Steady State Film Profiles of Viscous Drainage over a Naturally Permeable Wall

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The Navier-Stokes equation of viscous drainage over a naturally permeable wall is investigated. It is found that as an effect of porosity, the film falls faster than that in the clear zone.

Recently Annapurna and Ramanaiah have investigated the effect of the inertial contribution to the Jeffreys' solution corresponding to the steady state film profiles of viscous drainage. In this note we examine the steady state problem and solve it subject to the slip-boundary conditions

\[
\frac{\partial V}{\partial x} \bigg|_{x=0} = \frac{\alpha}{\sqrt{k}} (V_B - V_D), \quad \frac{\partial V}{\partial x} \bigg|_{x=h} = 0,
\]

stipulating that the film surface \(x=0\) is permeable as proposed by Beavers and Joseph and the film surface \(x=h(y)\) stress free. Here \(k\) is the permeability of the material, \(\alpha\) is a dimensionless quantity depending only on the material parameters, which characterise the structure of the permeable material within the boundary region, \(V_B\) is the slip velocity at the permeable surface and \(V_D\) is the Darcy velocity given by

\[
V_D = -\frac{g k}{v}.
\]

We then obtain

\[
V = -\frac{g h^2}{v} \left[ \frac{1}{\alpha \sigma} + \frac{1}{\sigma^2} + \frac{x}{h} + \frac{x^2}{2 h^2} \right],
\]

where \(\sigma = h/\sqrt{k}\).

In terms of the non-dimensional quantities defined by Eq. (8) of [1], we obtain

\[
y = -H^2 T \left[ 1 + (a + 2 \sigma)/a \sigma^2 \right].
\]

In the above equation we realize the well-known Jeffreys' solution

\[
y = -H^2 T,
\]

by allowing \(\alpha, \sigma \to \infty\) independently.

The effect of porosity on the liquid-film-profile is illustrated in Fig. 1, taking \(\alpha=1.45, \sigma=2\). It is noticed that the film falls much faster than the Jeffreys-film. A physical explanation of this result could be that the effects of viscous shear appear to penetrate into the permeable material in a boundary layer region. This confirms the experimental observation of Beavers and Joseph.

![Fig. 1. Steady state film profiles at T=0 and \(\alpha=1.45\).](image)

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