The Anisotropy Function $\eta(T)$ in Al-Cu and Al-Zn Alloys

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The anisotropy function $\eta(T)$ for Al-Cu and Al-Zn alloys is determined experimentally and compared with the theoretical results of Kagan and Zhernov.

Recent resistivity measurements on Al-Zn$^1,2$ and Al-Cu$^3$ alloys have shown that the deviation $\Delta(c, T)$ from Matthiessen’s rule is positive at low temperatures and increases with $T$, passing through a maximum at intermediate temperatures. At high temperatures, in the Al-Zn alloys investigated by Kawata et al.$^1$ the deviation increases approximately linearly with $T$, while in the Al-Zn alloys investigated by Papastaikoudis et al.$^2$ it decreases linearly with $T$. For the Al-Cu alloys the deviation remains almost constant$^3$.

It is the main feature of these measurements that the maximum of $\Delta(c, T)$ shifts with Zn and Cu concentration. In Al-Cu alloys it has been shown$^3$ that the temperature $T_{\text{max}}$, corresponding to the maximum of the deviation, follows very well a $c^{1/5}$ law. The same behaviour is also followed by the Al-Zn system as shown in Fig. 1, where the $T_{\text{max}}$ is plotted as a function of concentration $c$ in a log-log plot.

![Fig. 1. $T_{\text{max}}$ as a function of the solute concentration $c$.](image)

The $c^{1/5}$ law for $T_{\text{max}}$ was initially explained by Kagan and Zhernov$^4$ who in their calculation have taken into account the effect of the anisotropy of the electron-phonon scattering. This anisotropy is connected with Umklapp processes and the anisotropy of the vibrational spectrum of metals. In this case, and with the assumption that the Fermi surface is spherical, they have found that the resistivity of a pure metal has the form

$$\rho_\nu'(T) = \rho_\nu(T) [1 - \eta(T)],$$

where $\rho_\nu(T)$ is the resistivity of the pure metal in the first (isotropic) approximation and the factor $\eta(T)$ is a function of $T$ due to the anisotropy. In the presence of small amounts of impurities in the metal, they found an expression for the departure from Matthiessen’s rule $\Delta(c, T)$, which has the form

$$\Delta(c, T) = \Delta(c, T) + \rho_\nu(T) \eta(T) \left[ 1 - \frac{1}{1 + c R/P(T)} \right].$$

The quantity $\Delta(c, T)$ was calculated by the use of an isotropic distribution function, $R$ is an integral arising from the elastic scattering of the electrons from foreign atoms and $P(T)$ is an integral associated with the electron-phonon scattering in the pure metal. In the region of low temperatures the ratio $c R/P(T)$ is large, while at high temperatures it is small. In the intermediate temperature range for which $P(T) \sim c R$, that is when $\rho_\nu(T)$ is comparable in magnitude with the residual resistivity $\rho_\nu$, the magnitude of the impurity resistivity itself turns out to be large, namely

$$\Delta(c, T)/\rho_\nu(c) \sim \eta(T).$$

Kagan and Zhernov have defined a characteristic temperature $\tilde{T}$ for $P(T) = c R$, which, due to the proportionality of $P(T)$ to $T^5$ obeys the relation

$$\tilde{T} \sim c^{1/5}.$$  

$\tilde{T}$ is nearly equal to the temperature $T_{\text{max}}$ corresponding to the maximum of the deviation.

In Fig. 2 the dashed curve shows the theoretically calculated factor $\eta(T)$ as a function of $T/\Theta_D$.

![Fig. 2. $(\Delta/\rho_\nu)T_{\text{max}}$ against $T/\Theta_D$, together with the calculated $\eta(T)$ function.](image)
for the case \( k_F/q_0 = 0.9 \). The parameter \( \xi \), characterizing the phonon spectrum anisotropy was set at 1/2. \( \Theta_D \) is the Debye temperature, \( k_F \) is the radius of the Fermi sphere and \( q_0 = \pi \cdot b_{\text{min}} \), where \( b_{\text{min}} \) is the nearest vector of the reciprocal lattice. The same figure also shows our experimental values of \( \Lambda(c, T)/q_0 \) for the Al-Cu and Al-Zn alloy systems, which are taken at \( T_{\text{max}} \), together with those of Kawata et al. A comparison between the calculated \( \eta(T) \)-curve and the experimental \( \Lambda(c, T)/q_0 \) results shows that there is relatively a good agreement. This agreement points out that the calculation of Kagan and Zhernov provides an explanation for the observed maximum of the deviation from Matthiessen’s rule at intermediate temperatures and its shift with the concentration of the impurities.

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