega_{pe}^2} \int \frac{q \cdot \partial N_k^0 / \partial k}{(\Omega - q \cdot v_e)} dk,
\]

where the summation over \( k \) has been replaced by an integration in the usual manner (i.e., \( \Sigma \mathcal{d}k \rightarrow L/2\pi \int \mathcal{d}k \), where \( L \) is the size of the system). Equa-
tion (8) is the most general dispersion relation describing the interaction of upper-hybrid turbulence with low-frequency perturbations. For $|\Omega| \ll q v_T e$, we have $\chi_e = (q J_e)^{-2}$. We now let

$$N_{\theta}^6 \approx (2 \pi)^{1/2} \left( \frac{W}{4 \pi \omega_{pe}} \right) (k_\parallel L)^{-1} \cdot \exp \left[ - \frac{(k - k_0)^2}{2 k_\parallel^2} \right]$$

in (8) and assume that the spectrum of upper-hybrid turbulence is sufficiently peaked around $k_0$ [i.e., $|\Omega - q_\perp u_0| > 3 q_z \lambda_e k_\parallel v_T e \omega_{pe}/(\omega_{pe}^2 + \omega_{ce}^2)^{1/2}$, where $u_0 = v_p(k = k_0)$], we then obtain from (8)

$$\Omega - q_\perp u_0 = \pm i \left( \frac{3}{8 \pi} \right)^{1/2} q_\perp \lambda_e \omega_{pe}$$

$$\cdot \left( \frac{W}{n_0 T_e} \right)^{1/2} \left( \frac{1 + \chi_e}{\epsilon} \right)^{1/2} k_\parallel,$$

where $k_\parallel$ is the spread, $k_0$ is the mean wave vector, $W$ is the total energy of the turbulence spectrum, and $4 A^2 = \omega_{he} \omega_{pe}/(\omega_{pe}^2 - 3 \omega_{ce}^2)$ with $\omega_{he} = \omega_{pe}^2 + \omega_{ce}^2$.

Letting $\Omega = q_\perp u_0 + i \gamma$, we obtain the growth rate

$$\gamma = \left( \frac{3}{8 \pi} \right)^{1/2} q_\perp \lambda_e \left( \frac{W}{n_0 T_e} \right)^{1/2} \omega_{pe} A R_e,$$

where the frequency shift caused by the turbulence is neglected. When the argument of $\chi_e$ is near unity, that is $3 k_0 \lambda_e v_T e \approx \omega_{pe}^2$, which can occur for $k_0 \lambda_e \ll 1$, we find for $T_e \approx T_i$

$$\gamma \approx 0.9 \left( \frac{3}{8 \pi} \right)^{1/2} q_\perp \lambda_e \left( \frac{W}{n_0 T_e} \right)^{1/2} \omega_{pe} A.$$

In conclusion, we have shown that a spectrum of upper-hybrid mode turbulence is unstable when the group velocity of the turbulence mode is approximately equal to the ion thermal speed.

**Acknowledgement**

This work has been performed under the auspices of the Sonderforschungsbereich 162 Plasmaphysik Bochum/Jülich and the Humboldt Foundation.

---